Closed Form Designs of Complex Orthogonal Space-Time Block Codes of Rates $(k+1)/(2k)$ for $2k-1$ or $2k$ Transmit Antennas

Kejie Lu, Shengli Fu, Xiang-Gen Xia

Abstract

In this correspondence, we present a systematic and closed form construction of complex orthogonal space-time block codes from complex orthogonal designs of rates $(k+1)/2k$ for $2k-1$ or $2k$ transmit antennas for any positive integer k.

Index Terms

Complex orthogonal designs, orthogonal space-time block codes, Hurwitz-Radon theory

I. Introduction

Since the pioneering work of Alamouti [1] in 1998, orthogonal design has become an effective technique for the design of space-time block codes (STBC). The importance of this class of codes comes from the fact that they achieve full diversity and have the fast maximum-likelihood (ML) decoding. In this paper, we are interested in complex orthogonal designs (CODs).

Let B_n be a $p \times n$ matrix. It is a COD of variables $x_1, x_2, \cdots, x_u$ if entries of B_n are complex linear combinations of these variables and their complex conjugates, i.e., $x_1, x_2, \cdots, x_u, x_1^*, x_2^*, \cdots, x_u^*$, such that

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the following orthogonality holds for any complex values of x_i:

$$B_n^H B_n = (|x_1|^2 + |x_2|^2 + \cdots + |x_u|^2)I \tag{1}$$

where ^{H} denotes the complex conjugate transpose and I denotes the $n \times n$ identity matrix. The u variables $x_1, x_2, \cdots, x_u$ represent information symbols from a signal constellation, such as QPSK, to be transmitted through n transmit antennas in p time slots. There are two criteria for the evaluation of a COD code:

- Rate R: $R = u/p$, higher rate means more information carried by the code;
- Block length p: given n and R, smaller p results less delay in en/decoding.

Considerable efforts have been made on the search of CODs for different numbers of antennas. Alamouti [1] proposed the following scheme for $n = 2$ with $R = 1$ and $p = 2$:

$$\begin{bmatrix} x_1 & x_2^* \\ x_2 & -x_1^* \end{bmatrix}. \tag{2}$$

It is known that real orthogonal designs (ROD) of maximum rate 1 with minimum delay for real variables, such as PAM, can be systematically constructed for any number n of transmit antennas from Hurwitz-Radon theory [14], [15], [2]. In [7], a comprehensive study on complex orthogonal space-time codes, CODs, and some historical background can be found. Tutorials on this subject can be also found in [13], [9]. In [2], a systematic COD construction with rate $1/2$ for any number of transmit antennas is proposed by using rate 1 ROD. The question is then how to construct CODs of rates above $1/2$. For $n = 3$ and $n = 4$, CODs of rate $R = 3/4$ and $p = 4$ are developed in [2]-[5]. It is shown in [11] that a rate $R = 1$ COD when $n > 2$ does not exist no matter how large p is. In [7], it is shown that rate $3/4$ is an upper bound for rates of CODs without linear processing of symbols 0, $\pm x_i$, or $\pm x_i^*$ when $n > 2$ for any p, where x_i or x_i^* is forced to appear at most once in each column of a COD. It is shown in [12] that rate $3/4$ is an upper bound for rates of CODs when $n > 2$ for any p when the entries of CODs may have linear processing of $\pm x_i$ and $\pm x_i^*$ as in the definition of a COD used in this paper. Furthermore, in [12], it is conjectured that the rate R of a COD is upper bounded by $(k+1)/(2k)$ for $2k-1$ or $2k$ transmit antennas no matter how large p is. For $n = 5$ and $n = 6$, rates $7/11$ and $3/5$ generalized CODs are constructed in [6], respectively. In [8], a rate $2/3$ and size 15×5 COD is reported for 5 transmit antennas. While individual discoveries for different numbers of antennas appear interesting and motivating, it is highly desirable that a systematic method can be applied for the construction of CODs of rates above $1/2$ for arbitrary n. In [10], a systematic and computer-aided method (not with closed forms) is proposed to design CODs for any number n of transmit antennas. Although a computer algorithm for any n and CODs of rates $(k+1)/(2k)$ for $n = 2k-1,\ 2k \leq 18$ are presented in [10], the computer algorithm is prohibitive when n is large and furthermore it is not proved that a COD generated from the algorithm has its rate always above $1/2$. In addition, the algorithm may generate a block size p that may be too large to be necessary for the given n. For example, the size p of the COD for $n = 4$ is 8 but not 4 as existed and mentioned previously. So far, we have not seen any systematic closed form construction of CODs with rates above $1/2$ for an arbitrary number of transmit antennas in any literature.

As a note, after we submitted this paper in the August of 2003, we came across a later published paper [9]. In [9], a systematic construction of CODs with rates $(k+1)/(2k)$ for any number $n = 2k-1$ or $n = 2k$ of transmit antennas is proposed. However, the design method in [9] does not have a *closed-form*. In [9], the definition of a COD does not allow the entries of a COD to have a linear processing of 0, $\pm x_i$, or $\pm x_i^*$ and therefore, symbols x_i or x_i^* can not repeat in any column of a COD, which is different with the definition of a COD used in [8] where linear processings are allowed. With the assumption of *no linear processing*, it is shown in [9] that the maximum rate of a COD is $(k+1)/(2k)$ for $2k-1$ or $2k$ transmit antennas no matter how large p is, which coincides with the conjecture presented in [12]. However, the conjecture for a COD with linear processing is still *open* for a general k.

The goal of this correspondence is to present a systematic closed form construction of CODs of rates $(k+1)/(2k)$ for $n = 2k-1$ or $n = 2k$ transmit antennas for any positive integer k. In this construction, the entries of a COD B_n are from the set $\{0, \pm x_1, \pm x_1^*, \pm x_2, \pm x_2^*, \cdots, \pm x_u, \pm x_u^*\}$ where for each i, x_i or x_i^* occurs once and only once in each column. Starting from a given COD B_n for an odd n, we construct CODs of closed forms for $n+1$ and $n+2$ with the above mentioned rates R. Another closed form construction is also presented when n is a multiple of 4, where the delay size p is only half of the first construction and the designs obtained in [9], [10]. It should be emphasized here that our constructions are closed-form constructions while the ones in [9], [10] are not closed-form constructions.

This paper is organized as follows. In Section II, we present the COD constructions including the orthogonality property. In Section III, we give the CODs result for $n = 8$ as a design example, where the delay size p is only half of the one in [9], [10] while their rates are the same.

II. Constructions of Complex Orthogonal Designs

In this section, we first present notations in Section II-A. The closed form CODs are proposed in Section II-B. The rates and sizes of these CODs are discussed in Section II-C. In Section II-D we propose another construction to achieve smaller sizes when the number of transmit antennas is a multiple of 4.

A. Notation

To construct CODs, we define the following matrices:

- For any matrix A, its all entries are of forms 0, or $\pm x_i$, or $\pm x_i^*$ for $i = \zeta, \cdots, \zeta+u-1$ for some positive integers u and ζ, where for each i, x_i or x_i^* occurs once and only once in each column;

- B_n is a $p_n \times n$ COD for n antennas and the number of nonzero complex variables in B_n is u_n;
- $\overline{B}_n$ is a $p_n \times 1$ column vector that contains the same set of complex variables as B_n and the number of nonzero complex variables is u_n;
- $\widehat{B}_n$ is a $q_{1,n} \times 1$ column vector that contains the same set of complex variables as B_n and the number of nonzero complex variables is u_n;
- $Q_{m,n}$ is a $q_{m,n} \times n$ COD for n antennas, where we let $m \leq n$ and $Q_{0,n} = B_n$, and the number of nonzero complex variables is $v_{m,n} \leq u_n$;
- $\overline{Q}_{m,n}$ is a $q_{m-1,n} \times 1$ column vector that contains the same set of complex variables as $Q_{m,n}$, and $\overline{Q}_{0,n} = \overline{B}_n$, and the number of nonzero complex variables is $v_{m,n}$;
- $\widehat{Q}_{m,n}$ is a $q_{m+1,n} \times 1$ column vector that contains the same set of complex variables as $Q_{m,n}$, and $\widehat{Q}_{0,n} = \widehat{B}_n$, and the number of nonzero complex variables is $v_{m,n}$,

where the parameters are specified later. In the rest of this paper, we use different indices in brackets to distinguish different sets of nonzero complex variables, which is specified and explained as follows:

- For any matrix A, matrix $A(i)$ has the same structure but different variables as A when $i > 1$ and $A(1) = A$;
- Consider any w submatrices $A_1(1), \cdots, A_w(w)$ appearing in a bigger matrix simultaneously, where A_l has nonzero complex variables $x_1, \cdots, x_{u_l}$ for $l = 1, \cdots, w$. Then, the nonzero complex variables in these submatrices appear consecutively in the bigger matrix without any overlaps, i.e., the indices of the nonzero complex variables in $A_l(l)$ are from $u_1 + \cdots + u_{l-1} + 1$ to $u_1 + \cdots + u_{l-1} + u_l$ for $l = 2, \cdots, w$.

As an example, consider B_3 of the following form

$$B_3 = \begin{bmatrix} x_1 & x_2^* & x_3^* \\ x_2 & -x_1^* & 0 \\ x_3 & 0 & -x_1^* \\ 0 & x_3 & -x_2 \end{bmatrix}. \tag{3}$$

In the above matrix, $u_3 = 3$, and the indices of the nonzero complex variables in B_3 are from 1 to 3. Consider a bigger matrix

$$\begin{bmatrix} B_3(1) \\ \vdots \\ B_3(i) \end{bmatrix}.$$

Then, the indices of the nonzero complex variables in $B_3(i)$ are from $3(i-1)+1$ to $3i$ and $B_3(i)$ is as follows:

$$B_3(i) = \begin{bmatrix} x_{3(i-1)+1} & x_{3(i-1)+2}^* & x_{3(i-1)+3}^* \\ x_{3(i-1)+2} & -x_{3(i-1)+1}^* & 0 \\ x_{3(i-1)+3} & 0 & -x_{3(i-1)+1}^* \\ 0 & x_{3(i-1)+3} & -x_{3(i-1)+2} \end{bmatrix}. \tag{4}$$

Note that, the structure of $B_3(i)$ is the same as that of B_3, but the subscripts of the variables in the matrices are different. For example, the subscript of the term in the first row and the first column in B_3 is 1, while that of $B_3(i)$ is $3(i-1)+1$.

B. *Closed Form Construction of CODs*

The main idea of our method is to construct CODs inductively. Specifically, given a set of CODs for n antennas, where n is an odd integer, we can construct codes B_{n+1} and B_{n+2} for $n+1$ and $n+2$ antennas. Based on the definitions in Section II-A, we provide the following method to construct CODs for $n+1$ and $n+2$ antennas through B_n, $\overline{B}_n$, $\hat{B}_n$, $Q_{1,n}$, and $\overline{Q}_{1,n}$ where $n = 2k-1$, $k = 1, 2, \cdots$:

$$B_{n+1} = \begin{bmatrix} B_n(1) & \overline{B}_n(2) \\ B_n(2) & (-1)^k \overline{B}_n(1) \end{bmatrix}, \tag{5}$$

and

$$B_{n+2} = \begin{bmatrix} B_n(1) & \overline{B}_n(2) & \overline{B}_n(3) \\ B_n(2) & (-1)^k \overline{B}_n(1) & \overline{Q}_{1,n}(4) \\ B_n(3) & -\overline{Q}_{1,n}(4) & (-1)^k \overline{B}_n(1) \\ Q_{1,n}(4) & \hat{B}_n(3) & -\hat{B}_n(2) \end{bmatrix}. \tag{6}$$

Therefore, through inductive construction of B_n, $\overline{B}_n$, $\hat{B}_n$, $Q_{1,n}$, and $\overline{Q}_{1,n}$ we can construct CODs for any $n+1$ and $n+2$ antennas.

The inductive construction starts with the following initial settings:

- $B_1 = [x_1]$;
- $\overline{B}_1 = [x_1^*]$;
- $\hat{B}_1 = [x_1]$;
- $Q_{1,1} = [0]$;
- $\overline{Q}_{1,1} = [0]$;
- $\overline{Q}_{2,1} = [0]$;
- $\hat{Q}_{0,1} = [0]$;
- $\hat{Q}_{m,1} = \phi$, i.e., empty (does not appear), for $m > 0$;
- $Q_{m,1} = \phi$, i.e., empty (does not appear), for $m > 1$;
- $\overline{Q}_{m,1} = \phi$, i.e., empty (does not appear), for $m > 2$.

To complete the inductive method, we also provide the construction scheme for $\overline{B}_{n+2}$, $\hat{B}_{n+2}$, $Q_{m,n+2}$, $\overline{Q}_{m,n+2}$, and $\hat{Q}_{m,n+2}$ as follows:

$$\overline{B}_{n+2} = \begin{bmatrix} (-1)^k \overline{Q}_{1,n}(4) \\ \overline{B}_n(3) \\ -\overline{B}_n(2) \\ \hat{B}_n(1) \end{bmatrix}, \tag{7}$$

$$\widehat{B}_{n+2} = \begin{bmatrix} (-1)^k\overline{B}_n(1) \\ \widehat{B}_n(2) \\ \widehat{B}_n(3) \\ -\widehat{Q}_{1,n}(4) \end{bmatrix}, \tag{8}$$

$$Q_{m,n+2} = \begin{bmatrix} Q_{m-1,n}(1) & \overline{Q}_{m,n}(2) & \overline{Q}_{m,n}(3) \\ Q_{m,n}(2) & -\widehat{Q}_{m-1,n}(1) & \overline{Q}_{m+1,n}(4) \\ Q_{m,n}(3) & -\overline{Q}_{m+1,n}(4) & -\widehat{Q}_{m-1,n}(1) \\ Q_{m+1,n}(4) & \widehat{Q}_{m,n}(3) & -\widehat{Q}_{m,n}(2) \end{bmatrix}, \tag{9}$$

$$\overline{Q}_{m,n+2} = \begin{bmatrix} -\overline{Q}_{m-1,n}(1) \\ \overline{Q}_{m,n}(2) \\ \overline{Q}_{m,n}(3) \\ -\overline{Q}_{m+1,n}(4) \end{bmatrix}, \tag{10}$$

and

$$\widehat{Q}_{m,n+2} = \begin{bmatrix} \widehat{Q}_{m-1,n}(1) \\ -\widehat{Q}_{m,n}(2) \\ -\widehat{Q}_{m,n}(3) \\ \widehat{Q}_{m+1,n}(4) \end{bmatrix}, \tag{11}$$

where $m > 0$, $Q_{0,n} = B_n$, $\overline{Q}_{0,n} = \overline{B}_n$, and $\widehat{Q}_{0,n} = \widehat{B}_n$ as described in the definitions in the beginning of this section.

Before the proof of the construction, we have a general property on the orthogonality.

Theorem 1: Let A be a complex orthogonal design and has the form

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

where

(i) A_{11} and A_{22} have the same set of nonzero complex variables;

(ii) A_{21} and A_{12} have the same set of nonzero complex variables;

(iii) A_{11} and A_{12} do not share any common nonzero complex variable.

Then, the following $\overline{A}$

$$\overline{A} = \begin{bmatrix} (-1)^k A_{11} & (-1)^l A_{12} \\ (-1)^m A_{21} & (-1)^n A_{22} \end{bmatrix}$$

is also a complex orthogonal design if $k + l + m + n$ is even.

Proof: See Appendix I.

From Theorem 1, the following corollary is immediate.

Corollary 1: Let

$$\overline{A} = \begin{bmatrix} (-1)^k A_{11} & (-1)^l A_{12} \\ (-1)^m A_{21} & (-1)^n A_{22} \end{bmatrix}$$

be a COD and conditions (i)-(iii) in Theorem 1 hold. Then,

$$\widehat{A} = \begin{bmatrix} (-1)^p A_{11} & (-1)^q A_{12} \\ (-1)^r A_{21} & (-1)^s A_{22} \end{bmatrix}$$

is also a COD if $k+l+m+n+p+q+r+s$ is even.

To validate our method, we also have following theorem.

Theorem 2: For $n = 2k-1$, $k = 1, 2, \cdots$, if B_n, $\overline{B}_n$, $\widehat{B}_n$, $Q_{m,n}$, $\overline{Q}_{m,n}$, and $\widehat{Q}_{m,n}$ are inductively constructed from (6)-(11), then the following matrices are complex orthogonal designs:

$$B_{n+1} = \begin{bmatrix} B_n(i) & \overline{B}_n(j) \\ B_n(j) & (-1)^k \overline{B}_n(i) \end{bmatrix}, \tag{12}$$

$$\begin{bmatrix} \overline{B}_n(i) & \overline{B}_n(j) \\ \widehat{B}_n(j) & -\widehat{B}_n(i) \end{bmatrix}, \tag{13}$$

$$\begin{bmatrix} B_n(i) & \overline{Q}_{1,n}(j) \\ Q_{1,n}(j) & -\widehat{B}_n(i) \end{bmatrix}, \tag{14}$$

$$\begin{bmatrix} \overline{B}_n(i) & \overline{Q}_{1,n}(j) \\ \overline{Q}_{1,n}(j) & -\overline{B}_n(i) \end{bmatrix}, \tag{15}$$

$$\begin{bmatrix} \overline{Q}_{m,n}(i) & \overline{Q}_{m,n}(j) \\ \widehat{Q}_{m,n}(j) & -\widehat{Q}_{m,n}(i) \end{bmatrix}, \tag{16}$$

$$\begin{bmatrix} Q_{m,n}(i) & \overline{Q}_{m+1,n}(j) \\ Q_{m+1,n}(j) & -\widehat{Q}_{m,n}(i) \end{bmatrix}, \tag{17}$$

$$\begin{bmatrix} \widehat{B}_n(i) & \overline{Q}_{2,n}(j) \\ \overline{Q}_{2,n}(j) & -\widehat{B}_n(i) \end{bmatrix}, \tag{18}$$

$$\begin{bmatrix} \widehat{Q}_{m-1,n}(i) & \overline{Q}_{m+1,n}(j) \\ \overline{Q}_{m+1,n}(j) & -\widehat{Q}_{m-1,n}(i) \end{bmatrix}. \tag{19}$$

Proof: We can easily see that when the initial set of codes is used, all matrices in (12)-(19) are CODs. Now we show that if the matrices in (12)-(19) are CODs for some $n = 2k-1$, $k = 1, 2, \cdots$, they are still CODs for $n+2$ given the construction scheme in (6)-(11) by using Theorem 1 and Corollary 1 as follows.

For (12), we have

$$B_{n+3} = \begin{bmatrix} B_{n+2}(i) & \overline{B}_{n+2}(j) \\ B_{n+2}(j) & (-1)^{k+1} \overline{B}_{n+2}(i) \end{bmatrix} =$$

$$\left[\begin{array}{ccc|c} B_n(1) & \overline{B}_n(2) & \overline{B}_n(3) & (-1)^k \overline{Q}_{1,n}(8) \\ B_n(2) & (-1)^k \overline{B}_n(1) & \overline{Q}_{1,n}(4) & \overline{B}_n(7) \\ B_n(3) & -\overline{Q}_{1,n}(4) & (-1)^k \overline{B}_n(1) & -\overline{B}_n(6) \\ Q_{1,n}(4) & \widehat{B}_n(3) & -\widehat{B}_n(2) & \widehat{B}_n(5) \\ \hline B_n(5) & \overline{B}_n(6) & \overline{B}_n(7) & -\overline{Q}_{1,n}(4) \\ B_n(6) & (-1)^k \overline{B}_n(5) & \overline{Q}_{1,n}(8) & (-1)^{k+1} \overline{B}_n(3) \\ B_n(7) & -\overline{Q}_{1,n}(8) & (-1)^k \overline{B}_n(5) & (-1)^k \overline{B}_n(2) \\ Q_{1,n}(8) & \widehat{B}_n(7) & -\widehat{B}_n(6) & (-1)^{k+1} \widehat{B}_n(1) \end{array}\right]. \tag{20}$$

To show (20) is a COD, we need to prove that any column in (20) is orthogonal to all the other columns in (20). For simplicity, we only show that the first column of (20) is orthogonal to the last column. From (14) we know that

$$\begin{bmatrix} B_n(1) & \overline{Q}_{1,n}(8) \\ Q_{1,n}(8) & -\widehat{B}_n(1) \end{bmatrix}$$

is a COD. Combining with Corollary 1, we have that

$$\begin{bmatrix} B_n(1) & (-1)^k\overline{Q}_{1,n}(8) \\ Q_{1,n}(8) & (-1)^{k+1}\widehat{B}_n(1) \end{bmatrix}$$

is also a COD. From (12) we know that

$$\begin{bmatrix} B_n(2) & \overline{B}_n(7) \\ B_n(7) & (-1)^k\overline{B}_n(2) \end{bmatrix}$$

is a COD. From (12) and Corollary 1, we have that

$$\begin{bmatrix} B_n(3) & -\overline{B}_n(6) \\ B_n(6) & (-1)^{k+1}\overline{B}_n(3) \end{bmatrix}$$

is a COD. And from (14) and Corollary 1 we have that

$$\begin{bmatrix} B_n(5) & -\overline{Q}_{1,n}(4) \\ Q_{1,n}(4) & \widehat{B}_n(5) \end{bmatrix}$$

is also a COD. Thus the first column in (20) is orthogonal to the last column in (20). Similarly, we can verify that any column in (20) is orthogonal to all the other columns, which, thus, shows that (20) is a COD. With the same approach we can show that the matrices in (13)-(19) are also CODs for $n+2$ by using our construction scheme as follows.

$$\begin{bmatrix} \overline{B}_{n+2}(i) & \overline{B}_{n+2}(j) \\ \widehat{B}_{n+2}(j) & -\widehat{B}_{n+2}(i) \end{bmatrix} = \left[\begin{array}{r|r} (-1)^k\overline{Q}_{1,n}(4) & (-1)^k\overline{Q}_{1,n}(8) \\ \overline{B}_n(3) & \overline{B}_n(7) \\ -\overline{B}_n(2) & -\overline{B}_n(6) \\ \widehat{B}_n(1) & \widehat{B}_n(5) \\ \hline (-1)^k\overline{B}_n(5) & (-1)^{k+1}\overline{B}_n(1) \\ \widehat{B}_n(6) & -\widehat{B}_n(2) \\ \widehat{B}_n(7) & -\widehat{B}_n(3) \\ -\widehat{Q}_{1,n}(8) & \widehat{Q}_{1,n}(4) \end{array}\right]. \tag{21}$$

$$
\left[\begin{array}{rr} B_{n+2}(i) & \overline{Q}_{1,n+2}(j) \\ Q_{1,n+2}(j) & -\widehat{B}_{n+2}(i) \end{array}\right] =
$$

$$
\left[\begin{array}{rrr|r}
B_n(1) & \overline{B}_n(2) & \overline{B}_n(3) & -\overline{B}_n(5) \\
B_n(2) & (-1)^k \overline{B}_n(1) & \overline{Q}_{1,n}(4) & \overline{Q}_{1,n}(6) \\
B_n(3) & -\overline{Q}_{1,n}(4) & (-1)^k \overline{B}_n(1) & \overline{Q}_{1,n}(7) \\
Q_{1,n}(4) & \widehat{B}_n(3) & -\widehat{B}_n(2) & -\widehat{Q}_{2,n}(8) \\
\hline
B_n(5) & \overline{Q}_{1,n}(6) & \overline{Q}_{1,n}(7) & (-1)^{k+1} \overline{B}_n(1) \\
Q_{1,n}(6) & -\widehat{B}_n(5) & \overline{Q}_{2,n}(8) & -\widehat{B}_n(2) \\
Q_{1,n}(7) & -\overline{Q}_{2,n}(8) & -\widehat{B}_n(5) & -\widehat{B}_n(3) \\
Q_{2,n}(8) & \widehat{Q}_{1,n}(7) & -\widehat{Q}_{1,n}(6) & \widehat{Q}_{1,n}(4)
\end{array}\right]. \quad (22)
$$

$$
\left[\begin{array}{rr} \overline{B}_{n+2}(i) & \overline{Q}_{1,n+2}(j) \\ \overline{Q}_{1,n+2}(j) & -\overline{B}_{n+2}(i) \end{array}\right] =
$$

$$
\left[\begin{array}{r|r}
(-1)^k \overline{Q}_{1,n}(4) & -\overline{B}_n(5) \\
\overline{B}_n(3) & \overline{Q}_{1,n}(6) \\
-\overline{B}_n(2) & \overline{Q}_{1,n}(7) \\
\widehat{B}_n(1) & -\overline{Q}_{2,n}(8) \\
\hline
-\overline{B}_n(5) & (-1)^{k+1} \overline{Q}_{1,n}(4) \\
\overline{Q}_{1,n}(6) & -\overline{B}_n(3) \\
\overline{Q}_{1,n}(7) & \overline{B}_n(2) \\
-\overline{Q}_{2,n}(8) & -\widehat{B}_n(1)
\end{array}\right]. \quad (23)
$$

$$
\left[\begin{array}{rr} \overline{Q}_{m,n+2}(i) & \overline{Q}_{m,n+2}(j) \\ \widehat{Q}_{m,n+2}(j) & -\widehat{Q}_{m,n+2}(i) \end{array}\right] =
$$

$$
\left[\begin{array}{r|r}
-\overline{Q}_{m-1,n}(1) & -\overline{Q}_{m-1,n}(5) \\
\overline{Q}_{m,n}(2) & \overline{Q}_{m,n}(6) \\
\overline{Q}_{m,n}(3) & \overline{Q}_{m,n}(7) \\
-\overline{Q}_{m+1,n}(4) & -\overline{Q}_{m+1,n}(8) \\
\hline
-\widehat{Q}_{m-1,n}(5) & \widehat{Q}_{m-1,n}(1) \\
\widehat{Q}_{m,n}(6) & -\widehat{Q}_{m,n}(2) \\
\widehat{Q}_{m,n}(7) & -\widehat{Q}_{m,n}(3) \\
-\widehat{Q}_{m+1,n}(8) & \widehat{Q}_{m+1,n}(4)
\end{array}\right]. \quad (24)
$$

$$\begin{bmatrix} Q_{m,n+2}(i) & \overline{Q}_{m+1,n+2}(j) \\ Q_{m+1,n+2}(j) & -\widehat{Q}_{m,n+2}(i) \end{bmatrix} =$$

$$\left[\begin{array}{ccc|c} Q_{m-1,n}(1) & \overline{Q}_{m,n}(2) & \overline{Q}_{m,n}(3) & -\overline{Q}_{m,n}(5) \\ Q_{m,n}(2) & -\widehat{Q}_{m-1,n}(1) & \overline{Q}_{m+1,n}(4) & \overline{Q}_{m+1,n}(6) \\ Q_{m,n}(3) & -\overline{Q}_{m+1,n}(4) & -\widehat{Q}_{m-1,n}(1) & \overline{Q}_{m+1,n}(7) \\ Q_{m+1,n}(4) & \widehat{Q}_{m,n}(3) & -\widehat{Q}_{m,n}(2) & -\overline{Q}_{m+2,n}(8) \\ \hline Q_{m,n}(5) & \overline{Q}_{m+1,n}(6) & \overline{Q}_{m+1,n}(7) & \widehat{Q}_{m-1,n}(1) \\ Q_{m+1,n}(6) & -\widehat{Q}_{m,n}(5) & \overline{Q}_{m+2,n}(8) & -\widehat{Q}_{m,n}(2) \\ Q_{m+1,n}(7) & -\overline{Q}_{m+2,n}(8) & -\widehat{Q}_{m,n}(5) & -\widehat{Q}_{m,n}(3) \\ Q_{m+2,n}(8) & \widehat{Q}_{m+1,n}(7) & -\widehat{Q}_{m+1,n}(6) & \widehat{Q}_{m+1,n}(4) \end{array}\right]. \tag{25}$$

$$\begin{bmatrix} \widehat{B}_{n+2}(i) & \overline{Q}_{2,n+2}(j) \\ \overline{Q}_{2,n+2}(j) & -\widehat{B}_{n+2}(i) \end{bmatrix} =$$

$$\left[\begin{array}{c|c} (-1)^k\overline{B}_n(1) & -\overline{Q}_{1,n}(5) \\ \widehat{B}_n(2) & \overline{Q}_{2,n}(6) \\ \widehat{B}_n(3) & \overline{Q}_{2,n}(7) \\ -\widehat{Q}_{1,n}(4) & -\overline{Q}_{3,n}(8) \\ \hline -\overline{Q}_{1,n}(5) & (-1)^{k+1}\overline{B}_n(1) \\ \overline{Q}_{2,n}(6) & -\widehat{B}_n(2) \\ \overline{Q}_{2,n}(7) & -\widehat{B}_n(3) \\ -\overline{Q}_{3,n}(8) & \widehat{Q}_{1,n}(4) \end{array}\right]. \tag{26}$$

$$\begin{bmatrix} \widehat{Q}_{m-1,n+2}(i) & \overline{Q}_{m+1,n+2}(j) \\ \overline{Q}_{m+1,n+2}(j) & -\widehat{Q}_{m-1,n+2}(i) \end{bmatrix} =$$

$$\left[\begin{array}{c|c} \widehat{Q}_{m-2,n}(1) & -\overline{Q}_{m,n}(5) \\ -\widehat{Q}_{m-1,n}(2) & \overline{Q}_{m+1,n}(6) \\ -\widehat{Q}_{m-1,n}(3) & \overline{Q}_{m+1,n}(7) \\ \widehat{Q}_{m,n}(4) & -\overline{Q}_{m+2,n}(8) \\ \hline -\overline{Q}_{m,n}(5) & -\widehat{Q}_{m-2,n}(1) \\ \overline{Q}_{m+1,n}(6) & \widehat{Q}_{m-1,n}(2) \\ \overline{Q}_{m+1,n}(7) & \widehat{Q}_{m-1,n}(3) \\ -\overline{Q}_{m+2,n}(8) & -\widehat{Q}_{m,n}(4) \end{array}\right]. \tag{27}$$

As a conclusion, Theorem 2 is proved inductively. **q.e.d.**

Thus, by the induction, for any odd number n, we can inductively construct B_n, $\overline{B}_n$, $\widehat{B}_n$, $Q_{1,n}$, and $\overline{Q}_{1,n}$. Because of the orthogonalities in (12)-(19), B_{n+1}, B_{n+2} in (5) and (6) are also CODs, which completes the proof of our construction.

C. Rate Formula

For the sizes of matrices B_l and Q_{l_1,l_2}, we have following partial difference equations from (6) and (9)[1]:

$$\begin{cases} q_{0,n+2} = 3q_{0,n} + q_{1,n}, \\ q_{m,n+2} = q_{m-1,n} + 2q_{m,n} + q_{m+1,n}, \ m > 0, \end{cases} \tag{28}$$

with initial conditions

$$\begin{cases} q_{0,1} = 1, \\ q_{1,1} = 1, \\ q_{m,1} = 0, \ m > 1. \end{cases}$$

Similarly, we have the following partial difference equations for numbers of the complex variables in B_l and Q_{l_1,l_2}:

$$\begin{cases} v_{0,n+2} = 3v_{0,n} + v_{1,n}, \\ v_{m,n+2} = v_{m-1,n} + 2v_{m,n} + v_{m+1,n}, \ m > 0, \end{cases} \tag{29}$$

with initial conditions

$$\begin{cases} v_{0,1} = 1, \\ v_{m,1} = 0, \ m > 0. \end{cases}$$

The solutions of the above two partial difference equations are given as follows:

$$\begin{cases} q_{m,2k-1} = 0 & m > k \\ q_{m,2k-1} = \frac{(2k)! \times \left[k + \frac{m(m+1)}{k}\right]}{(k+m+1)!(k-m)!} & 0 \leq m \leq k \end{cases} \tag{30}$$

and

$$\begin{cases} v_{m,2k-1} = 0 & m > k-1 \\ v_{m,2k-1} = \frac{(2k-1)!}{(k+m)!(k-m-1)!} & 0 \leq m \leq k-1 \end{cases} \tag{31}$$

Both (30) and (31) can be shown by induction. Therefore, when $m = 0$, we have the rate formula

$$R_{2k-1} = \frac{v_{0,2k-1}}{q_{0,2k-1}} = \frac{k+1}{2k} > \frac{1}{2}. \tag{32}$$

It is interesting to note that it is conjectured in [12] that the rate of a COD for $2k-1$ or $2k$ antennas is upper bounded by $(k+1)/(2k)$. And in [9] Liang showed that when there is no linear processing in a COD, the rate upper bound for $2k-1$ or $2k$ transmit antennas is $(k+1)/(2k)$. However, it is still open for a COD with linear processing.

D. Construction of Smaller Size COD for $4l$ Transmit Antennas

Let us consider the case of $n = 2k-1$, where n is the number of transmit antennas and k is an odd integer. Based on the construction procedure in Section II-B, we can construct a COD B_{n+2} from (6). From B_{n+2} we can construct a COD B_{n+3} from (5), which is rewritten as follows:

$$B_{n+3} = \begin{bmatrix} B_{n+2}(1) & \overline{B}_{n+2}(2) \\ B_{n+2}(2) & (-1)^k \overline{B}_{n+2}(1) \end{bmatrix}. \tag{33}$$

[1]Note that $Q_{0,n} = B_n$ by definition, we use $q_{0,n} = p_n$ and $v_{0,n} = u_n$.

To compute the rate of the COD B_{n+3}, we have

$$p_{n+3} = 2q_{0,n+2} = 2q_{0,2(k+1)-1} = 2\frac{(2(k+1))! \times (k+1)}{(k+2)!(k+1)!} \tag{34}$$

and

$$u_{n+3} = 2v_{0,n+2} = 2v_{0,2(k+1)-1} = 2\frac{(2(k+1)-1)!}{(k+1)!(k)!}. \tag{35}$$

Therefore, the rate is:

$$R_{n+3} = \frac{u_{n+3}}{p_{n+3}} = \frac{k+2}{2(k+1)}. \tag{36}$$

However, we find that when k is odd, there exists one smaller size COD B'_{n+3} than B_{n+3}, which is constructed directly from B_n:

$$B'_{n+3} = \left[\begin{array}{ccc|c} B_n(1) & \overline{B}_n(2) & \overline{B}_n(3) & -\overline{Q}_{1,n}(4) \\ B_n(2) & -\overline{B}_n(1) & \overline{Q}_{1,n}(4) & \overline{B}_n(3) \\ B_n(3) & -\overline{Q}_{1,n}(4) & -\overline{B}_n(1) & -\overline{B}_n(2) \\ Q_{1,n}(4) & \widehat{B}_n(3) & -\widehat{B}_n(2) & \widehat{B}_n(1) \end{array}\right] \tag{37}$$

Based on the COD assumptions in (12)-(19) and Theorem 1, we can see that B'_{n+3} is a COD. Since

$$p'_{n+3} = 3q_{0,n} + q_{1,n} = 3\frac{(2k)! \times k}{(k+1)!(k)!} + \frac{(2k)! \times \left[k + \frac{2}{k}\right]}{(k+2)!(k-1)!} \tag{38}$$

and

$$u'_{n+3} = 3v_{0,n} + v_{1,n} = 3\frac{(2k-1)!}{k!(k-1)!} + \frac{(2k-1)!}{(k+1)!(k-2)!}, \tag{39}$$

we have that the rate of B'_{n+3} is:

$$R'_{n+3} = \frac{u'_{n+3}}{p'_{n+3}} = \frac{k+2}{2(k+1)}, \tag{40}$$

while B'_{n+3} has a smaller size:

$$p'_{n+3} = \frac{1}{2}p_{n+3}.$$

Note that in this case, since k is an odd number, the number of antennas $n+3 = 2(k+1)$ is in the form of $4l$, $l = 1, 2, \cdots$. This shows that the COD construction (37) for $n+3 = 4l$ for any positive integer l has only half delay of the delay of COD B_{n+3} in the first construction and also the one in the literature [9], [10].

III. Design Examples

In this section, we will give some design examples. From the initial settings with $n = 2k-1 = 1$ and $k = 1$, B_2 and B_3 can be easily constructed from (5), (6) and have the form in (2), (3), respectively. We can also construct the following matrices:

- $\overline{Q}_{2,1}$

$$\overline{Q}_{2,1} = [0] \tag{41}$$

From the definition of $\overline{Q}_{m,n}$, we can see that $\overline{Q}_{2,1}$ is a $q_{1,1} \times 1$ column vector, which contains the same set of nonzero complex variables as $Q_{2,1}$. Since $Q_{2,1} = \phi$ (initial setting) and $q_{1,1} = 1$, there is no nonzero complex variables in both $Q_{2,1}$ and $\overline{Q}_{2,1}$. As a result the only entry of $\overline{Q}_{2,1}$ is zero.

- $Q_{1,3}$

$$Q_{1,3} = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & -x_1 & 0 \\ 0 & 0 & -x_1 \end{bmatrix} \tag{42}$$

Note that in matrix (42), we eliminate the last row in matrix (9) since all the terms of this row are ϕ, i.e., do not appear.

- $\overline{B}_3$

$$\overline{B}_3 = \begin{bmatrix} 0 \\ x_3^* \\ -x_2^* \\ x_1 \end{bmatrix} \tag{43}$$

- $\widehat{B}_3$

$$\widehat{B}_3 = \begin{bmatrix} -x_1^* \\ x_2 \\ x_3 \end{bmatrix} \tag{44}$$

In (44), we eliminate the last row for the same reason as that of (42).

- $\overline{Q}_{1,3}$

$$\overline{Q}_{1,3} = \begin{bmatrix} -x_1^* \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{45}$$

- $\widehat{Q}_{1,3}$

$$\widehat{Q}_{1,3} = \begin{bmatrix} x_1 \end{bmatrix} \tag{46}$$

- $Q_{2,3}$

$$Q_{2,3} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{47}$$

- $\overline{Q}_{2,3}$

$$\overline{Q}_{2,3} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{48}$$

With all matrices above, we can construct a COD for 4 antennas as:

$$B_4 = \left[\begin{array}{ccc|c} x_1 & x_2^* & x_3^* & 0 \\ x_2 & -x_1^* & 0 & x_6^* \\ x_3 & 0 & -x_1^* & -x_5^* \\ 0 & x_3 & -x_2 & x_4 \\ \hline x_4 & x_5^* & x_6^* & 0 \\ x_5 & -x_4^* & 0 & x_3^* \\ x_6 & 0 & -x_4^* & -x_2^* \\ 0 & x_6 & -x_5 & x_1 \end{array}\right], \tag{49}$$

which has the same rate $R = 3/4$ and delay $p = 8$ as the ones constructed in [9], [10]. However, from (37), the COD for 4 antennas can be constructed as:

$$B'_4 = \begin{bmatrix} x_1 & x_2^* & x_3^* & 0 \\ x_2 & -x_1^* & 0 & x_3^* \\ x_3 & 0 & -x_1^* & -x_2^* \\ 0 & x_3 & -x_2 & x_1 \end{bmatrix}, \tag{50}$$

which coincides with the existing one for 4 antennas and has only half delay as that in [9], [10].

Also we can construct a COD for 5 antennas as:

$$B_5 = \left[\begin{array}{ccc|c|c} x_1 & x_2^* & x_3^* & 0 & 0 \\ x_2 & -x_1^* & 0 & x_6^* & x_9^* \\ x_3 & 0 & -x_1^* & -x_5^* & -x_8^* \\ 0 & x_3 & -x_2 & x_4 & x_7 \\ \hline x_4 & x_5^* & x_6^* & 0 & -x_{10}^* \\ x_5 & -x_4^* & 0 & x_3^* & 0 \\ x_6 & 0 & -x_4^* & -x_2^* & 0 \\ 0 & x_6 & -x_5 & x_1 & 0 \\ \hline x_7 & x_8^* & x_9^* & x_{10}^* & 0 \\ x_8 & -x_7^* & 0 & 0 & x_3^* \\ x_9 & 0 & -x_7^* & 0 & -x_2^* \\ 0 & x_9 & -x_8 & 0 & x_1 \\ \hline x_{10} & 0 & 0 & -x_7^* & x_4^* \\ 0 & -x_{10} & 0 & x_8 & -x_5 \\ 0 & 0 & -x_{10} & x_9 & -x_6 \end{array}\right], \tag{51}$$

which has the same parameters as the one constructed by Liang in [8]. To construct a COD for 8 antennas, we have following CODs:

- $Q_{1,5}$

$$Q_{1,5} = \left[\begin{array}{ccc|c|c} x_1 & x_2^* & x_3^* & -x_4^* & -x_5^* \\ x_2 & -x_1^* & 0 & 0 & 0 \\ x_3 & 0 & -x_1^* & 0 & 0 \\ 0 & x_3 & -x_2 & 0 & 0 \\ \hline x_4 & 0 & 0 & x_1^* & 0 \\ 0 & -x_4 & 0 & -x_2 & 0 \\ 0 & 0 & -x_4 & -x_3 & 0 \\ \hline x_5 & 0 & 0 & 0 & x_1^* \\ 0 & -x_5 & 0 & 0 & -x_2 \\ 0 & 0 & -x_5 & 0 & -x_3 \\ \hline 0 & 0 & 0 & x_5 & -x_4 \end{array}\right] \tag{52}$$

- $\overline{B}_5$, $\widehat{B}_5$, and $\overline{Q}_{1,5}$

$$
\overline{B}_5 = \begin{bmatrix} -x_{10}^* \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ x_9^* \\ -x_8^* \\ x_7 \\ \hline 0 \\ -x_6^* \\ x_5^* \\ -x_4 \\ \hline -x_1^* \\ x_2 \\ x_3 \end{bmatrix}, \quad \widehat{B}_5 = \begin{bmatrix} 0 \\ x_3^* \\ -x_2^* \\ x_1 \\ \hline -x_4^* \\ x_5 \\ x_6 \\ \hline -x_7^* \\ x_8 \\ x_9 \\ \hline -x_{10} \end{bmatrix}, \quad \overline{Q}_{1,5} = \begin{bmatrix} 0 \\ -x_3^* \\ x_2^* \\ -x_1 \\ \hline -x_4^* \\ 0 \\ 0 \\ 0 \\ \hline -x_5^* \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}
$$

Since $k = 3$ when $n = 2k - 1 = 5$, we can construct a COD B_8' shown at the end of the paper for $n + 3 = 8$ antennas based on (37). The delay size and the number of variables in the above COD are 56 and 35, respectively. These values can be predicted through the solutions of the difference equations (30) and (31), where $m = 0$ and $k = 4$:

$$
p_8 = p_7 = q_{0,7} = \frac{8! \times 4}{5! \times 4!} = 56
$$

and

$$
u_8 = u_7 = v_{0,7} = \frac{7!}{4! \times 3!} = 35.
$$

Note that the block length or delay size p is 56, while in [9], [10], it is 112.

To further compare our construction schemes with that of [9], [10], we list the design examples from 1 to 16 antennas in Table I where "Liang" and "Su-Xia-Liu" are for the schemes in [9] and [10], respectively, and "Lu-Fu-Xia" is for our new closed form designs. One can clearly see that all the three constructions have the same rate. However, when the number of transmit antennas is any multiple of 4, the CODs in our scheme have only half delays of that of [9], [10].

IV. Conclusion

In this correspondence, a novel inductive and closed-form method has been proposed to systematically construct a complex orthogonal design (COD) of rate $(k + 1)/(2k)$ for $2k - 1$ or $2k$ transmit antennas for any positive integer k. These rates are conjectured optimal in [12] with or without linear processing. Although it is shown in [9] that when there is no linear processing the upper bound of the rate for $2k - 1$ or $2k$ transmit antennas is $(2k - 1)/(2k)$, it is still open for CODs with linear processing. Another closed form COD construction for $4l$ transmit antennas has also been presented with smaller delay sizes that

TABLE I

COMPARISON OF DESIGN EXAMPLES

# of Tx	**Liang & Su-Xia-Liu**		**Lu-Fu-Xia**		**Rate**
n	u	p	u	p	u/p
1	1	1	1	1	1
2	2	2	2	2	1
3	3	4	3	4	3/4
4	6	8	**3**	**4**	3/4
5	10	15	10	15	2/3
6	20	30	20	30	2/3
7	35	56	35	56	5/8
8	70	112	**35**	**56**	5/8
9	126	210	126	210	6/10
10	252	420	252	420	6/10
11	462	792	462	792	7/12
12	924	1584	**462**	**792**	7/12
13	1716	3003	1716	3003	8/14
14	3432	6006	3432	6006	8/14
15	6435	11440	6435	11440	9/16
16	12870	22880	**6435**	**11440**	9/16

are only half of the ones of the first construction and the ones appeared in [9], [10] while their rates are the same. However, the optimal delay for any number of transmit antennas is also an open question for future research.

As we mentioned in Introduction, after we submitted this correspondence in the August of 2003, we came across with [9] where also a systematic construction of CODs of rate $(k+1)/(2k)$ for $2k-1$ or $2k$ transmit antennas for any positive integer k was proposed. Comparing to [9], our designs have closed-forms while the method in [9] is not a closed-form method, and furthermore, our designs have only half of the delays of the ones in [9] when the number of transmit antennas is any multiple of 4, such as $4, 8, 12, 16, 20, \cdots$.

Appendix I

Proof of Theorem 1

Define A_1, A_2, $\overline{A}_1$, and $\overline{A}_2$ as:

$$\begin{aligned} A_1 &= \begin{bmatrix} A_{11} \\ A_{21} \end{bmatrix}, \\ A_2 &= \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix}, \\ \overline{A}_1 &= \begin{bmatrix} (-1)^k A_{11} \\ (-1)^m A_{21} \end{bmatrix}, \\ \overline{A}_2 &= \begin{bmatrix} (-1)^l A_{12} \\ (-1)^n A_{22} \end{bmatrix}. \end{aligned}$$

Since A is a COD, we can see that A_1 and A_2 are both CODs. Also, since A_{11} and A_{21} have disjoint sets of nonzero complex variables, matrix $\overline{A}_1$ is a COD too regardless of the numbers k and m. Similarly, matrix $\overline{A}_2$ is a COD for any values of l and n. Thus, we see that $\overline{A}$ is a COD if and only if any column in $\overline{A}_1$ is orthogonal to all columns in $\overline{A}_2$. Without lose of generality, we consider two columns i and j in matrix $\overline{A}$, where column i belongs to $\overline{A}_1$ and column j belongs to $\overline{A}_2$.

Write matrix $A = [a_{\zeta\xi}]$. For any row p in $\overline{A}$, we let $x = a_{pi}$. According to the definition in the theorem, if $x \neq 0$, we can find another row q such that:

$$|a_{qj}| = |x|.$$

Now let $\widehat{x} = a_{qj}$, $y = a_{pj}$, and $z = a_{qi}$. Since matrix A is a COD, we have $p \neq q$ and

$$xy^* + z\widehat{x}^* = 0,$$

which is because the remaining of the inner product of the ith and the jth column of A do not contain variable x due to the assumption in beginning of this section that each complex variable or its conjugate appears once and only once in each column of A.

In matrix $\overline{A}$, the left hand side of the above equation becomes

$$(-1)^k x \times (-1)^l y^* + (-1)^m z \times (-1)^n \widehat{x}^*$$

Note that if $k + l + m + n$ is even, then $(-1)^{k+l} = (-1)^{m+n}$. Therefore,

$$(-1)^k x \times (-1)^l y^* + (-1)^m z \times (-1)^n \widehat{x}^* = 0,$$

which proves that the ith column is orthogonal to the jth column and therefore $\overline{A}$ is a COD. **q.e.d.**

ACKNOWLEDGMENT

The authors would like to thank anonymous reviewers for helpful suggestions to improve the presentation of the paper.

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$$
B_8^{'} = \left(\begin{array}{ccccc|c|c|c}
x_1 & x_2^* & x_3^* & 0 & 0 & -x_{20}^* & -x_{30}^* & 0 \\
x_2 & -x_1^* & 0 & x_6^* & x_9^* & 0 & 0 & x_{33}^* \\
x_3 & 0 & -x_1^* & -x_5^* & -x_8^* & 0 & 0 & -x_{32}^* \\
0 & x_3 & -x_2 & x_4 & x_7 & 0 & 0 & x_{31} \\
x_4 & x_5^* & x_6^* & 0 & -x_{10}^* & 0 & 0 & x_{34}^* \\
x_5 & -x_4^* & 0 & x_3^* & 0 & x_{19}^* & x_{29}^* & 0 \\
x_6 & 0 & -x_4^* & -x_2^* & 0 & -x_{18}^* & -x_{28}^* & 0 \\
0 & x_6 & -x_5 & x_1 & 0 & x_{17} & x_{27} & 0 \\
x_7 & x_8^* & x_9^* & x_{10}^* & 0 & 0 & 0 & x_{35}^* \\
x_8 & -x_7^* & 0 & 0 & x_3^* & -x_{16}^* & -x_{26}^* & 0 \\
x_9 & 0 & -x_7^* & 0 & -x_2^* & x_{15}^* & x_{25}^* & 0 \\
0 & x_9 & -x_8 & 0 & x_1 & -x_{14} & -x_{24} & 0 \\
x_{10} & 0 & 0 & -x_7^* & x_4^* & -x_{11}^* & -x_{21}^* & 0 \\
0 & -x_{10} & 0 & x_8 & -x_5 & x_{12} & x_{22} & 0 \\
0 & 0 & -x_{10} & x_9 & -x_6 & x_{13} & x_{23} & 0 \\
\hline
x_{11} & x_{12}^* & x_{13}^* & 0 & 0 & x_{10}^* & 0 & -x_{30}^* \\
x_{12} & -x_{11}^* & 0 & x_{16}^* & x_{19}^* & 0 & -x_{33}^* & 0 \\
x_{13} & 0 & -x_{11}^* & -x_{15}^* & -x_{18}^* & 0 & x_{32}^* & 0 \\
0 & x_{13} & -x_{12} & x_{14} & x_{17} & 0 & -x_{31} & 0 \\
x_{14} & x_{15}^* & x_{16}^* & 0 & -x_{20}^* & 0 & -x_{34}^* & 0 \\
x_{15} & -x_{14}^* & 0 & x_{13}^* & 0 & -x_9^* & 0 & x_{29}^* \\
x_{16} & 0 & -x_{14}^* & -x_{12}^* & 0 & x_8^* & 0 & -x_{28}^* \\
0 & x_{16} & -x_{15} & x_{11} & 0 & -x_7 & 0 & x_{27} \\
x_{17} & x_{18}^* & x_{19}^* & x_{20}^* & 0 & 0 & -x_{35}^* & 0 \\
x_{18} & -x_{17}^* & 0 & 0 & x_{13}^* & x_6^* & 0 & -x_{26}^* \\
x_{19} & 0 & -x_{17}^* & 0 & -x_{12}^* & -x_5^* & 0 & x_{25}^* \\
0 & x_{19} & -x_{18} & 0 & x_{11} & x_4 & 0 & -x_{24} \\
x_{20} & 0 & 0 & -x_{17}^* & x_{14}^* & x_1^* & 0 & -x_{21}^* \\
0 & -x_{20} & 0 & x_{18} & -x_{15} & -x_2 & 0 & x_{22} \\
0 & 0 & -x_{20} & x_{19} & -x_{16} & -x_3 & 0 & x_{23} \\
\hline
x_{21} & x_{22}^* & x_{23}^* & 0 & 0 & 0 & x_{10}^* & x_{20}^* \\
x_{22} & -x_{21}^* & 0 & x_{26}^* & x_{29}^* & x_{33}^* & 0 & 0 \\
x_{23} & 0 & -x_{21}^* & -x_{25}^* & -x_{28}^* & -x_{32}^* & 0 & 0 \\
0 & x_{23} & -x_{22} & x_{24} & x_{27} & x_{31} & 0 & 0 \\
x_{24} & x_{25}^* & x_{26}^* & 0 & -x_{30}^* & x_{34}^* & 0 & 0 \\
x_{25} & -x_{24}^* & 0 & x_{23}^* & 0 & 0 & -x_9^* & -x_{19}^* \\
x_{26} & 0 & -x_{24}^* & -x_{22}^* & 0 & 0 & x_8^* & x_{18}^* \\
0 & x_{26} & -x_{25} & x_{21} & 0 & 0 & -x_7 & -x_{17} \\
x_{27} & x_{28}^* & x_{29}^* & x_{30}^* & 0 & x_{35}^* & 0 & 0 \\
x_{28} & -x_{27}^* & 0 & 0 & x_{23}^* & 0 & x_6^* & x_{16}^* \\
x_{29} & 0 & -x_{27}^* & 0 & -x_{22}^* & 0 & -x_5^* & -x_{15}^* \\
0 & x_{29} & -x_{28} & 0 & x_{21} & 0 & x_4 & x_{14} \\
x_{30} & 0 & 0 & -x_{27}^* & x_{24}^* & 0 & x_1^* & x_{11}^* \\
0 & -x_{30} & 0 & x_{28} & -x_{25} & 0 & -x_2 & -x_{12} \\
0 & 0 & -x_{30} & x_{29} & -x_{26} & 0 & -x_3 & -x_{13} \\
\hline
x_{31} & x_{32}^* & x_{33}^* & -x_{34}^* & -x_{35}^* & 0 & 0 & 0 \\
x_{32} & -x_{31}^* & 0 & 0 & 0 & x_{23}^* & -x_{13}^* & x_3^* \\
x_{33} & 0 & -x_{31}^* & 0 & 0 & -x_{22}^* & x_{12}^* & -x_2^* \\
0 & x_{33} & -x_{32} & 0 & 0 & x_{21} & -x_{11} & x_1 \\
x_{34} & 0 & 0 & x_{31}^* & 0 & -x_{24}^* & x_{14}^* & -x_4^* \\
0 & -x_{34} & 0 & -x_{32} & 0 & x_{25} & -x_{15} & x_5 \\
0 & 0 & -x_{34} & -x_{33} & 0 & x_{26} & -x_{16} & x_6 \\
x_{35} & 0 & 0 & 0 & x_{31}^* & -x_{27}^* & x_{17}^* & -x_7^* \\
0 & -x_{35} & 0 & 0 & -x_{32} & x_{28} & -x_{18} & x_8 \\
0 & 0 & -x_{35} & 0 & -x_{33} & x_{29} & -x_{19} & x_9 \\
0 & 0 & 0 & x_{35} & -x_{34} & -x_{30} & x_{20} & -x_{10}
\end{array}\right)
$$