

Some Super-Orthogonal Space-Time Trellis Codes Based on Non-PSK MTCM

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Abstract—Super-orthogonal space-time trellis codes recently proposed in the literature are space-time trellis codes of full diversity and high rates systematically constructed by concatenating orthogonal space-time codes and multiple trellis coded modulation (MTCM). However, the existing MTCM only has designs for phase-shift keying (PSK) signals. In this paper, MTCM is extended from PSK signals to non-PSK signals in some special cases. With obtained constellations of MTCM, several super-orthogonal space-time trellis codes for two transmit antennas are presented. The 2- and 4-state codes have a simple mathematical expression for the coding gain distance (CGD), or diversity products. At rates 2.5, 3, 3.5, 4 bits/s/Hz, the newly proposed codes outperform the existing ones.

Index Terms—Alamouti's scheme, coding gain distance (CGD), lattices, multiple trellis coded modulation (MTCM), orthogonal design, space-time trellis codes.

I. INTRODUCTION

SPACE-TIME coding techniques are widely discussed to combat fading in wireless communication links, [1]–[3], etc. However, due to the intractabilities of the design criteria for space-time codes, systematic designs of good properties are of particular interest. Recently, [4]–[6], and proposed a systematic design of space-time trellis codes. In [4]–[6], it is shown that super-orthogonal space-time trellis codes can be obtained from the existing multiple trellis coded modulations (MTCM) if orthogonal designs are used as the modulators. Furthermore, [4]–[6] introduced the concepts of constellation expansion and set-partitioning [7] into space-time coding to achieve high rates and quality performance. In [8] and [4], the authors proposed the equal eigenvalue criterion to tackle the design problem.

On the other hand, the existing MTCM [9] only has designs from phase-shift keying (PSK) signals and PSK signals are inefficient at high rates. In this paper, we first present MTCM designs from spectrally efficient constellations (non-PSK constellations). Basically, these MTCM constellations are obtained from the partitioning of lattices as in [10], [11]. However, a new method is adopted to increase the minimum intracoset squared

distance. Based on the new MTCM designs, several super-orthogonal space-time trellis codes are then proposed. The resultant codes show significant gains at high rates. In addition, for error events of arbitrary length, our 2- and 4-state codes are the optimal in the sense of the equal eigenvalue criterion.

In what follows, the following notations are adopted. \mathbf{C}^T , \mathbf{C}^H denote the transpose of matrix \mathbf{C} , the complex conjugate transpose of matrix \mathbf{C} , respectively; $\det\{\mathbf{C}\}$, $\text{tr}\{\mathbf{C}\}$ denote the determinant of matrix \mathbf{C} , the trace of matrix \mathbf{C} , respectively. $\Re\{c\}$ and c^* denote the real part and the complex conjugate of a complex number c , respectively. $\mathbf{E}\{c\}$ denotes the expectation of random variable c . \mathbb{N} , \mathbb{Z} , and \mathbb{R} denote natural numbers, integers, and real numbers, respectively. \mathbf{I}_n represents an $n \times n$ identity matrix.

II. SIGNAL AND SYSTEM MODEL

Consider a wireless communication system with 2 transmit and N_r receive antennas over a flat fading channel. Let $h_{m,n}$ be the fading coefficient of the channel between the m th transmit and the n th receive antenna. It is assumed that $h_{m,n}$ is quasi-static, i.e., constant over a frame of length L_t and independent from one frame to another. As in [3], it is modeled as independent complex Gaussian variable with zero mean and unit variance, i.e., $h_{m,n} \sim CN(0, 1)$. In the k th trellis transition, the codeword matrix label is

$$\mathbf{C}(k) = \begin{bmatrix} c_{2k+1,1} & c_{2k+1,2} \\ c_{2k+2,1} & c_{2k+2,2} \end{bmatrix} \quad (1)$$

where $c_{t,n}$ is the signal transmitted on the n th antenna at time t . It is assumed that

$$\frac{1}{2} \mathbf{E} \left\{ \sum_{l=1,2} |c_{2k+l,i}|^2 \right\} = E_s, \quad i = 1, 2 \quad (2)$$

which ensures normalized energy per channel use. It is assumed that $E_s = 1$ in the paper. Therefore, the $L_t \times 2$ transmitted signal matrix in a frame is

$$\mathbf{C} = [\mathbf{C}^T(0) \quad \mathbf{C}^T(1) \quad \dots \quad \mathbf{C}^T(N_t)]^T \quad (3)$$

where N_t is the total number of the trellis transitions in a frame and $N_t = L_t/2$, with an assumption of even L_t .

At the receiver side

$$\mathbf{X} = \mathbf{C}\mathbf{H} + \mathbf{W}. \quad (4)$$

In (4), \mathbf{X} is the received signal $L_t \times N_r$ matrix composed by the received signal $x_{t,n}$ at the n th antenna at time t ; $\mathbf{H} = (h_{m,n})_{2 \times N_r}$ is the channel coefficient matrix;

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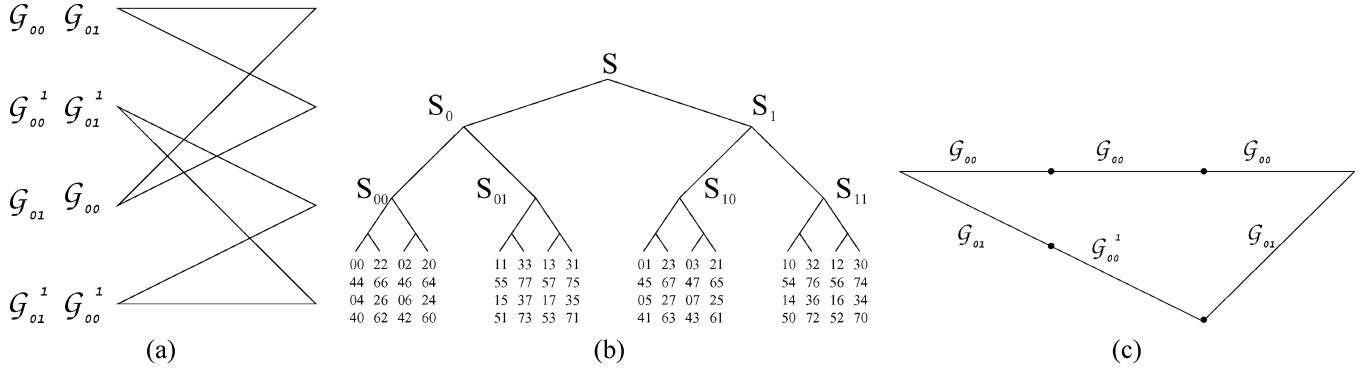


Fig. 1. (a) A 4-state, 2.5 bits/s/Hz code if 8-PSK signals are used. It also represents rate 2.5, 3, and 3.5 bits/s/Hz codes if the three constellations in Table I and the transform \mathbf{J} in (9) are used. (b) Set partitioning of 8-PSK signals in multiplicity two MTCM. The two integers represent a vector from the concatenated PSK constellations, i.e., pq refers to $(e^{j(2\pi p/8)}, e^{j(2\pi q/8)})$. (c) Typical paths different at three trellis transitions.

$\mathbf{W} = (w_{t,n})_{L_t \times N_r}$, where $w_{t,n}$ is the independent complex Gaussian noise at the n th receive antenna at time t and $w_{t,n} \sim CN(0, \sigma^2)$. Therefore, the signal-to-noise ratio (SNR) at each receive antenna is $2E_s/\sigma^2$.

In multiple antenna systems, there are the rank criterion and the coding gain distance (CGD) criterion as guidelines for the design of the space-time codes. Given two codewords \mathbf{C}_l and \mathbf{C}_l' , the rank criterion requires the difference matrix $\mathbf{B} = \mathbf{C}_l - \mathbf{C}_l'$ be full rank since the rank of \mathbf{B} decides the slope of the pairwise error probability (PEP) curve at high SNRs. Also the PEP of full rank codes decreases as the increase of the CGD of \mathbf{C} and \mathbf{D} , defined as $\sqrt{\det\{\mathbf{B}^H \mathbf{B}\}}$, [3], [5]. The minimum CGD, ξ , of the code is the overall minimum among all possible codeword pairs.

In [4]–[6], it is shown that the classical set partitioning in MTCM [9] can be used in the design of space-time trellis codes if Alamouti's scheme [12]

$$\mathbf{G}(u_1, u_2) \triangleq \begin{bmatrix} u_1 & u_2 \\ -u_2^* & u_1^* \end{bmatrix}$$

serves as the modulator in the system. That is, the codeword matrix label at trellis transition k is

$$\mathbf{C}(k) \in \mathcal{G} = \{\mathbf{G}(u_1, u_2) | u_1, u_2 \text{ are PSK signals}\}.$$

Under such a configuration, given two codeword matrix labels at a trellis transition, $\mathbf{C} = \mathbf{G}(u_1, u_2)$ and $\mathbf{D} = \mathbf{G}(v_1, v_2)$, the CGD is

$$\sqrt{\det\{(\mathbf{C} - \mathbf{D})^H(\mathbf{C} - \mathbf{D})\}} = |u_1 - v_1|^2 + |u_2 - v_2|^2.$$

This means that the CGD for each trellis transition in space-time trellis codes is equivalent to the Euclidean distance for MTCM. In other words, the partition criterion of \mathcal{G} is equivalent to that of the two concatenated PSK constellations. Thus, the set-partitioning in MTCM [9] can be applied here. For the set-partitioning of 8-PSK signals in MTCM, shown in Fig. 1(b), the corresponding set-partitioning of \mathcal{G} is

$$\mathcal{G}_{mn} = \{\mathbf{G}(u, v) | (u, v) \in S_{mn}\}, \quad \mathcal{G} = \bigcup_{m,n} \mathcal{G}_{mn} \quad (5)$$

where S_{mn} is a subconstellation in multiplicity two MTCM. Let $S = \bigcup_{m,n} S_{mn}$. Actually, S is a Cartesian product of two 8-PSK constellations. The average power of the constellation S is defined as

$$P \triangleq \frac{1}{|S|} \sum_{(u_1, u_2) \in S} (|u_1|^2 + |u_2|^2). \quad (6)$$

To achieve high rates while introducing redundancy, signal constellation \mathcal{G} is expanded to $\{\mathcal{G} \cup \mathcal{G}^1 \cup \mathcal{G}^2 \dots\}$ in [4]–[6]. Each set \mathcal{G}^i is a transformed orthogonal space-time constellation, defined as

$$\mathcal{G}^i = \{\mathbf{G}\mathbf{U}_i | \mathbf{G} \in \mathcal{G}, \mathbf{U}_i \mathbf{U}_i^H = \mathbf{I}\}, \quad i = 1, 2, \dots \quad (7)$$

where $\mathbf{U}_1, \mathbf{U}_2, \dots$ are unitary matrices. Space-time trellis codes can be designed based on the super-orthogonal space-time constellation $\{\mathcal{G} \cup \mathcal{G}^1 \cup \mathcal{G}^2 \dots\}$. This is analogous to the constellation expansion concept in the classical trellis coded modulations (TCM). Within each transformed space-time constellation \mathcal{G}^i , the set-partitioning is correspondent with that of PSK constellations in MTCM, as \mathcal{G} does. For example, in Fig. 1(a)

$$\mathcal{G}_{mn}^i = \{\mathbf{G}(u, v)\mathbf{U}_i | (u, v) \in S_{mn}\}, \quad i = 1 \quad (8)$$

where $\mathbf{U}_i = \text{diag}[1, e^{j(2\pi i/L)}]$ and $L = 8$ is the constellation size of PSK signals in use. The performance of codes in Fig. 1 depends on the structure of \mathcal{G} and its transformed constellation \mathcal{G}^1 .

Using the above constellation expansion and set-partitioning, a number of super-orthogonal space-time trellis codes were proposed in [4]–[6]. Although for codeword matrix labels at each trellis transition $\mathbf{C}(k) \in \mathcal{G}^i$, and $\mathbf{D}(k) \in \mathcal{G}^l$, $i \neq l$, the difference matrix $\mathbf{B}(k) = \mathbf{C}(k) - \mathbf{D}(k)$ might not be full rank, the difference matrix of two paths diverging from a state and re-merging to another state can be designed to be full rank.

Let us see the code of 4-state, 2.5 bits/s/Hz in Fig. 1. Fig. 1(b) shows the partitioning of S , a Cartesian product of two 8-PSK signals. \mathcal{G}_{mn} is the corresponding subconstellation, generated by (5), of the space-time constellation. \mathcal{G}_{mn}^1 is the transformed subconstellation obtained from (8). Fig. 1(c) shows typical paths different at three transitions. The difference matrix of two codeword matrix labels at the second trellis transition, $\mathbf{C}(2) = \mathbf{G}(u_3, u_4)$, $(u_3, u_4) \in S_{00}$, and $\mathbf{D}(2) = \mathbf{G}(v_3, v_4)\mathbf{U}$,

$(v_3, v_4) \in S_{00}$, is not full rank. However, the difference matrix of the following two paths is full rank:

$$\mathbf{C} = [\mathbf{G}^T(u_1, u_2) \quad \mathbf{G}^T(u_3, u_4) \quad \mathbf{G}^T(u_5, u_6)]^T,$$

$$(u_1, u_2), (u_3, u_4), (u_5, u_6) \in S_{00},$$

$$\mathbf{D} = [\mathbf{G}^T(v_1, v_2) \quad \mathbf{U}^T \mathbf{G}^T(v_3, v_4) \quad \mathbf{G}^T(v_5, v_6)]^T,$$

$$(v_1, v_2), (v_5, v_6) \in S_{01}, (v_3, v_4) \in S_{00}$$

both of which start from state zero and end at state zero. [4]–[6] proposed a set of rules to assign proper subconstellations to the proper states and branches to guarantee the full rank of paths that start from a state and remerge at another state. Thus full rank property of the code is guaranteed.

III. IMPROVED SPACE-TIME TRELLIS CODES

It is known that PSK signals are inefficient at high rates. The main effort in this paper is to exploit more efficient constellations to design super-orthogonal space-time trellis codes.

To design super-orthogonal space-time trellis codes, the first step is to partition the multidimensional constellation S into $N_g = 2^k$, $k \in \mathbb{N}$, subconstellations S_i , where index i can be thought of as a binary represented integer. Each S_i has size of $|S_i| = N_s$. In other words, it has N_s pairs (u_1, u_2) of complex numbers u_1 and u_2 . Therefore, N_g is the number of subconstellations. N_s is the size of subconstellations. The average power of the constellation S , the union of all S_i , is P defined in (6). Define

$$\tau^2(S_i, S_j) = \min_{(u_1, u_2) \in S_i, (v_1, v_2) \in S_j} |u_1 - v_1|^2 + |u_2 - v_2|^2$$

as the minimum squared *interdistance* between subconstellations S_i and S_j , and

$$\tau^2(S_i) = \min_{(u_1, u_2), (v_1, v_2) \in S_i, (u_1, u_2) \neq (v_1, v_2)} |u_1 - v_1|^2 + |u_2 - v_2|^2$$

as the minimum squared *intradistance* of the subconstellations. In most cases, all $\tau^2(S_i)$, $1 \leq i \leq N_g$, are the same. Therefore, in the paper, the minimum squared intradistance is referred to as τ^2 .

In [4]–[6], the vector (u_1, u_2) is chosen from the Cartesian product of two PSK constellations, as in [9]. Actually, (u_1, u_2) can also be designed from four-dimensional (4-D) lattices in \mathbb{R}^4 . The first two dimensions of a point in the lattice form u_1 , as the real and the imaginary part, respectively. The last two dimensions of the point are mapped to u_2 in the same way. In the following, designs from the 4-D half integer grid $\mathbb{Z}^4 + (1/2, 1/2, 1/2, 1/2)$ are shown. Each subconstellation S_i is constructed by points in the 4-D subsets, or cosets. The 4-D grid can be denoted as the Cartesian product of two two-dimensional (2-D) half integer grid, (Λ, Λ) , where Λ denotes the 2-D half integer grid, shown in Fig. 2. The partition of the 4-D grid can be performed through the partition of the constituent 2-D grids.

In Λ , the minimum squared Euclidean distance is 1. We can make a partition for Λ as for the 2-D square lattice in [13], shown in Fig. 2. The partition, denoted as Λ/Λ_1 , is two way. In the coset

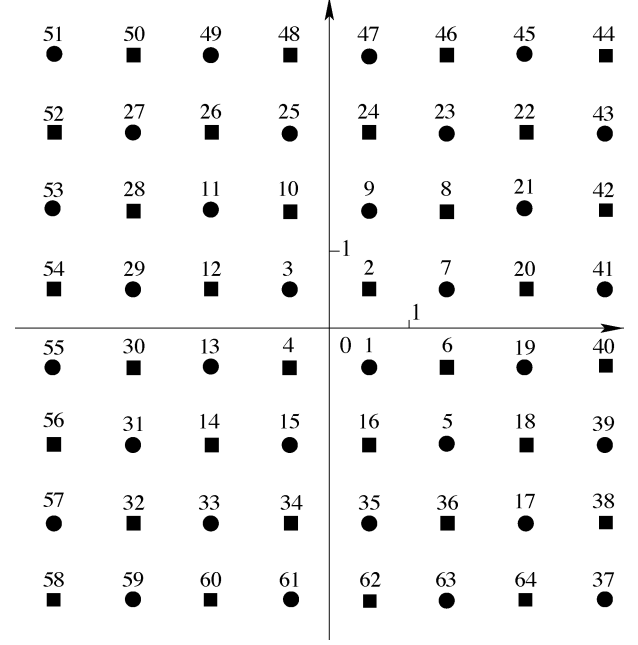


Fig. 2. The two-way partition of the 2-D half integer grid Λ . The coset Λ_1 is represented by points of circle and the coset Λ_2 is represented by points of square.

Λ_1 or Λ_2 , the minimum squared intradistance is 2. In the same way, Λ_1 , or Λ_2 , can be further partitioned.

With the partition method for Λ , the partition of the 4-D grid (Λ, Λ) can be obtained by alternatively partitioning the constituent grid Λ until N_g cosets are obtained. For example, if the desired $N_g = 4$, the cosets are (Λ_1, Λ_1) , (Λ_2, Λ_1) , (Λ_1, Λ_2) , and (Λ_2, Λ_2) . In each coset, the minimum squared intradistance τ^2 is 2. The final constellation can be obtained by taking N_s points of the least powers as in [10].

Since the obtained constellations are used in super-orthogonal space-time trellis codes and as shown later, the minimum CGD is upper-bounded by $2\tau^2/P$, $2\tau^2/P$ is to be maximized. Our method is to choose a threshold τ_d^2 for minimum squared intradistance τ^2 . The N_s points of the least powers and of at least τ_d^2 far away from each other are taken. Then, we vary τ_d^2 until the constellation with maximum $2\tau^2/P^2$ is found. The detailed algorithm is shown as follows.

- Step 1) Initialization. $P^{(0)} = 0$. Take kN_s points of the least powers from each coset to form subset C_{mn} , $k = 4$ to 6. Set $\tau_{(1)}^2 = \tau_{(0)}^2 = \tau^2(C_{mn})$ and $i = 1$.
- Step 2) Choose point \mathbf{p} of the least power from C_{mn} as the first point in $S_{mn}^{(i)}$ and mark the point \mathbf{p} .
- Step 3) Choose point \mathbf{q} of the least power among the unmarked points in C_{mn} . Mark this point \mathbf{q} in C_{mn} . If \mathbf{q} is at least $\tau_{(i)}^2$ far away from all points in $S_{mn}^{(i)}$, add \mathbf{q} to $S_{mn}^{(i)}$. Otherwise, just discard \mathbf{q} . Repeat Step 3 until N_s points are found for each $S_{mn}^{(i)}$.
- Step 4) Calculate $\tau_{(i)}^2/P^{(i)}$, where $P^{(i)}$ is the average power of $S^{(i)} = \bigcup_{m,n} S_{mn}^{(i)}$. If $(\tau_{(i)}^2/P^{(i)}) > (\tau_{(i-1)}^2/P^{(i-1)})$, proceed to Step 2 with $i = i + 1$ and $\tau_{(i)}^2 = \tau_{(i-1)}^2 + 1$. Otherwise, take $S^{(i-1)}$ as the final constellation S .

TABLE I
 NON-PSK MTCM CONSTELLATIONS

Constellations	N_s	P	τ^2	d_{free}^2 in 2-state MTCM	d_{free}^2 of designs in [9]	PAPR (dB)
I	16	3.5	4	1.7142	1.7574	1.55
II	32	5	4	1.2001	-	2.55
III	64	6.75	4	0.8889	0.4566	1.24

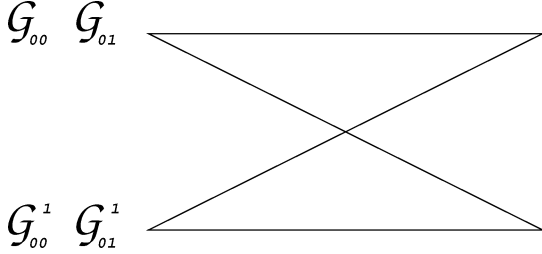


Fig. 3. 2-state codes: 2.5 bits/s/Hz, 3 bits/s/Hz, 3.5 bits/s/Hz, constructed from the three constellations in Table I.

If $\tau_{(i)}^2$ becomes large, then $P^{(i)}$ increases dramatically and the algorithm stops quickly. Therefore, the computation load is light. In fact, the constellations in Table I are all obtained within five iterations.

Using this searching algorithm, we obtain several constellations for MTCM, shown in Table I. The peak-to-average power ratio (PAPR) might be of concerns when the signal constellations change. However, for the three constellations listed, the PAPR are all within 3 dB as shown from Table I.

Since, for these three constellations in Table I, N_g and τ^2 are the same, Constellation I and Constellation II are subsets of Constellation III. Constellation III is enumerated in Appendix. Constellations I and II are composed by the N_s points of the least powers in S_{mn} of Constellation III. As shown in Table I, Constellation I is not superior to the corresponding MTCM design in [9] if it is used in the 2-state MTCM. However, since it has larger $2\tau^2/P$ than the constellations from 8-PSK signals, super-orthogonal space-time trellis codes of larger minimum CGDs can be obtained from it. Constellation III has larger d_{free}^2 with respect to the design in [9] if it is used in 2-state MTCM.

The new 2-state codes are shown in Fig. 3, where

$$\mathcal{G}_{mn}^1 = \{\mathbf{G}(u, v)\mathbf{J} | (u, v) \in S_{mn}\}$$

where

$$\mathbf{J} = \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix} \quad (9)$$

and S_{mn} is the constellation referred to in Table III. \mathbf{J} is obtained by maximizing the minimum CGD of length two paths, which start from a state and remerge at another state.

Fig. 1(a) can also represent the new 4-state 2.5, 3, and 3.5 bits/s/Hz codes if the new constellations in Table I and the transform \mathbf{J} in (9) are used. \mathbf{J} provides a simple expression for the CGDs of the 2- and 4-state codes in Fig. 3 and Fig. 1, as shown in the next subsection.

Fig. 4 shows the 8-state codes of rates 3 and 4 bits/s/Hz if Constellation I and III in Table I are used. In Fig. 4

$$\mathcal{G}_{mn}^i = \{\mathbf{G}(u, v)\mathbf{K}_i | (u, v) \in S_{mn}\}, \quad i = 1, 2, 3$$

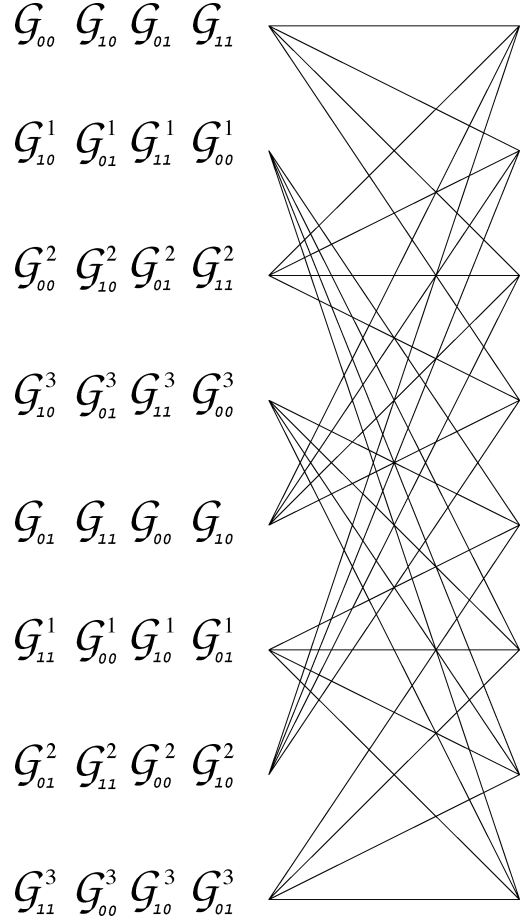


Fig. 4. 8-state codes: 3, 4 bits/s/Hz, constructed from Constellation I and Constellation III in Table I.

where

$$\mathbf{K}_1 = \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix}, \quad \mathbf{K}_2 = \begin{bmatrix} -j & 0 \\ 0 & j \end{bmatrix}, \quad \mathbf{K}_3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad (10)$$

and S_{mn} is the constellation referred to in Table III.

Since the new constellations are not PSK signals, for the energy normalization in (2), the transmitted signal matrix at trellis transition k is $\mathbf{C}(k) = \sqrt{2/P}\mathbf{G}(u_1, u_2)\mathbf{V}$, where $\mathbf{V} = \mathbf{I}, \mathbf{J}$, or \mathbf{K}_i according to the design in Fig. 3, Fig. 1(a), and Fig. 4.

A. CGDs of the New Codes

It is commonly known that the distance between two codewords along two trellis paths in a conventional TCM is the sum of the distances of the codewords in all the corresponding trellis branches. This additivity plays a key role in analyzing properties of a TCM. However, this additivity no longer holds for CGDs in a general space-time trellis code. The following result states that the additivity is indeed true for some of our newly proposed super-orthogonal space-time trellis codes.

Proposition 1: For two paths different at k trellis transitions

$$\begin{aligned} \mathbf{C} &= \left[(\mathbf{G}(u_1, u_2)\mathbf{V}_1^1)^T (\mathbf{G}(u_3, u_4)\mathbf{V}_2^1)^T \cdots \right. \\ &\quad \left. (\mathbf{G}(u_{2k-1}, u_{2k})\mathbf{V}_k^1)^T \right]^T \\ \mathbf{D} &= \left[(\mathbf{G}(v_1, v_2)\mathbf{V}_1^2)^T (\mathbf{G}(v_3, v_4)\mathbf{V}_2^2)^T \cdots \right. \end{aligned} \quad (11)$$

$$(\mathbf{G}(v_{2k-1}, v_{2k})\mathbf{V}_k^2)^T]^T \quad (12)$$

where \mathbf{V}_t^i is either \mathbf{I} or \mathbf{J} in (9), the CGD is

$$\sqrt{\det\{\mathbf{A}\}} = \sum_{i=1}^k \sqrt{\det\{\mathbf{A}_i\}} \quad (13)$$

where $\mathbf{A} = (\mathbf{C} - \mathbf{D})^H(\mathbf{C} - \mathbf{D})$, $\sqrt{\det\{\mathbf{A}_i\}}$ is the CGD of the two codeword matrix labels in the transition i , and

$$\mathbf{A}_i = (\mathbf{G}(u_{2i-1}, u_{2i})\mathbf{V}_i^1 - \mathbf{G}(v_{2i-1}, v_{2i})\mathbf{V}_i^2)^H \times (\mathbf{G}(u_{2i-1}, u_{2i})\mathbf{V}_i^1 - \mathbf{G}(v_{2i-1}, v_{2i})\mathbf{V}_i^2). \quad (14)$$

Proof: The CGD, by the definition, is

$$\sqrt{\det\{\mathbf{A}\}} = \left(\det \left\{ \sum_{i=1}^k \mathbf{A}_i \right\} \right)^{\frac{1}{2}}. \quad (15)$$

Since \mathbf{V}_t^i is unitary

$$\mathbf{A}_i = \left(\mathbf{G}(u_{2i-1}, u_{2i}) - \mathbf{G}(v_{2i-1}, v_{2i})\mathbf{V}_i^2 (\mathbf{V}_i^1)^H \right)^H \times \left(\mathbf{G}(u_{2i-1}, u_{2i}) - \mathbf{G}(v_{2i-1}, v_{2i})\mathbf{V}_i^2 (\mathbf{V}_i^1)^H \right). \quad (16)$$

Furthermore, \mathbf{V}_t^i can only be one of the two matrices, \mathbf{I} and \mathbf{J} . Therefore, $\mathbf{V}_i^2 (\mathbf{V}_i^1)^H$ also only has three possible outcomes \mathbf{I} , \mathbf{J} , \mathbf{J}^H .

Since \mathbf{I} , \mathbf{J} , and \mathbf{J}^H are diagonal, we may assume $\mathbf{V}_i^2 (\mathbf{V}_i^1)^H = \mathbf{T} = \text{diag}[t_1 \ t_2]$. Therefore

$$\begin{aligned} \mathbf{A}_i &= (|u_{2i-1}|^2 + |u_{2i}|^2 + |v_{2i-1}|^2 + |v_{2i}|^2) \mathbf{I} \\ &\quad - \mathbf{T}^H \mathbf{G}^H(v_{2i-1}, v_{2i}) \mathbf{G}(u_{2i-1}, u_{2i}) \\ &\quad - \mathbf{G}^H(u_{2i-1}, u_{2i}) \mathbf{G}(v_{2i-1}, v_{2i}) \mathbf{T} \\ &= (|u_{2i-1}|^2 + |u_{2i}|^2 + |v_{2i-1}|^2 + |v_{2i}|^2) \mathbf{I} - \mathbf{A}_{i'}, \end{aligned} \quad (17)$$

where (see the equation at bottom of page).

If $\mathbf{T} = \mathbf{I}$, \mathbf{J} , or \mathbf{J}^H , $t_1 = t_2^*$. Therefore

$$\mathbf{A}_i = (|u_{2i-1} - t_1 v_{2i-1}|^2 + |u_{2i} t_1 - v_{2i}|^2) \mathbf{I}. \quad (18)$$

Therefore

$$\mathbf{A} = \frac{1}{2} \sum_{i=1}^k \text{tr} \{ \mathbf{A}_i^H \mathbf{A}_i \} \mathbf{I} \quad (19)$$

and

$$\sqrt{\det\{\mathbf{A}\}} = \sum_{i=1}^k \sqrt{\det\{\mathbf{A}_i\}}.$$

In [8] and [4], the authors proposed the equal eigenvalue criterion, in which the optimal \mathbf{A} should be semi-unitary, i.e., $\mathbf{A} = (1/2)\text{tr}\{\mathbf{A}^H \mathbf{A}\} \mathbf{I}$, to maximize the CGD. From (19), it is

shown that, for event events of any length, the new 2- and 4-state codes are optimal in the sense of the equal eigenvalue criterion. Also, via Proposition 1, the minimum CGDs of the new 2- and 4-state codes can be derived.

Proposition 2: For the 2-state codes in Fig. 3, the minimum CGD, ξ , of the codes is $2\tau^2/P$, where P is the average power of the constellation and τ^2 is the minimum squared intradistance of S_{mn} , listed in Table III.

Proof: For the codes in Fig. 3, the paths diverging from state zero and remerging at state zero are considered as a typical case. The other cases can be dealt with in the same way. If the two paths differ only at one trellis transition, two different codewords are of the forms

$$\begin{aligned} \mathbf{C} &= \sqrt{\frac{2}{P}} \mathbf{G}(u_1, u_2), \\ \mathbf{D} &= \sqrt{\frac{2}{P}} \mathbf{G}(v_1, v_2), \\ &(u_1, u_2), (v_1, v_2) \in S_{00}, \text{ but } (u_1, u_2) \neq (v_1, v_2). \end{aligned}$$

Therefore

$$\mathbf{A} = \frac{2}{P} (|u_1 - v_1|^2 + |u_2 - v_2|^2) \mathbf{I} \quad (20)$$

and, thus, the minimum CGD of the parallel paths is $2\tau^2/P$.

If the two paths differ only at two trellis transitions, the two codewords are

$$\begin{aligned} \mathbf{C} &= \sqrt{\frac{2}{P}} [\mathbf{G}^T(u_1, u_2) \ \mathbf{G}^T(u_3, u_4)]^T, \\ &(u_1, u_2), (u_3, u_4) \in S_{00}, \\ \mathbf{D} &= \sqrt{\frac{2}{P}} [\mathbf{G}^T(v_1, v_2) \ \mathbf{J}^T \mathbf{G}^T(v_3, v_4)]^T, \\ &(v_1, v_2), (v_3, v_4) \in S_{01}. \end{aligned}$$

So $\mathbf{A}_1 = (2/P)(|u_1 - v_1|^2 + |u_2 - v_2|^2) \mathbf{I}$ and $\mathbf{A}_2 = (2/P)(|ju_3 - v_3|^2 + |u_4 - jv_4|^2) \mathbf{I}$. It is easy to show that

$$\min_{(u_1, u_2) \in S_{00}, (v_1, v_2) \in S_{01}} \sqrt{\det\{\mathbf{A}_1\}} = \frac{2\tau^2(S_{00}, S_{01})}{P} \quad (21)$$

where $\tau^2(S_{00}, S_{01})$ is the minimum squared interdistance between S_{00} and S_{01} .

To study the property of \mathbf{A}_2 , we assume $u_3 = (p+0.5+j(q-0.5))$, $p+q = 2k$, and $v_3 = (\nu+0.5+j(\mu-0.5))$, $\nu+\mu = 2m$, $p, q, \nu, \mu, k, m \in \mathbb{Z}$, because $u_3, v_3 \in \Lambda_1$. Therefore

$$|ju_3 - v_3|^2 = |j(p - \mu + 1) - q - \nu|^2. \quad (22)$$

If (22) equals zero, $p + q = 2k$ and $\nu + \mu = 2m$ can not hold. Furthermore, p, q, ν, μ are integers, so $|ju_3 - v_3|^2 \geq 1$. With the same argument, $|u_4 - jv_4|^2 \geq 1$. Therefore,

$$\min_{(u_3, u_4), (v_3, v_4) \in S_{00}} \sqrt{\det\{\mathbf{A}_2\}} = \frac{4}{P}. \quad (23)$$

$$\mathbf{A}_{i'} = \begin{bmatrix} 2\Re\{t_1 u_{2i-1}^* v_{2i-1} + t_1 u_{2i} v_{2i}^*\} & (t_2 - t_1^*)(u_{2i-1}^* v_{2i} - u_{2i} v_{2i-1}^*) \\ (t_2^* - t_1)(u_{2i-1} v_{2i}^* - u_{2i}^* v_{2i-1}) & 2\Re\{t_2^* u_{2i-1}^* v_{2i-1} + t_2^* u_{2i} v_{2i}^*\} \end{bmatrix}.$$

So, from (21), (23), and (13), the minimum CGD of length two paths is

$$\frac{2}{P} (\tau^2(S_{00}, S_{01}) + 2). \quad (24)$$

For paths different at $k \geq 3$ trellis transitions, two codewords take the forms

$$\begin{aligned} \mathbf{C} &= \sqrt{\frac{2}{P}} [\mathbf{G}^T(u_1, u_2) \quad \cdots \quad \mathbf{G}^T(u_{2k-1}, u_{2k})]^T, \\ &\quad (u_1, u_2), (u_{2k-1}, u_{2k}) \in S_{00}, \\ \mathbf{D} &= \sqrt{\frac{2}{P}} [\mathbf{G}^T(v_1, v_2) \quad \cdots \quad \mathbf{J}^T \mathbf{G}^T(v_{2k-1}, v_{2k})]^T, \\ &\quad (v_1, v_2), (v_{2k-1}, v_{2k}) \in S_{01}. \end{aligned}$$

Therefore, from Proposition 1

$$\sqrt{\det\{\mathbf{A}\}} = \sum_{i=1}^k \sqrt{\det\{\mathbf{A}_i\}} \geq \sqrt{\det\{\mathbf{A}_1\}} + \sqrt{\det\{\mathbf{A}_k\}} \quad (25)$$

since all $\sqrt{\det\{\mathbf{A}_i\}} \geq 0$, for $1 \leq i \leq k$. Again, when the same argument for (24) is applied to (25), the minimum CGD of length $k \geq 3$ paths is

$$\xi_{3+} \geq \frac{2}{P} (\tau^2(S_{00}, S_{01}) + 2). \quad (26)$$

Therefore, ξ_{3+} is always no less than that of length two paths. As a result, the minimum CGD should be the minimum between the minimum CGD of parallel paths and the minimum CGD of length two paths, i.e.,

$$\xi = \frac{2}{P} \min \{ \tau^2, \tau^2(S_{00}, S_{01}) + 2 \}.$$

For the three constellations in Table I, $\tau^2(S_{00}, S_{01}) = 2$ and $\tau^2 = 4$. Therefore, $\tau^2 = \tau^2(S_{00}, S_{01}) + 2$. That is, $\xi = 2\tau^2/P$. ■

For the new 4-state codes in Fig. 1(a), we also consider the paths diverging from state zero and remerging at state zero. The minimum CGD of parallel paths is still $2\tau^2/P$ as in the proof for Proposition 2. There are no error events of length two for the trellis in Fig. 1. For the paths of length k , two codewords are in the forms of

$$\begin{aligned} \mathbf{C} &= \sqrt{\frac{2}{P}} [\mathbf{G}^T(u_1, u_2) \quad \cdots \quad \mathbf{G}^T(u_{2k-1}, u_{2k})]^T, \\ &\quad (u_1, u_2), (u_{2k-1}, u_{2k}) \in S_{00}, \\ \mathbf{D} &= \sqrt{\frac{2}{P}} [\mathbf{G}^T(v_1, v_2) \quad \cdots \quad \mathbf{G}^T(v_{2k-1}, v_{2k})]^T, \\ &\quad (v_1, v_2), (v_{2k-1}, v_{2k}) \in S_{01}. \end{aligned}$$

The CGD of the two codewords is

$$\sqrt{\det\{\mathbf{A}\}} \geq \sqrt{\det\{\mathbf{A}_1\}} + \sqrt{\det\{\mathbf{A}_k\}}. \quad (27)$$

TABLE II
MINIMUM CGDS, ξ , OF NEW CODES

Code	Rate(bits/s/Hz)	Number of states	ξ	ξ of [4] [5] [6]
Fig. 3	2.5	2	2.29	-
Fig. 1	2.5	4	2.29	2
Fig. 3	3	2	1.60	-
Fig. 1	3	4	1.60	1.64
Fig. 4	3	8	2.29	2
Fig. 3	3.5	2	1.19	-
Fig. 1	3.5	4	1.19	0.58 ¹
Fig. 4	4.0	8	1.19	0.58 ¹

Since

$$\begin{aligned} \mathbf{A}_1 &= \frac{2}{P} (|u_1 - v_1|^2 + |u_2 - v_2|^2) \mathbf{I}, \\ &\quad (u_1, u_2) \in S_{00}, (v_1, v_2) \in S_{01}, \\ \mathbf{A}_k &= \frac{2}{P} (|u_{2k-1} - v_{2k-1}|^2 + |u_{2k} - v_{2k}|^2) \mathbf{I}, \\ &\quad (u_{2k-1}, u_{2k}) \in S_{00}, (v_{2k-1}, v_{2k}) \in S_{01}, \end{aligned}$$

we have

$$\begin{aligned} &\min_{(u_1, u_2) \in S_{00}, (v_1, v_2) \in S_{01}} \det\{\mathbf{A}_1\} \\ &= \min_{(u_{2k-1}, u_{2k}) \in S_{00}, (v_{2k-1}, v_{2k}) \in S_{01}} \det\{\mathbf{A}_k\}. \quad (28) \end{aligned}$$

Furthermore

$$\min_{(u_1, u_2) \in S_{00}, (v_1, v_2) \in S_{01}} \sqrt{\det\{\mathbf{A}_1\}} = \frac{2\tau^2(S_{00}, S_{01})}{P}. \quad (29)$$

So, from (13), (27)–(29), the minimum CGD of the length k paths $\xi_{3+} \geq (4\tau^2(S_{00}, S_{01})/P)$. For all three constellations in Table I, $\tau^2 = 4$ and $\tau^2(S_{00}, S_{01}) = 2$. Therefore, $2\tau^2/P = (4\tau^2(S_{00}, S_{01})/P) \leq \xi_{3+}$. Thus, we have the following result.

Proposition 3: For the new 4-state codes, the minimum CGD of the code is $2\tau^2/P$, where P is the average power of the constellation and τ^2 is the minimum squared intradistance of S_{mn} , listed in Table III.

For all the new codes, the minimum CGDs are tabulated in Table II. Except the 3 bits/s/Hz code in Fig. 1, the new codes have larger minimum CGDs than the existing codes in [4]–[6]. Also, as the partitioning of MTCM [9], if N_g is larger, constellations with larger $2\tau^2/P$ can be obtained. However, more states are needed to construct super-orthogonal space-time trellis codes with corresponding minimum CGDs.

IV. SIMULATION RESULTS

The simulation results are presented in this section to show the performance of the proposed codes. The channels in the simulation are quasi-static. The length of each frame is 130 symbols. The curves of frame error rate (FER) are obtained by averaging over 50 000 frames. The simulated systems use two transmit antennas and one receive antenna.

At 2.5 bits/s/Hz as shown in Fig. 5, our code gains 0.5 dB over the same rate code in [4]–[6] with the same number of states. At 3 bits/s/Hz shown in Fig. 6, our 4-state code shows 1 dB gain over the existing 8-state code in [5] and [6]. At 3.5, 4 bits/s/Hz shown in Fig. 7, our code has about 2 dB gain over the same rate code in [4]–[6] with 16-PSK signals. Admittedly, our codes

TABLE III
MTCM CONSTELLATION III, (p, q) REFERS TO A PAIR OF POINTS INDEXED BY p AND q IN FIG. 2. CONSTELLATIONS I, II ARE COMPOSED BY THE 16, 32 POINTS OF THE LEAST POWERS IN S_{mn} OF CONSTELLATION III, RESPECTIVELY

$S_{00} \subset \Lambda_1 \otimes \Lambda_1$	$S_{01} \subset \Lambda_2 \otimes \Lambda_2$	$S_{10} \subset \Lambda_1 \otimes \Lambda_2$	$S_{11} \subset \Lambda_2 \otimes \Lambda_1$
(3,3),(1,1),(3,7),(3,15)	(2,2),(4,4),(10,4),(12,2)	(3,2),(1,4),(3,12),(3,16)	(2,3),(4,1),(10,1),(12,3)
(9,1),(7,3),(15,3),(13,1),(1,9),(1,13)	(2,12),(2,16),(4,10),(4,6),(16,2),(6,4)	(9,4),(7,2),(15,2),(13,4),(1,10),(1,6)	(2,7),(2,15),(4,9),(4,13),(16,3),(6,1)
(11,1),(3,5),(5,3),(1,11),(9,9),(9,13)	(8,4),(2,14),(14,2),(4,8),(10,10),(10,6)	(11,4),(3,14),(5,2),(1,8),(9,10),(9,6)	(8,1),(2,5),(14,3),(4,11),(10,9),(10,13)
(7,7),(7,15),(15,7),(15,15),(13,9),(13,13)	(12,12),(12,16),(16,12),(16,16),(6,10),(6,6)	(7,12),(7,16),(15,12),(15,16),(13,10),(13,6)	(12,7),(12,15),(16,7),(16,15),(6,9),(6,13)
(25,3),(29,3),(3,25),(3,29),(35,1),(19,1)	(24,2),(20,2),(2,24),(2,20),(34,4),(20,4)	(25,2),(29,2),(3,24),(3,20),(35,4),(19,4)	(24,3),(20,3),(2,25),(2,29),(34,1),(20,1)
(1,35),(1,19),(3,23),(3,31),(23,3),(21,1)	(4,34),(4,20),(26,2),(28,4),(2,26),(2,18)	(1,34),(1,20),(3,26),(3,18),(23,2),(21,4)	(4,35),(4,19),(26,3),(28,1),(2,23),(2,31)
(33,1),(31,3),(1,21),(1,33),(11,9),(11,13)	(4,28),(4,36),(36,4),(18,2),(10,8),(12,14)	(33,4),(31,2),(1,28),(1,36),(11,10),(11,6)	(4,21),(4,33),(36,1),(18,3),(10,11),(12,5)
(9,11),(7,5),(15,5),(13,11),(5,7),(5,15)	(8,10),(8,6),(14,12),(14,16),(16,14),(6,8)	(9,8),(7,14),(15,14),(13,8),(5,12),(5,16)	(8,9),(8,13),(14,7),(14,15),(16,5),(6,11)
(25,7),(25,15),(29,7),(29,15),(9,35),(9,19)	(10,34),(10,20),(12,24),(12,20),(24,12),(24,16)	(25,12),(25,16),(29,12),(29,16),(9,34),(9,20)	(10,35),(10,19),(12,25),(12,29),(24,7),(24,15)
(7,25),(7,29),(15,25),(15,29),(13,35),(13,19)	(20,12),(20,16),(34,10),(34,6),(20,10),(20,6)	(7,24),(7,20),(15,24),(15,20),(13,34),(13,20)	(20,7),(20,15),(34,9),(34,13),(20,9),(20,13)
(35,9),(35,13),(19,9),(19,13),(11,11),(5,5)	(16,24),(16,20),(6,34),(6,20),(8,8),(14,14)	(35,10),(35,6),(19,10),(19,6),(11,8),(5,14)	(16,25),(16,29),(6,35),(6,19),(8,11),(14,5)

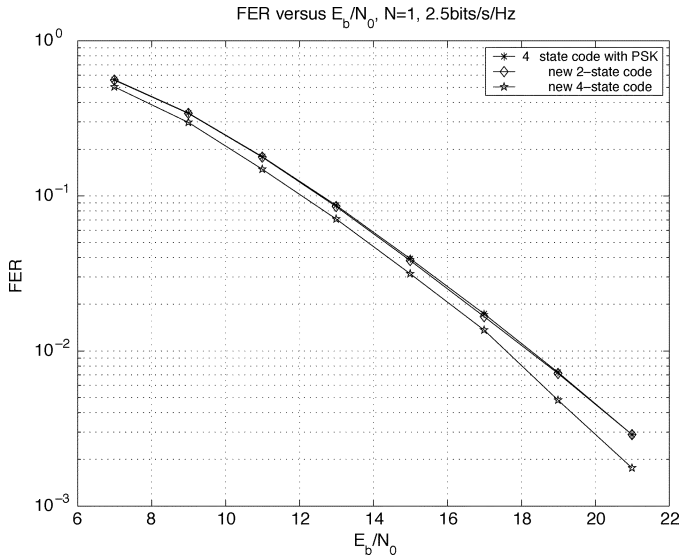


Fig. 5. Performance comparison of codes: rate 2.5 bits/s/Hz.

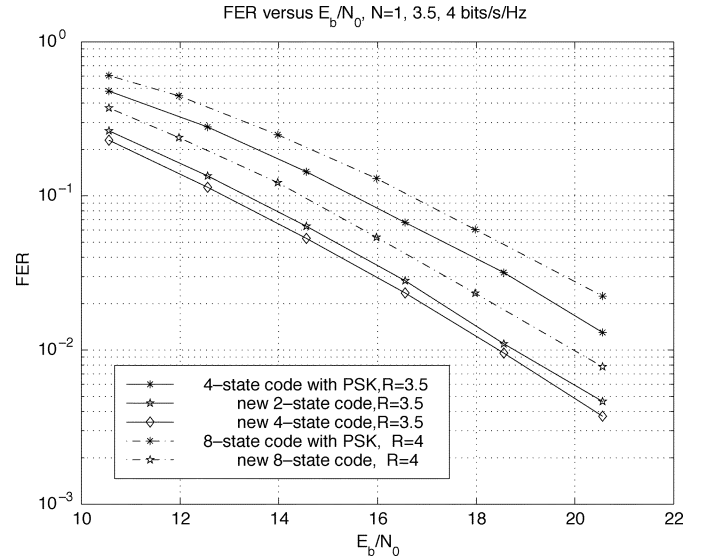


Fig. 7. Performance comparison of codes: rates 3.5 and 4 bits/s/Hz.

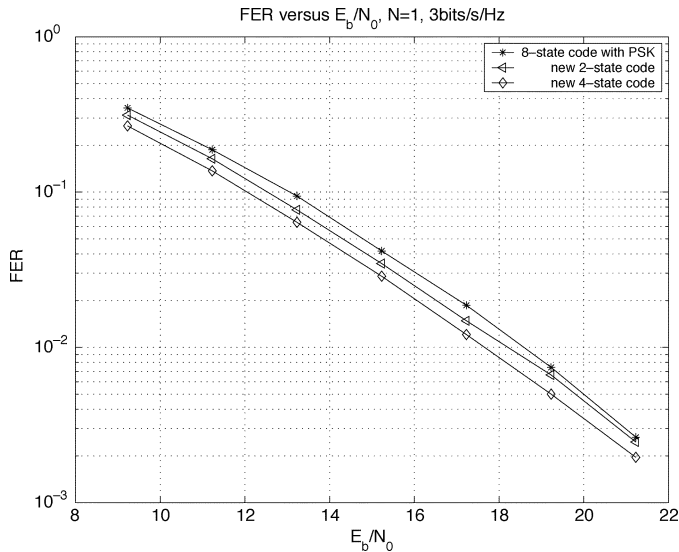


Fig. 6. Performance comparison of codes: rate 3 bits/s/Hz.

have higher decoding complexity compared with the codes with PSK constellations when using same number of states.

It is also shown the distance spectrum is important for the space-time trellis codes. For example, the new 2.5 bits/s/Hz codes of 2- and 4-states have the same minimum CGD. But the 4-state code is about 0.5 dB better in terms of FER versus E_b/N_0 . This observation coincides with those in [5] and [6].

V. CONCLUSION

The MTCM designs from non-PSK signals were proposed in this paper. Based on the MTCM designs, some super-orthogonal space-time trellis codes were presented. Furthermore, it was shown that the new 2- and 4-state codes have a simple mathematical expression for their CGDs. The proposed new space-time trellis codes of 3.5 and 4 bits/s/Hz show significant gains over the existing ones.

APPENDIX
CONSTELLATIONS FOR NON-PSK MTCM

See Table III.

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