Corrections

Correction to "Bounds on Packings of Spheres in the Grassmann Manifold"

Alexander Barg, *Senior Member, IEEE,* and Dmitry Yu. Nogin

In [1], the density of the Haar measure for the case of $G_{k,n}(\mathbb{C})$ was cited incorrectly from [2, eq. (A18)]. The corrections affect the last displayed equation before (10) which should have the form

$$K(k,n) = 2^k \prod_{i=1}^{k} \frac{(n-i)!}{((i-1)!)^2 (n-k-i)!}$$

and (11) which in the complex case should read

$$J_k = \underset{\substack{0<x_k<\cdots<x_1<1 \\ \|x\|_2\le\delta}}{\int\cdots\int} (x_1x_2\dots x_k)^{2(n-2k)+1} \times \prod_{i<j} \left(x_i^2 - x_j^2\right)^2 dx_1 \dots dx_k. \quad (11)$$

Thus, the volume of the ball of radius δ in $G_{k,n}(\mathbb{C})$ equals $K(k,n)J_k$. Applying Theorem 3 to (11), we again obtain the complex case of Theorems 1 and 2 of [1], so this error does not affect the main results of the paper.

This error was pointed out to us independently by David Love and Oliver Henkel.

References

[1] A. Barg and D. Nogin, "Bounds on packings of spheres in the Grassmann manifold," *IEEE Trans. Inf. Theory*, vol. 48, no. 9, pp. 2450–2454, Sep. 2002.

[2] B. Hassibi, B. Hochwald, and T. Marzetta, "Space-time autocoding," *IEEE Trans. Inf. Theory*, vol. 47, no. 7, pp. 2761–2781, Nov. 2001.

Manuscript received December 18, 2004.

A. Barg is with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742 USA (e-mail: abarg@umd.edu)

D. Yu. Nogin is with the IPPI RAN, Bol'shoj Karetnyj 19, Moscow 101447, Russia (e-mail nogin@iitp.ru).

Communicated by C. Carlet, Associate Editor for Coding Theory.

Digital Object Identifier 10.1109/TIT.2005.850231

Correction to the Definition of Diversity Product in "On Optimal Multilayer Cyclotomic Space–Time Code Designs"

Genyuan Wang and Xiang-Gen Xia, *Senior Member, IEEE*

I. A Corrected Definition

With the definition of diversity product $d_{\min}(G_1,\dots,G_L)$ in (13) of an L-layer cyclotomic space–time code $X(G_1,\dots,G_L)$ from a composed complex lattice $\Gamma_{nL}(G_1,\dots,G_L)$ in [1]

$$d_{\min}(G_1,\dots,G_L) = \min_{[\mathbf{x}_1,\dots,\mathbf{x}_{nL}]^T \ne [0,\dots,0]^T} |\det(X(G_1,\dots,G_L))|, \quad (1)$$

Lemma 1 and Lemma 2 in [1] do not hold due to the normalization problem, i.e., a scaled lattice $\Gamma_{nL}(aG_1,\dots,aG_L)$ by a constant a may not be the same as itself $\Gamma_{nL}(G_1,\dots,G_L)$

$$\frac{d_{\min}(aG_1,\dots,aG_L)}{\prod_{l=1}^{L} |\det(aG_l)|\cdot|\det(K_l)|^{n/2}} \ne \frac{d_{\min}(G_1,\dots,G_L)}{\prod_{l=1}^{L} |\det(G_l)|\cdot|\det(K_l)|^{n/2}} \quad (2)$$

which is certainly not proper. In order for the following ratio for an L-layer cyclotomic space–time code $X(G_1,\dots,G_L)$

$$\frac{d_{\min}(G_1,\dots,G_L)}{\prod_{l=1}^{L} |\det(G_l)|\cdot|\det(K_l)|^{n/2}} \quad (3)$$

to have the scale invariability (normalization), the above diversity product definition in (1) used in [1] can be changed into

$$d_{\min}(G_1,\dots,G_L) \triangleq \min_{[\mathbf{x}_1,\dots,\mathbf{x}_{nL}]^T \ne [0,\dots,0]^T} |\det(X(G_1,\dots,G_L))|^L. \quad (4)$$

With the above corrected definition of $d_{\min}(G_1,\dots,G_L)$, Lemma 1 and Lemma 2 in [1] hold and criterion (3) for an L-layer cyclotomic space–time code $X(G_1,\dots,G_L)$ does not change in terms of a constant scaling factor, i.e., $X(G_1,\dots,G_L)$ and $aX(G_1,\dots,G_L) = X(aG_1,\dots,aG_L)$ for any nonzero constant a are the same in terms of criterion (3).

Manuscript received March 25, 2005. This work was supported in part by the Air Force Office of Scientific Research (AFOSR) under Grants F49620-02-1-0157 and FA9550-05-1-0161, and the National Science Foundation under Grant CCR-0097240 and CCR-0325180.

The authors are with the Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716 USA (e-mail: gwang@ee.udel.edu; xxia@ee.udel.edu).

Communicated by Ø. Ytrehus, Associate Editor for Coding Techniques.

Digital Object Identifier 10.1109/TIT.2005.850232

II. The Results and Proofs in [1] Still Hold

It is clear that the above corrected $d_{\min}(G_1, \ldots, G_L)$ in (4) and the one in (1) used in [1] coincide for a single layer code, i.e., for the case when $L = 1$. Furthermore, all optimal L-layer cyclotomic space–time codes presented in [1] are over either Eisenstein lattices or Gaussian lattices and their diversity products are always 1, i.e., $d_{\min}(G_1, \ldots, G_L) = 1$. On the other hand, it is not hard to see that diversity products of L-layer cyclotomic space–time codes over other cyclotomic lattices are not above 1, i.e., $d_{\min}(G_1, \ldots, G_L) \leq 1$. Therefore, raising the power in the corrected definition in (4) compared to the one in (1) used in [1] does not change any optimality result (or proof) obtained (or presented) in [1].

In conclusion, with the above changed definition (4) of diversity product (for convenience, we still call it the diversity product), all the optimality results and the corresponding proofs in [1] still hold. The change does not affect any other results or proofs in [1] either.

Acknowledgment

The authors want to thank Haiquan Wang for noticing the scaling problem.

References

[1] G. Wang and X.-G. Xia, "On optimal multilayer cyclotomic space–time code designs," *IEEE Trans. Inf. Theory*, vol. 51, no. 3, pp. 1102–1135, Mar. 2005.