Correction to "On Full Diversity Space-Time Block Codes with Partial Interference Cancellation Group Decoding"

Xiaoyong Guo, *Student Member, IEEE,* and Xiang-Gen Xia, *Fellow, IEEE*

In this letter, we want to make a correction to the criterion for a linear dispersion space-time block code to achieve full diversity with the partial interference cancellation (PIC) group decoding recently published in [1].

By following all the notations in [1], the main result (criterion) obtained in [1] is as follows.

Theorem 1 (Main Theorem, [1])**.** *Let $\mathcal{X}$ be a linear dispersion STBC. There are n_t transmit and n_r receive antennas. The channel matrix is $\boldsymbol{H} \in \mathbb{C}^{n_r \times n_t}$. The received signal is decoded using the PIC group decoding with a grouping scheme $\{\mathcal{I}_0, \mathcal{I}_1, \ldots, \mathcal{I}_{N-1}\}$. The equivalent channel is $\boldsymbol{G}(\boldsymbol{h})$, where $\boldsymbol{h} = \mathrm{vec}(\boldsymbol{H}) = \{h_0, h_1, \ldots, h_{n_r \cdot n_t - 1}\} \in \mathbb{C}^{n_r \cdot n_t}$. Then, each of the following dimension-reduced systems (i.e., the STBC with the PIC group decoding),*

$$\boldsymbol{z}_{\mathcal{I}_k} = \sqrt{\mathsf{SNR}}\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{G}_{\mathcal{I}_k}\boldsymbol{x}_{\mathcal{I}_k} + \boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{w}, \ k = 0, 1, \ldots, N-1, \quad (1)$$

has power gain order $n_r \cdot n_t$ if and only if the following two conditions are satisfied:

- *for any two different codewords $\boldsymbol{X}, \tilde{\boldsymbol{X}} \in \mathcal{X}$, $\Delta\boldsymbol{X} \triangleq \boldsymbol{X} - \tilde{\boldsymbol{X}}$ has the full rank property, i.e., the code $\mathcal{X}$ achieves full diversity with the ML receiver;*
- *$\boldsymbol{G}_{\mathcal{I}_0}, \boldsymbol{G}_{\mathcal{I}_1}, \ldots, \boldsymbol{G}_{\mathcal{I}_{N-1}}$ defined in (10) in [1] from $\boldsymbol{G} = \boldsymbol{G}(\boldsymbol{h})$ are linearly independent vector groups as long as $\boldsymbol{h} \neq \boldsymbol{0}$.*

When the received signals are decoded using the PIC-SIC group decoding with the ordering (22) in [1] each dimension-reduced system derived during the decoding process (i.e., the STBC with the PIC-SIC group decoding) has power gain order $n_r \cdot n_t$ if and only if

- *for any two different codewords $\boldsymbol{X}, \tilde{\boldsymbol{X}} \in \mathcal{X}$, $\Delta\boldsymbol{X} \triangleq \boldsymbol{X} - \tilde{\boldsymbol{X}}$ has the full rank property, i.e., the code $\mathcal{X}$ achieves full diversity with the ML receiver;*
- *at each decoding stage, $\boldsymbol{G}_{\mathcal{I}_{i_k}}$, which corresponds to the current to-be decoded symbol group $\boldsymbol{x}_{i_k}$, and $[\boldsymbol{G}_{\mathcal{I}_{i_{k+1}}}, \ldots, \boldsymbol{G}_{\mathcal{I}_{i_{N-1}}}]$ are linearly independent vector groups as long as $\boldsymbol{h} \neq \boldsymbol{0}$.*

An error in the proof of this result in [1] occurs in the sufficiency proof, the part below (31) on page 4375 in [1]. As shown in [1], the column vectors of $\boldsymbol{G}_{\mathcal{I}_k}$ are linearly independent over $\Delta\mathcal{A}$, i.e., for any $a_0, a_1, \ldots, a_{n_k-1} \in \Delta\mathcal{A}$, $a_j, j = 0, 1, \ldots, n_k - 1$, not all zero, we have

$$\sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}} \neq \boldsymbol{0}. \quad (2)$$

Although vector groups $\boldsymbol{G}_{\mathcal{I}_0}, \boldsymbol{G}_{\mathcal{I}_1}, \ldots, \boldsymbol{G}_{\mathcal{I}_{N-1}}$ are linearly independent and the column vectors $\boldsymbol{g}_{i_{k,j}}, j = 0, 1, \ldots, n_k - 1$, in $\boldsymbol{G}_{\mathcal{I}_k}$ do not belong to the vector space $\mathbb{V}_{\mathcal{I}_k}$ defined in (12) in [1], their linear combination in (2) may belong to $\mathbb{V}_{\mathcal{I}_k}$. In other words, we may not obtain

$$\boldsymbol{Q}_{\mathcal{I}_k}\left(\sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}}\right) \neq \sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}},$$

i.e., we may not obtain that the column vectors of $\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{G}_{\mathcal{I}_k}$ are also linearly independent over $\Delta\mathcal{A}$.

Based on the above observation, we need to revise the second condition for the above main result that is corrected as follows.

Theorem 2 (Correction to Theorem 1)**.** *Let $\mathcal{X}$ be a linear dispersion STBC. There are n_t transmit and n_r receive antennas. The channel matrix is $\boldsymbol{H} \in \mathbb{C}^{n_r \times n_t}$. The received signal is decoded using the PIC group decoding with a grouping scheme $\{\mathcal{I}_0, \mathcal{I}_1, \ldots, \mathcal{I}_{N-1}\}$. The equivalent channel is $\boldsymbol{G}(\boldsymbol{h})$, where $\boldsymbol{h} = \mathrm{vec}(\boldsymbol{H}) = \{h_0, h_1, \ldots, h_{n_r \cdot n_t - 1}\} \in \mathbb{C}^{n_r \cdot n_t}$. Then, each of the following dimension-reduced systems (i.e., the STBC with the PIC group decoding),*

$$\boldsymbol{z}_{\mathcal{I}_k} = \sqrt{\mathsf{SNR}}\boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{G}_{\mathcal{I}_k}\boldsymbol{x}_{\mathcal{I}_k} + \boldsymbol{P}_{\mathcal{I}_k}\boldsymbol{w}, \ k = 0, 1, \ldots, N-1, \quad (3)$$

has power gain order $n_r \cdot n_t$ if and only if the following two conditions are satisfied:

- *for any two different codewords $\boldsymbol{X}, \tilde{\boldsymbol{X}} \in \mathcal{X}$, $\Delta\boldsymbol{X} \triangleq \boldsymbol{X} - \tilde{\boldsymbol{X}}$ has the full rank property, i.e., the code $\mathcal{X}$ achieves full diversity with the ML receiver;*
- *for a fixed k, $0 \leq k \leq N-1$, any non-zero linear combination over $\Delta\mathcal{A}$ of the vectors in the kth group $\boldsymbol{G}_{\mathcal{I}_k}$ does not belong to the space linearly spanned by all the vectors in the remaining vector groups, which is $\mathbb{V}_{\mathcal{I}_k}$ defined in (12) in [1], i.e., for any $a_0, a_1, \ldots, a_{n_k-1} \in \Delta\mathcal{A}$, not all zero,*

  $$\sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}} \notin \mathbb{V}_{\mathcal{I}_k},$$

  as long as $\boldsymbol{h} \neq \boldsymbol{0}$.

The authors are with the Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716, USA (e-mail: {guo,xxia}@ee.udel.edu). This work was supported in part by the Air Force Office of Scientific Research (AFOSR) under Grant No. FA9550-08-1-0219.

When the received signals are decoded using the PIC-SIC group decoding with the ordering (22) in [1], each dimension-reduced system derived during the decoding process (i.e., the STBC with the PIC-SIC group decoding) has power gain order $n_r \cdot n_t$ if and only if the following two conditions are satisfied:

- *for any two different codewords $\boldsymbol{X}, \tilde{\boldsymbol{X}} \in \mathcal{X}$, $\Delta \boldsymbol{X} \triangleq \boldsymbol{X} - \tilde{\boldsymbol{X}}$ has the full rank property, i.e., the code $\mathcal{X}$ achieves full diversity with the ML receiver;*
- *at each decoding stage, for $\boldsymbol{G}_{\mathcal{I}_{i_k}}$, which corresponds to the current to-be decoded symbol group $\boldsymbol{x}_{i_k}$, any non-zero linear combination over $\Delta\mathcal{A}$ of the vectors in $\boldsymbol{G}_{\mathcal{I}_{i_k}}$ does not belong to the space linearly spanned by all the vectors in the group $[\boldsymbol{G}_{\mathcal{I}_{i_{k+1}}}, \ldots, \boldsymbol{G}_{\mathcal{I}_{i_{N-1}}}]$ as long as $\boldsymbol{h} \neq \mathbf{0}$.*

For the PIC group decoding, the revised second condition is that, for any $a_0, a_1, \ldots, a_{n_k-1} \in \Delta\mathcal{A}$, $a_j, j = 0, 1, \ldots, n_k-1$, not all zero, we have that

$$\sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}} \notin \mathbb{V}_{\mathcal{I}_k}.$$

Thus, we have

$$\boldsymbol{Q}_{\mathcal{I}_k} \left(\sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}} \right) \neq \sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}}.$$

Then the remaining of the proof follows the proof of Theorem 1 in [1]. The necessity proof is similar to that in the proof of Theorem 1 in [1]. Similar argument for the PIC-SIC group decoding part holds too.

Note that the first condition, i.e., the full rank property, in Theorem 2 is equivalent to the linear independence of all the column vectors in all the groups $\boldsymbol{G}_{\mathcal{I}_0}, \boldsymbol{G}_{\mathcal{I}_1}, \ldots, \boldsymbol{G}_{\mathcal{I}_{N-1}}$ over $\Delta\mathcal{A}$ for a signal constellation $\mathcal{A}$ as shown in [1]. This implies that the above linear combination $\sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}}$ is not a linear combination of the vectors in the remaining vector groups over $\Delta\mathcal{A}$. However, this does not imply that $\sum_{j=0}^{n_k-1} a_j \boldsymbol{g}_{i_{k,j}}$ is not a linear combination of the vectors in the remaining vector groups over the complex field $\mathbb{C}$, i.e., it may belong to $\mathbb{V}_{\mathcal{I}_k}$. Clearly, the revised second condition in Theorem 2 is stronger than the vector group independence, the original second condition, when a group has more than one element and there are more than one groups. Also, for the linear receiver, i.e., every group has only one element, and the ML decoding, i.e., there is only one group, the revised and the original second conditions are the same.

It is not difficult to check that all the existing design examples in [1] and other related ones do satisfy the above new second condition. This implies that they do achieve full diversity when the PIC (or PIC-SIC) group decoding is used at the receiver.

References

[1] X. Guo and X.-G. Xia, “On full diversity space-time block codes with partial interference cancellation group decoding,” *IEEE Trans. on Information Theory*, vol. 55, no. 10, pp. 4366-4385, Oct. 2009.