Unitary Space–Time Codes From Alamouti's Scheme With APSK Signals

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***Abstract*—Unitary space–time codes have been used in differential space–time modulation, when neither the transmitter nor the receiver of a multiple antenna system knows the channel state information in Rayleigh fading channels. Among the codes in literature, unitary orthogonal space–time codes, constructed from Alamouti's scheme, have the advantage of fast maximum likelihood (ML) decoding but they require signal constellations to be phase-shift keying (PSK). In this paper, unitary space–time codes are constructed from Alamouti's scheme with amplitude/phase-shift keying (APSK) constellations. We show that the unitary space–time codes from Alamouti's scheme with APSK signals have larger diversity products than those with PSK signals while the complexity of their ML decoding algorithm is comparable. Our newly proposed 4 b/s/Hz code has about 2 dB gain over the same rate code with PSK signals at bit-error rate (BER) of 10^{-3} with one receive antenna. We also propose a noncoherent scheme of rate 5 b/s/Hz, which has the same BER performance as the 4 b/s/Hz unitary orthogonal space–time code in DSTM while having comparable decoding complexity.**

***Index Terms*—Alamouti's scheme, amplitude/phase shift keying (APSK) signals, differential space–time modulation (DSTM), fast maximum-likelihood (ML) decoding algorithm, orthogonal space–time codes, unitary space–time codes.**

I. Introduction

DIFFERENTIAL space–time modulation (DSTM) has been proposed in [1]–[3] for multiple antenna systems in Rayleigh fading channels, when the multiple-input and multiple-output channel state information is not available or hard/costly to obtain. As a differential phase-shift keying (DPSK) scheme for a single transmit antenna system, DSTM allows the receiver to decode without the channel state information. In such a differentially encoded system, unitary space–time codes are necessary to ensure average transmission power to be constant in each time block (called *block-mean power*). There are many constructions of unitary space–time codes in the literature, for example, diagonal codes [1], [2], dicyclic codes [2], [4] fixed-point-free group unitary codes [5], parametric codes [6], unitary codes using Cayley transform [7], and unitary orthogonal space–time codes from Alamouti's scheme with PSK signals [3]. However, for general unitary space–time codes, decoding complexity will go up exponentially with the number of transmit antennas and with the rate (bandwidth efficiency). Even for a reasonable rate in a two transmit antenna system, the size of unitary space–time codes may be large and, therefore, the development of fast decoding algorithm becomes a critical issue. One of the remarkable advantages of unitary orthogonal space–time codes in [3] over others is the existence of fast maximum likelihood (ML) decoding algorithm.

In differential orthogonal space–time modulation [3], the mean power of transmit signal matrix, i.e., block-mean power, is constant over time. This is particularly important for differential modulation, as we can see from [1] and [2]. There are two useful measures of signal power for differential orthogonal space–time modulation. One is the individual information symbol power and the other is the block-mean power. In [3], two independent information symbols in Alamouti's scheme [8] are PSK signals in order to ensure constant block-mean power. Therefore, the power of individual information symbols is constant. As shown later, this may degrade the diversity products of the codes.

The main goal of this paper is to relax this constraint to allow the information symbols to have different power levels and keep the fast ML decoding algorithm. We design unitary space–time codes from Alamouti's scheme with amplitude/phase shift keying (APSK) signals rather than PSK signals. While the ML decoding complexity is slightly higher, the resultant unitary codes have better diversity products than the unitary orthogonal space–time codes with PSK signals at 1.5, 2.5, 3, 3.5, 4, and 4.5 b/s/Hz. In [5], via parameterizing Alamouti's scheme, Hamiltonian codes have constraints similar to those of our proposed codes. However, the fast decoding algorithm of Hamiltonian codes is not ML. The maximal decoding time depends on the structure of the employed spherical codes. As other unitary space–time codes, our codes in DSTM do not need channel state information for decoding. We further show that the peak-to-average power ratio of the proposed codes in DSTM does not increase with respect to that of the unitary orthogonal space–time codes.

For the proposed codes, the block-mean power is constant over time. The one-level block-mean power in the differential orthogonal space–time modulation in [3] has been generalized in [9] to two-level block-mean power. In [9], an additional bit of information is carried by the two levels of block-mean power. This can be considered as a generalization of the differential

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APSK modulation in a single transmit antenna system [10], [11]. Note that in [12], an alternative noncoherent block encoding scheme using APSK signals was proposed. In [13], a rectangular noncoherent space–time coding scheme was proposed. In [9], the information symbol power in a codeword matrix is constant, i.e., information symbols are PSK signals. In this paper, we combine the scheme proposed in [9] with the newly designed unitary codes to increase data throughput. Therefore, two levels of the block-mean power along intermatrix blocks and multiple levels of information symbol power in an intramatrix of a codeword are used.

Our simulations confirm the performance advantage of the proposed codes and scheme. In particular, with one receive antenna, the 4 b/s/Hz code has about 2 dB gain over the same rate code with PSK signals at bit-error rate (BER) of 10^{-3}. Furthermore, the decoding complexity of our 3, 4 b/s/Hz codes is only about twice that of the unitary orthogonal space–time codes with PSK signals. The proposed 4.5 b/s/Hz code has 1 dB gain over the 4 b/s/Hz code with PSK signals with one receive antenna. By using the proposed 4.5 b/s/Hz code, the two-level block-mean power differential modulation provides a 5 b/s/Hz noncoherent scheme. This scheme has the same BER performance as the 4 b/s/Hz unitary orthogonal space–time code with PSK signals.

This paper is organized as follows. In Section II, the space–time modulation system model, DSTM, and unitary orthogonal space–time codes are briefly reviewed. In Section III, unitary space–time codes from Alamouti's scheme with APSK signals are presented. Also some properties of the proposed codes are investigated. In Section IV, a two-level block-mean power differential modulation using the proposed codes is given. Finally, in Section IV, simulation results are presented to show the performance of the proposed codes and scheme.

In what follows, the following notations are adopted. $\mathbf{c}^H$ and $\mathbf{c}^{-1}$ denote the complex conjugate transpose of matrix $\mathbf{c}$, the inverse of $\mathbf{c}$, respectively; $\det\{\mathbf{c}\}$ denotes the determinant of matrix $\mathbf{c}$; $\|\mathbf{c}\|$ denotes Frobenius norm of matrix $\mathbf{c} = (c_{m,n})_{M\times N}$, i.e.,

$$\|\mathbf{c}\| = \sqrt{\sum_{m=1}^{M}\sum_{n=1}^{N}|c_{m,n}|^2}.$$

$\Re\{c\}$ and c^* denote the real part and the complex conjugate of a complex number c, respectively. $\mathbf{E}\{c\}$ denotes the expectation of random variable c; Prob$\{c\}$ denotes the probability of the event c. $\mathbf{R}$ denotes the real number domain. $\mathbf{I}_n$ represents an $n\times n$ identity matrix; $\mathbf{0}_{m\times n}$ represents an $m\times n$ matrix with all zero elements. For convenience, unitary orthogonal space–time codes from Alamouti's scheme with PSK signals in [3] are shortened as PSK-UA codes. The proposed unitary space–time codes from Alamouti's scheme with APSK signals are called APSK-UA codes.

II. Review of DSTM and PSK-UA Codes

In this section, we briefly review the system model commonly used in the space–time modulation literature, DSTM [1], [2], Alamouti's scheme, and PSK-UA codes in [3].

A. System Model and Differential Encoding

Consider a wireless communication system with two transmit antennas and N receive antennas over a frequency-nonselective fading channel that is unknown to both the transmitter and the receiver. Let $s_{t,m}$ be the signal transmitted at the mth transmit antenna at time t, and let $h_{m,n,t}$ be the fading coefficient of the channel between the mth transmit and the nth receive antenna at time t. It is assumed that $h_{m,n,t}$ is quasi-static, i.e., it is constant over a frame of length T and independent from one frame to another. $w_{t,n}$ is the additive noise at the nth receive antenna at time t. It is assumed that $w_{t,n}$ is a complex Gaussian white noise with zero mean and unit variance, i.e., $w_{t,n}\sim CN(0,1)$ and $w_{t,n}$ are independent of each other with respect to both n and t.

The received signal $x_{t,n}$ at the nth receive antenna at time t is the superposition of the transmitted signals on two transmit antennas, i.e.,

$$x_{t,n} = \sqrt{\rho}\sum_{m=1}^{2} h_{m,n,t}s_{t,m} + w_{t,n}, \quad t = 1,2,\ldots \tag{1}$$

where ρ is signal-to-noise ratio (SNR) at each receive antenna. When $h_{m,n,t}$ is constant within a frame

$$\mathbf{x}_\tau = \sqrt{\rho}\mathbf{s}_\tau\mathbf{h}_\tau + \mathbf{w}_\tau, \quad \tau = 1,2,\ldots \tag{2}$$

where $\mathbf{x}_\tau$ is the τth block of the received signal matrix $(x_{t,n})_{2\times N}$, $\mathbf{s}_\tau = (s_{t,m})_{2\times 2}$ is the transmitted signal matrix in the τth block, $\mathbf{w}_\tau = (w_{t,n})_{2\times N}$ is the noise matrix in the τth block, and $\mathbf{h}_\tau = (h_{m,n})_{2\times N}$ is the channel coefficient matrix in the τth block. As in [3], $h_{m,n}$ is modeled as an independent complex Gaussian variable with zero mean and unit variance.

In the DSTM proposed in [1] and [2], the transmit signal matrix $\mathbf{s}_\tau$ is obtained by the differential encoding of $\mathbf{c}_\tau$

$$\mathbf{s}_\tau = \mathbf{c}_\tau\mathbf{s}_{\tau-1}, \quad \mathbf{c}_\tau\mathbf{c}_\tau^H = \mathbf{I}_2, \qquad \tau = 1,2,\ldots \tag{3}$$

and $\mathbf{s}_0 = \mathbf{I}_2$. In [3], the unitary space–time codeword $\mathbf{c}_\tau$ is from Alamouti's scheme

$$\mathbf{c}_\tau \in \mathcal{C}_{\text{PSK}} = \left\{\begin{bmatrix} c_1 & -c_2^* \\ c_2 & c_1^* \end{bmatrix}, c_1\in\mathbb{S}_1, c_2\in\mathbb{S}_2\right\} \tag{4}$$

where $\mathbb{S}_i$ is a PSK constellation, i.e., $\mathbb{S}_i = \{(1/\sqrt{2})e^{j2\pi l/L_i}\,|\,l = 0,1,\ldots,L_i-1\}, i = 1,2$, to ensure Frobenius norm of $\mathbf{s}_\tau$ to be constant. The two information symbols c_1 and c_2 are independent of each other. Therefore, the size of PSK-UA codes $\mathcal{C}_{\text{PSK}}$ is $L = L_1L_2$. The rate R of $\mathcal{C}_{\text{PSK}}$ is $R = (1/2)\log_2 L$ b/s/Hz. The available rates ($\le$4.5 b/s/Hz), depending on the PSK constellations for c_1 and c_2, are listed in Table I.

B. ML Decoding Algorithm

The noncoherent ML decoder, or demodulator, of $\mathbf{c}_\tau$ is [1]–[3]

$$\hat{\mathbf{c}}_\tau = \arg\min_{\mathbf{c}\in\mathcal{C}_{\text{PSK}}} \|\mathbf{x}_\tau - \mathbf{c}\mathbf{x}_{\tau-1}\|^2. \tag{5}$$

For general unitary space–time codes, the decoding, thus, requires an exhaustive search within the codes. Since $L = 2^{2R}$, the decoding complexity can be prohibitive at a rational rate. On the other hand, for PSK-UA codes, the complexity of ML decoding can be greatly reduced. Due to the orthogonality of $\mathbf{c}_\tau$, decoder (5) can be manipulated into

$$\hat{\mathbf{c}}_\tau = \arg \max_{(c_1,c_2)\in \mathbb{S}_1\times\mathbb{S}_2} \Re\Big\{c_1\mathbf{x}^1_{\tau-1}\left(\mathbf{x}^1_\tau\right)^H + c_1^*\mathbf{x}^2_{\tau-1}\left(\mathbf{x}^2_\tau\right)^H + c_2\mathbf{x}^2_{\tau-1}\left(\mathbf{x}^1_\tau\right)^H - c_2^*\mathbf{x}^1_{\tau-1}\left(\mathbf{x}^2_\tau\right)^H\Big\} \tag{6}$$

where $\mathbf{x}^i_\tau$ and $\mathbf{x}^i_{\tau-1}$ denote the ith row of the $\mathbf{x}_\tau$ and $\mathbf{x}_{\tau-1}$, respectively, $i = 1, 2$.

Since c_1 and c_2 are independent, they can be separately decoded

$$(\hat{c}_1,\hat{c}_2) = \left(\arg\max_{c_1\in\mathbb{S}_1}\Re\{c_1g_1\}, \arg\max_{c_2\in\mathbb{S}_2}\Re\{c_2g_2\}\right) \tag{7}$$

where

$$g_1 = \mathbf{x}^1_{\tau-1}\left(\mathbf{x}^1_\tau\right)^H + \mathbf{x}^2_\tau\left(\mathbf{x}^2_{\tau-1}\right)^H \tag{8}$$
$$g_2 = \mathbf{x}^2_{\tau-1}\left(\mathbf{x}^1_\tau\right)^H - \mathbf{x}^2_\tau\left(\mathbf{x}^1_{\tau-1}\right)^H. \tag{9}$$

Moreover, the decoder (7) can be further simplified because c_1 and c_2 are PSK signals. The complex plane may be divided into L_i equal sectors started from the origin and $\hat{c}_i$ is determined by the sector in which the complex number g_i^* falls, $i = 1, 2$. Thus, the decoding complexity is comparable to that of PSK demodulation in a single transmit antenna system. It is clear, from (7), that the decoding of PSK-UA codes does not need channel estimation.

As another fact, PSK-UA codes are full rank codes because $\det\{\mathbf{c}_l - \mathbf{c}_{l'}\} > 0$ for any two different codewords $\mathbf{c}_l$ and $\mathbf{c}_{l'}$ in $\mathcal{C}_{\rm PSK}$. The following *diversity product*:

$$\xi = \min_{0\le l<l'\le L-1}\frac{1}{2}\sqrt{|\det\{\mathbf{c}_l-\mathbf{c}_{l'}\}|}, \quad \mathbf{c}_l,\mathbf{c}_{l'}\in\mathcal{C} \tag{10}$$

is defined for the unitary space–time code $\mathcal{C} = \{\mathbf{c}_0 \;\; \mathbf{c}_1 \;\; \cdots \;\; \mathbf{c}_{L-1}\}$ in [1]. For full rank codes, maximization of diversity product is commonly used as a criterion for the design of unitary space–time codes because ξ is a good indicator for the block error rate (BLER) of unitary space–time codes in DSTM. For PSK-UA codes, as configured in Table I, the diversity product is

$$\xi = \frac{\sqrt{2}}{2}\sin\frac{\pi}{\max\{L_1,L_2\}}. \tag{11}$$

III. Unitary Space-Time Codes From Alamouti's Scheme With APSK Signals

From (11), we know the performance of the PSK-UA codes is limited by the larger constellation of c_1 and c_2's. For example, if the 1.5, 2.5, or 3.5 b/s/Hz codes in Table I are considered, the corresponding diversity products are limited by the L_2-PSK constellations. In this section, this disadvantage of the PSK-UA codes is overcome. We show that the two symbols in Alamouti's scheme can be chosen from an APSK signal constellation to construct unitary space–time codes. Our proposed unitary space–time codes with APSK signals have larger diversity products than those with PSK signals.

TABLE I
Rates of PSK-UA Codes

R	$\mathbb{S}_1$	$\mathbb{S}_2$
1	BPSK($L_1 = 2$)	BPSK($L_2 = 2$)
1.5	BPSK($L_1 = 2$)	QPSK($L_2 = 4$)
2	QPSK($L_1 = 4$)	QPSK($L_2 = 4$)
2.5	QPSK($L_1 = 4$)	8-PSK($L_2 = 8$)
3	8-PSK($L_1 = 8$)	8-PSK($L_2 = 8$)
3.5	8-PSK($L_1 = 8$)	16-PSK($L_2 = 16$)
4	16-PSK($L_1 = 16$)	16-PSK($L_2 = 16$)
4.5	16-PSK($L_1 = 16$)	32-PSK($L_2 = 32$)

A. Unitary Code Construction

We propose a unitary code construction from Alamouti's scheme via APSK signals as follows:

$$\mathbf{c}(a_1,\zeta_1,\zeta_2)\in\mathcal{C}_{\rm APSK} \triangleq \left\{\begin{bmatrix} a_1\zeta_1 & -a_2^*\zeta_2^* \\ a_2\zeta_2 & a_1^*\zeta_1^*\end{bmatrix}, a_1\in\mathbb{A}, \zeta_1,\zeta_2\in\mathbb{S}_0\right\}. \tag{12}$$

In (12), ζ_1 and ζ_2 are from the same PSK constellation $\mathbb{S}_0 = \{(1/\sqrt{2})e^{j2\pi l/L_0} | l = 0,1,\ldots,L_0-1\}$. a_1 is from set $\mathbb{A}$, defined as $\mathbb{A} = \{r_0e^{j\phi_0}, r_1e^{j\phi_1},\ldots,r_{k-1}e^{j\phi_{k-1}}\}$, where $0 \le \phi_i < (2\pi)/(L_0), r_i$ is a positive real number, $i = 0,1,\ldots,k-1$, and $r_0 \le r_1 \le \cdots \le r_{K-1}$. The value of a_2 is determined by the selection of a_1: when a_1 is assigned as $r_ie^{j\phi_i}$ for some i, a_2 takes $r_{k-i-1}e^{j\phi_{k-i-1}}$, where $i = 0,1,\ldots,k-1$. Theoretically, k can be any positive integer. But only $k = 2, 4$, and 8 are considered in this paper.

As for $r_ie^{j\phi_i}$ and $r_{k-i-1}e^{j\phi_{k-i-1}}$ in $\mathbb{A}$, the sum of their squared modulus is two, i.e.,

$$r_i^2 + r_{k-i-1}^2 = 2, \quad i = 0,1,\ldots,\frac{k}{2}-1$$

and their modulus ratio is defined as

$$\alpha_i \triangleq \frac{r_{k-i-1}}{r_i} \ge 1, \quad i = 0,1,\ldots,\frac{k}{2}-1.$$

Therefore, set $\mathbb{A}$ is determined by the following two vectors:

$$\boldsymbol{\alpha} \triangleq \left[\alpha_0 \quad \alpha_1 \quad \cdots \quad \alpha_{\frac{k}{2}-1}\right] \tag{13}$$
$$\boldsymbol{\phi} \triangleq [\phi_0 \quad \phi_i \quad \cdots \quad \phi_{k-1}]. \tag{14}$$

Let p denote the number of distinct components in vector $\boldsymbol{\phi}$ and Ψ denote the set composed by all distinct components in vector $\boldsymbol{\phi}$, i.e.,

$$\Psi = \{\psi_i \,|\, \psi_i = \phi_l, \text{for some } l, 0\le l\le k-1 \;\; \psi_i \ne \psi_j \text{ if } i\ne j \text{ for } 0\le i,j\le p-1\}. \tag{15}$$

Let $\mathbf{t} = [t_0 \;\; t_1 \;\; \cdots \;\; t_{k-1}]$ be the correspondence between the components ϕ_i in $\boldsymbol{\phi}$ and the elements ψ_l in Ψ as

$$\phi_i = \psi_{t_i}, \quad i = 0,1,\ldots,k-1. \tag{16}$$

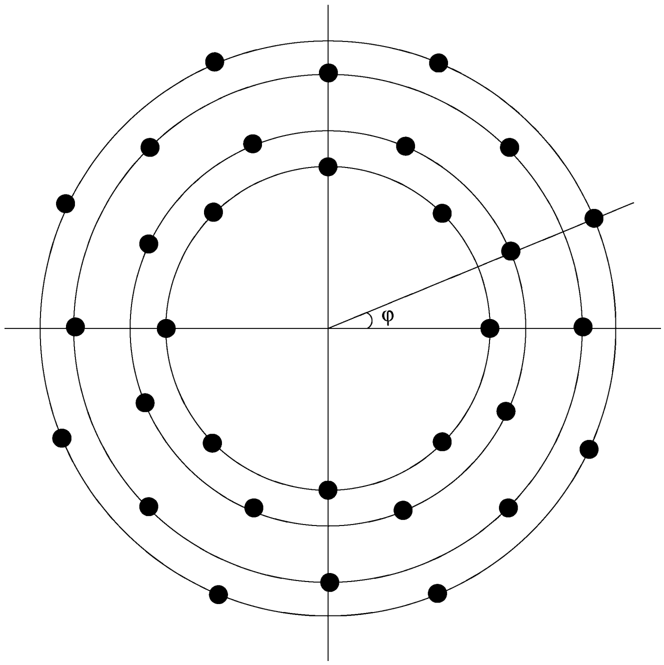

Fig. 1. The APSK constellation for c_1 and c_2 in the (8, 4, 2) APSK-UA code. In the graph, $\varphi = (\pi/8)$ and $\boldsymbol{\alpha} = [1.64\ 1.37]$.

Parameter p will be shown to have direct effects on the decoding complexity and to be relevant to the diversity product.

It is not hard to see that a_1, ζ_1, and ζ_2 in the code in (12) are independent of each other. Therefore, the size of $\mathcal{C}_{\rm APSK}$ is $L = kL_0^2$ and the rate is $R = (1/2)\log_2 k + \log_2 L_0$ b/s/Hz. In what follows, for convenience, an APSK-UA code in (12) is defined as an (L_0, k, p) APSK-UA code, or an (L_0, k, p) code for short.

If we let $c_i = a_i\zeta_i$ in the code (12), then the code follows Alamouti's scheme. The difference with Alamouti's scheme is that the two symbols are not independent while there is a relationship between their amplitudes $|c_1|$ and $|c_2|$ as

$$|c_1|^2+|c_2|^2 = |a_1|^2|\zeta_1|^2+|a_2|^2|\zeta_2|^2 = \frac{1}{2}\left(r_i^2 + r_{k-i-1}^2\right) = 1.$$

The above identity ensures that the matrices in the code in (12) are unitary, i.e., the code is unitary. Since the amplitude of c_i may have different levels, symbol c_i actually is an APSK signal. Fig. 1 shows an APSK constellation for c_1 and c_2 in an (8, 4, 2) APSK-UA code.

For two distinct codewords

$$\mathbf{c}_l = \begin{bmatrix} a_1\zeta_1 & -a_2^*\zeta_2^* \\ a_2\zeta_2 & a_1^*\zeta_1^* \end{bmatrix} \quad \text{and} \quad \mathbf{c}_{l'} = \begin{bmatrix} a_3\zeta_3 & -a_4^*\zeta_4^* \\ a_4\zeta_4 & a_3^*\zeta_3^* \end{bmatrix}$$

where $\zeta_i = (1/\sqrt{2})e^{j2\pi\nu_i/L_0}, i = 1,2,3,4$, the determinant of the difference matrix is

$$\det\{\mathbf{c}_l - \mathbf{c}_{l'}\} = |a_1\zeta_1 - a_3\zeta_3|^2 + |a_2\zeta_2 - a_4\zeta_4|^2. \quad (17)$$

Therefore, the diversity product does not change if all elements in $\mathbb{A}$ are multiplied by $e^{-j\phi_0}$. Without loss of generality, $\phi_0 = 0$ is assumed. Also, if no elements in $\mathbb{A}$ are congruent, which is true for all of our designed codes, $\det\{\mathbf{c}_l - \mathbf{c}_{l'}\} > 0$. In other words, APSK-UA codes are full rank codes.

In the following sections, the code (12) is designed under the diversity product criterion, i.e., maximizing the diversity product. This leads to the design of the parameter vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\phi}$ of $\mathbb{A}$.

B. Unitary Codes With Optimum Diversity Products When $k = 2$

If $k = 2$, $\mathbb{A} = \{r_0, r_1e^{j\phi_1}\}$. The rate is $R = \log_2 L_0 + 0.5$ b/s/Hz. To maximize the diversity product, the determinant in (17) is investigated in two cases: $a_1 = a_3$ and $a_1 \neq a_3$. The first case is $a_1 = a_3$. Since there are only two elements in $\mathbb{A}$, this means $a_2 = a_4$. Therefore

$$\det\{\mathbf{c}_l - \mathbf{c}_{l'}\} = |a_1|^2|\zeta_1 - \zeta_3|^2 + |a_2|^2|\zeta_2 - \zeta_4|^2. \quad (18)$$

Consequently, the minimum determinant in this case is

$$\begin{aligned} f_1 &\triangleq \min_{0\le l<l'\le L-1} \det\{\mathbf{c}_l - \mathbf{c}_{l'}\} = r_0^2 \min_{\zeta_1,\zeta_2\in\mathbb{S}_0} |\zeta_1 - \zeta_3|^2 \\ &= 2r_0^2\sin^2\frac{\pi}{L_0} = \frac{4}{1+\alpha_0^2}\sin^2\frac{\pi}{L_0}. \end{aligned} \quad (19)$$

In the second case, $a_1 \neq a_3$. So $a_1 = a_4$ and $a_2 = a_3$. Without loss of generality, we assume $a_1 = r_0$. Thus

$$\begin{aligned} \det\{\mathbf{c}_l - \mathbf{c}_{l'}\} &= |r_0|^2 + |r_1|^2 - 2r_0r_1\Re\left\{e^{-j\phi_1}(\zeta_1\zeta_3^* + \zeta_2\zeta_4^*)\right\} \\ &= 2 - \alpha_0 r_0^2\left(\cos\left(\frac{2\pi(\nu_1-\nu_3)}{L_0} - \phi_1\right)\right. \\ &\qquad \left. + \cos\left(\frac{2\pi(\nu_2-\nu_4)}{L_0} - \phi_1\right)\right). \end{aligned} \quad (20)$$

Define $\varphi \triangleq \min\{2\pi/L_0 - \phi_1, \phi_1\}$. Then, we have $0 \le \varphi \le \pi/L_0$ since $0 \le \phi_1 < 2\pi/L_0$. The determinant in (20) is minimized when $\cos((2\pi(\nu_1-\nu_3))/(L_0) - \phi_1)$ and $\cos((2\pi(\nu_2 - \nu_4))/(L_0) - \phi_1)$ both reach maximum. Therefore, when

$$\left|\frac{2\pi(\nu_1-\nu_3)}{L_0} - \phi_1\right| = \left|\frac{2\pi(\nu_2-\nu_4)}{L_0} - \phi_1\right| = \varphi, \quad (21)$$

we have

$$\begin{aligned} f_2 &\triangleq \min_{0\le l<l'\le L-1} \det\{\mathbf{c}_l - \mathbf{c}_{l'}\} \\ &= 2 - \frac{4\alpha_0}{1+\alpha_0^2}\cos\varphi. \end{aligned} \quad (22)$$

The diversity product of the code is

$$\xi(\alpha_0, \phi_1) = \frac{1}{2}\sqrt{\min\{f_1, f_2\}}.$$

Since $\alpha_0 \ge 1$ and $0 \le \varphi \le (\pi)/(L_0)$, f_1 is monotonically decreasing with respect to α_0 and f_2 is monotonically increasing with respect to α_0 and φ. Therefore, the maximum diversity product $\xi(\alpha_0, \phi_0)$, in terms of parameters α_0 and φ, is achieved when $f_1 = f_2$ and $\varphi = (\pi)/(L_0)$. This implies that

$$\alpha_0^2 - 2\alpha_0\cos\varphi + 1 - 2\sin^2\frac{\pi}{L_0} = 0, \quad \varphi = \frac{\pi}{L_0}. \quad (23)$$

TABLE II
APSK-UA CODES

R	L	ξ	(L_0, k, p)	$\boldsymbol{\alpha}$ and $\boldsymbol{\phi}$
1.5	8	0.7071	(2, 2, 2)	$\boldsymbol{\alpha} = [1]$, $\boldsymbol{\phi} = [0\ \ \frac{\pi}{2}]$
2.5	32	0.4082	(4, 2, 2)	$\boldsymbol{\alpha} = [1.4142]$, $\boldsymbol{\phi} = [0\ \ \frac{\pi}{4}]$
3	64	0.3162	(4, 4, 2)	$\boldsymbol{\alpha} = [2\ \ 2]$, $\boldsymbol{\phi} = [0\ \ 0\ \ 0\ \ \frac{\pi}{4}]$
		0.3362	(4, 4, 3)	$\boldsymbol{\alpha} = [2\ \ 2]$, $\boldsymbol{\phi} = [0\ \ 0\ \ \frac{\pi}{8}\ \ \frac{3\pi}{8}]$
3.5	128	0.2083	(8, 2, 1)	$\boldsymbol{\alpha} = [1.5412]$, $\boldsymbol{\phi} = [0\ \ 0]$
		0.2326	(8, 2, 2)	$\boldsymbol{\alpha} = [1.3066]$, $\boldsymbol{\phi} = [0\ \ \frac{\pi}{8}]$
		0.2646	(4, 8, 4)	$\boldsymbol{\alpha} = [2.45\ \ 2.45\ \ 1.5\ \ 1.5]$, $\boldsymbol{\phi} = [0\ \ 0\ \ \frac{2\pi}{8}\ \ \frac{2\pi}{8}\ \ \frac{\pi}{8}\ \ \frac{3\pi}{8}\ \ 0\ \ \frac{2\pi}{8}]$
4	256	0.1985	(8, 4, 2)	$\boldsymbol{\alpha} = [1.64\ \ 1.39]$, $\boldsymbol{\phi} = [0\ \ \frac{\pi}{8}\ \ 0\ \ \frac{\pi}{8}]$
		0.1991	(8, 4, 4)	$\boldsymbol{\alpha} = [1.64\ \ 1.37]$, $\boldsymbol{\phi} = [0\ \ \frac{4\pi}{32}\ \ \frac{\pi}{32}\ \ \frac{5\pi}{32}]$
4.5	512	0.1493	(8, 8, 2)	$\boldsymbol{\alpha} = [2.36\ \ 1.46\ \ 1.36\ \ 1.02]$, $\boldsymbol{\phi} = [0\ \ 0\ \ \frac{\pi}{8}\ \ \frac{\pi}{8}\ \ 0\ \ \frac{\pi}{8}\ \ 0\ \ \frac{\pi}{8}]$
		0.1584	(8, 8, 4)	$\boldsymbol{\alpha} = [2.2\ \ 2.2\ \ 1.4\ \ 1.1]$, $\boldsymbol{\phi} = [0\ \ 0\ \ \frac{2\pi}{16}\ \ \frac{2\pi}{16}\ \ 0\ \ \frac{2\pi}{16}\ \ \frac{\pi}{16}\ \ \frac{3\pi}{16}]$

Thus, the optimum α_0 can be solved from (23) and the solution is

$$\alpha_0 = \cos\frac{\pi}{L_0} + \sin\frac{\pi}{L_0} \tag{24}$$

since $\alpha_0 \geq 1$. When $\varphi = (\pi)/(L_0)$, the optimum ϕ_1 has to be $\phi_1 = (\pi)/(L_0)$. The optimum diversity product, therefore, is

$$\xi_{\mathrm{OPT},k=2} = \frac{\sin\frac{\pi}{L_0}}{\sqrt{2\left(1+\cos\frac{\pi}{L_0}\sin\frac{\pi}{L_0}\right)}}. \tag{25}$$

With the optimum α_0 and ϕ_1, it is easy to obtain the optimum $\mathbb{A}$ for code (12) when $k = 2$.

For these codes, $\boldsymbol{\phi} = [0\ (\pi)/(L_0)]$. Thus $\Psi = \{0, (\pi)/(L_0)\}$ and $p = 2$. In Table II, these codes are denoted as (2, 2, 2), (4, 2, 2), (8, 2, 2) codes. The parameters of $\mathbb{A}$ and the diversity products are also listed.

C. Unitary Codes When $k = 4$ and 8

When $k > 2$, analytic solutions tend to be more difficult. A family of codes is obtained, shown in Table II, by computer search under the diversity product criterion.

The (2, 2, 2), (4, 2, 2), and (8, 2, 2) codes in Table II are constructed in Section III-B. The (8, 2, 1) code is obtained by setting $\varphi = 0$ in (23). Except for the two codes of 4.5 b/s/Hz, the codes of the same rate have approximately the same BER performance with a small or median number of receive antennas. A smaller p in a code corresponds to a smaller diversity product, but a lower decoding complexity. Therefore, for 3, 3.5, 4 b/s/Hz, the codes with small p would be suggested for use in systems with a small or median number of receive antennas, or in systems of strict complexity requirement. For 4.5 b/s/Hz codes, the (8, 8, 4) code outperforms the (8, 8, 2) code by 1 dB with one receive antenna.

D. Peak-to-Average Power Ratio

When the constellation of c_i migrates from PSK to APSK, the peak-to-average power ratio of transmitted signals is a concern. In DSTM, the average power of the transmitted signals on ith antenna is

$$\varrho_{\mathrm{AVE}} = \frac{1}{2}\mathbf{E}\{|s_{2\tau+1,i}|^2 + |s_{2\tau+2,i}|^2\}, \quad i = 1 \text{ or } 2. \tag{26}$$

From (3) and (4), it is straightforward to show that $\mathbf{s}_\tau$ is a unitary matrix in the form of Alamouti's scheme if APSK-UA codes are used in (3), i.e.,

$$\mathbf{s}_\tau = \begin{bmatrix} s_{2\tau+1,1} & -s^*_{2\tau+2,1} \\ s_{2\tau+2,1} & s^*_{2\tau+1,1} \end{bmatrix}$$

where

$$|s_{2\tau+1,1}|^2 + |s_{2\tau+2,1}|^2 = 1.$$

So, $\varrho_{\mathrm{AVE}} = (1/2)$ and the peak power $\varrho_{\mathrm{MAX}} \leq 1$. The peak-to-average power ratio is $\gamma_{\mathrm{NC}} \leq 3$ dB. Suppose

$$\begin{aligned} \mathbf{c}_1 &= \begin{bmatrix} r_0\zeta_1 & -r_{k-1}e^{-j\phi_{k-1}}\zeta_2^* \\ r_{k-1}e^{j\phi_{k-1}}\zeta_2 & r_0\zeta_1^* \end{bmatrix} \\ \mathbf{c}_2 &= \begin{bmatrix} r_{k-1}e^{j\phi_{k-1}}\zeta_2 & -r_0\zeta_1 \\ r_0\zeta_1^* & r_{k-1}e^{-j\phi_{k-1}}\zeta_2^* \end{bmatrix} \end{aligned} \tag{27}$$

are to encode in a sequence, then $\mathbf{s}_2 = \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$. Under such circumstance, $\varrho_{\mathrm{MAX}} = 1$. Therefore, $\gamma_{\mathrm{NC}} = 3$ dB.

However, for PSK-UA codes, the peak-to-average power ratio is also 3 dB, which can be shown with same argument by letting $r_0 = 1$ in (27). In [3], unitary matrix

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

is used to preprocess $\mathbf{c}_\tau$. As a result, the transmitted power on each transmit antenna is constant at 1 bit/s/Hz. However, at other rates, differential encoding still leads to expansion of the constellation [2] and the peak-to-average power ratio is 3 dB. Therefore, the signal constellation change from PSK to APSK has no adverse effects on the peak-to-average power ratio of transmitted signals in DSTM.

E. A Posteriori Probabilities (APP)

For APSK-UA codes, it is possible to derive APP for a_1, ζ_1, and ζ_2, which can be used in soft decoding/demodulation if needed. For illustration purposes, we assume $N = 1$, and it is not hard to extend the result to the general case. From (2), we may assume $\mathbf{x}_\tau$ is complex Gaussian. Furthermore

$$\mu_{\mathbf{x}_\tau} = \mathbf{E}\{\mathbf{x}_\tau\} = 0_{2\times 1} \tag{28}$$

$$\Lambda_{\mathbf{x}_\tau} = \mathbf{E}\left\{\mathbf{x}_\tau\mathbf{x}_\tau^H\right\} = (\rho+1)\mathbf{I}_2. \tag{29}$$

Therefore, the probability density function (pdf) of $\mathbf{x}_\tau$ is

$$p_{\mathbf{x}_\tau}(\mathbf{x}) = \frac{1}{\pi^2 \det\{\Lambda_{\mathbf{x}_\tau}\}} \exp\left\{-\mathbf{x}^H \mathbf{\Lambda}_{\mathbf{x}_\tau}^{-1}\mathbf{x}\right\}. \tag{30}$$

With known past received signals $\mathbf{x}_{\tau-1}$, (2) can be written as

$$\mathbf{x}_\tau = \mathbf{c}_\tau \mathbf{x}_{\tau-1} + \mathbf{w}_\tau - \mathbf{c}_\tau \mathbf{w}_{\tau-1}. \tag{31}$$

Since $\mathbf{w}_\tau$ and $\mathbf{w}_{\tau-1}$ are complex Gaussian, $\mathbf{x}_\tau$ given a_1 is also complex Gaussian. From (31), the mean vector and correlation matrix of $\mathbf{x}_\tau$ given a_1 can be obtained as shown in (32) and (33) at the bottom of the page. Therefore, the pdf of $\mathbf{x}_\tau$ given a_1 is

$$p_{\mathbf{x}_\tau\,|\,a_1}(\mathbf{x}\,|\,a_1) = \frac{1}{\pi^2 \det\{\mathbf{\Lambda}_{\mathbf{x}_\tau\,|\,a_1}\}} \exp\left\{-\mathbf{x}^H \mathbf{\Lambda}_{\mathbf{x}_\tau\,|\,a_1}^{-1}\mathbf{x}\right\}. \tag{34}$$

From (30) and (34), APP for symbol a_1 is

$$\begin{aligned}\text{Prob}&\{a = a_1\,|\,\mathbf{x}_\tau\}\\ &= \frac{\text{Prob}\{a=a_1\} p_{\mathbf{x}_\tau\,|\,a_1}(\mathbf{x}\,|\,a_1)|_{\mathbf{x}=\mathbf{x}_\tau}}{p_{\mathbf{x}_\tau}(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_\tau}}\\ &= \frac{(\rho+1)^2}{k \det\{\mathbf{\Lambda}_{\mathbf{x}_\tau\,|\,a_1}\}} \exp\left(-\mathbf{x}_\tau^H \left(\mathbf{\Lambda}_{\mathbf{x}_\tau\,|\,a_1}^{-1} - \frac{1}{\rho+1}\mathbf{I}\right)\mathbf{x}_\tau\right).\end{aligned} \tag{35}$$

In the same way, APP for symbol ζ_i can be obtained as

$$\begin{aligned}\text{Prob}(\zeta = \zeta_i\,|\,\mathbf{x}_\tau) &= \frac{(\rho+1)^2}{L_0 \det\{\mathbf{\Lambda}_{\mathbf{x}_\tau\,|\,\zeta_i}\}}\\ &\times \exp\left(-\left(\mathbf{x}_\tau - \boldsymbol{\mu}_{\mathbf{x}_\tau\,|\,\zeta_i}\right)^H\right.\\ &\quad \left.\times\, \mathbf{\Lambda}_{\mathbf{x}_\tau\,|\,\zeta_i}^{-1}\left(\mathbf{x}_\tau - \boldsymbol{\mu}_{\mathbf{x}_\tau\,|\,\zeta_i}\right) + \frac{1}{\rho+1}\mathbf{x}_\tau^H\mathbf{x}_\tau\right),\\ &\qquad i = 1 \text{ or } 2\end{aligned} \tag{36}$$

where we have (37)–(40) shown at the bottom of the page. $\mathbf{E}\{a_1^2\}$ and $\mathbf{E}\{a_1\}$ can be computed according to the constellation $\mathbb{A}$ of the code.

F. ML Decoding Algorithm

The remarkable advantage of PSK-UA codes is their fast ML decoding algorithm. The proposed APSK-UA codes keep such an advantage, although the decoding complexity has a moderate increase as shown in this section.

By expanding the ML decoder (5), the optimal estimates of a_1, ζ_1, and ζ_2 can be found by

$$(\hat{a}_1, \hat{\zeta}_1, \hat{\zeta}_2) = \arg \max_{a_1 \in \mathbb{A}, \zeta_1, \zeta_2 \in \mathbb{S}_0} \Re\{a_1\zeta_1 g_1 + a_2 \zeta_2 g_2\} \tag{41}$$

where g_1 and g_2 are defined in (8) and (9), respectively.

If a_1 is fixed, i.e., $a_1 = r_i e^{j\phi_i}$, then a_2 is also determined in $\mathbb{A}$ by $a_2 = r_{k-i-1} e^{j\phi_{k-i-1}}$ according to the code design in Section III-A. Thus, ζ_1 and ζ_2 can be decoded separately as

$$(\hat{\zeta}_1, \hat{\zeta}_2) = \left(\arg\max_{\zeta_1 \in \mathbb{S}_0} \Re\left\{e^{j\phi_{a_1}} g_1 \zeta_1\right\},\ \arg\max_{\zeta_2 \in \mathbb{S}_0} \Re\left\{e^{j\phi_{a_2}} g_2 \zeta_2\right\}\right) \tag{42}$$

where $a_i = |a_i| e^{j\phi_{a_i}}, i = 1, 2$. In (42), the amplitude of a_i is discarded because it is not relevant in decoding $\zeta_i, i = 1, 2$. The final ML decoding algorithm is

$$(\hat{a}_1, \hat{\zeta}_1, \hat{\zeta}_2) = \arg\max_{a_1 \in \mathbb{A}} \left(\max_{\zeta_1, \zeta_2 \in \mathbb{S}_0} \Re\{a_1 \zeta_1 g_1 + a_2 \zeta_2 g_2\}\right). \tag{43}$$

The inner maximization in (43) can be simplified as in (42). Clearly, APSK-UA codes, as other unitary codes, in DSTM do not need channel state information in decoding.

Since, in (42), only phases ϕ_{a_i} of a_i affect the estimation of ζ_i and there are only p distinct phases ϕ_{a_i} in $\mathbb{A}$, there are only p many trials of estimating ζ_1 and ζ_2 by enumerating a_1. The ML estimation algorithm can be stated as follows.

Step 1) Obtain the estimates of ζ_1 and ζ_2 by

$$\hat{\zeta}_{1,l} = \arg\max_{\zeta_1 \in \mathbb{S}_0} \Re\{e^{j\phi_{a_1}} g_1 \zeta_1\} \qquad \phi_{a_1} = \psi_l, \quad l = 0, 1, \ldots, p-1 \tag{44}$$

$$\boldsymbol{\mu}_{\mathbf{x}_\tau|a_1} = \mathbf{E}\{\mathbf{x}_\tau | a_1\} = \mathbf{0}_{2\times 1} \tag{32}$$

$$\begin{aligned}\mathbf{\Lambda}_{\mathbf{x}_\tau|a_1} &= \mathbf{E}\left\{\mathbf{x}_\tau \mathbf{x}_\tau^H | a_1\right\}\\ &= \begin{bmatrix} 2 + |x_{2\tau,1}|^2 + \frac{1}{2}|a_1|^2(|x_{2\tau-1,1}|^2 - |x_{2\tau,1}|^2) & 0 \\ 0 & 2 + |x_{2\tau-1,1}|^2 + \frac{1}{2}|a_1|^2(|x_{2\tau,1}|^2 - |x_{2\tau-1,1}|^2)\end{bmatrix}\end{aligned} \tag{33}$$

$$\boldsymbol{\mu}_{\mathbf{x}_\tau\,|\,\zeta_1} = [\zeta_1 x_{2\tau-1,1}\mathbf{E}\{a_1\} \quad \zeta_1^* x_{2\tau,1}\mathbf{E}^*\{a_1\}]^T \tag{37}$$

$$\boldsymbol{\mu}_{\mathbf{x}_\tau\,|\,\zeta_2} = [-\zeta_2^* x_{2\tau,1}\mathbf{E}^*\{a_1\} \quad \zeta_2 x_{2\tau-1,1}\mathbf{E}\{a_1\}]^T \tag{38}$$

$$\mathbf{\Lambda}_{\mathbf{x}_\tau\,|\,\zeta_1} = \begin{bmatrix} 2 + \frac{1}{2}(|x_{2\tau-1,1}|^2 + |x_{2\tau,1}|^2) & \zeta_1^2 x_{2\tau-1,1} x_{2\tau,1}^* \mathbf{E}\{a_1^2\} \\ (\zeta_1^*)^2 x_{2\tau,1} x_{2\tau-1,1}^* \mathbf{E}^*\{a_1^2\} & 2 + \frac{1}{2}(|x_{2\tau,1}|^2 + |x_{2\tau-1,1}|^2)\end{bmatrix} \tag{39}$$

$$\mathbf{\Lambda}_{\mathbf{x}_\tau\,|\,\zeta_2} = \begin{bmatrix} 2 + \frac{1}{2}(|x_{2\tau-1,1}|^2 + |x_{2\tau,1}|^2) & -(\zeta_2^*)^2 x_{2\tau,1} x_{2\tau-1,1}^* \mathbf{E}^*\left\{a_1^2\right\} \\ -\zeta_2^2 x_{2\tau-1,1} x_{2\tau,1}^* \mathbf{E}\left\{a_1^2\right\} & 2 + \frac{1}{2}(|x_{2\tau,1}|^2 + |x_{2\tau-1,1}|^2)\end{bmatrix} \tag{40}$$

TABLE III
THE DECODING COMPLEXITY OF PSK-UA CODES AND APSK-UA CODES

Decoding of PSk-UA codes	Decoding of APSk-UA codes
Calculate g_1 and g_2.	Calculate g_1 and g_2.
Obtain $\hat{c}_1$ and $\hat{c}_2$ by two multiples of PSK demodulation.	Obtain p groups of $\hat{\zeta}_{1,l}$ and $\hat{\zeta}_{2,l}$, by $2p$ multiples of PSK demodulation, $1 \leq p \leq 4$.
	Obtain $\hat{a}_1$, $\hat{\zeta}_1$ and $\hat{\zeta}_2$ by a k element search, $k = 2$ to 8.

$$\hat{\zeta}_{2,l} = \arg\max_{\zeta_2 \in \mathbb{S}_0} \Re\{e^{j\phi_{a_2}} g_2 \zeta_2\}$$
$$\phi_{a_2} = \psi_l, \quad l = 0, 1, \ldots, p-1 \quad (45)$$

where $\psi_l \in \Psi$ is defined in (15).

Step 2) Form k candidate estimates of (a_1, ζ_1, ζ_2) as

$$(r_i e^{j\phi_i}, \hat{\zeta}_{1,t_i}, \hat{\zeta}_{2,t_{k-i-1}}), \quad i = 0, 1, \ldots, k-1 \quad (46)$$

where t_i is defined in (16).

Step 3) Select the optimum of the k candidates in (46) by maximizing

$$\Re\{a_1 \zeta_1 g_1 + a_2 \zeta_2 g_2\} \quad (47)$$

as the final decision for (a_1, ζ_1, ζ_2), where $a_2 = r_{k-i-1} e^{j\phi_{k-i-1}}$ when $a_1 = r_i e^{j\phi_i}$.

Let us take the decoding of the (8, 8, 4) code as an example. For the (8, 8, 4) code

$$\boldsymbol{\phi} = \begin{bmatrix} 0 & 0 & \frac{2\pi}{16} & \frac{2\pi}{16} & 0 & \frac{2\pi}{16} & \frac{\pi}{16} & \frac{3\pi}{16} \end{bmatrix}.$$

Thus, $\Psi = \{0, \ (2\pi/16), \ (\pi/16), \ (3\pi/16)\}$ in (15) and $\mathbf{t} = [0 \ \ 0 \ \ 1 \ \ 1 \ \ 0 \ \ 1 \ \ 2 \ \ 3]$ in (16). After $\hat{\zeta}_{1,l}$ and $\hat{\zeta}_{2,l}$ are obtained from (44) and (45), the eight estimate candidates for (a_1, ζ_1, ζ_2) are

$$(r_0, \hat{\zeta}_{1,0}, \hat{\zeta}_{2,3}), \quad (r_1, \hat{\zeta}_{1,0}, \hat{\zeta}_{2,2})$$
$$\left(r_2 e^{j\frac{2\pi}{16}}, \hat{\zeta}_{1,1}, \hat{\zeta}_{2,1}\right), \quad \left(r_3 e^{j\frac{2\pi}{16}}, \hat{\zeta}_{1,1}, \hat{\zeta}_{2,0}\right)$$
$$(r_4, \hat{\zeta}_{1,0}, \hat{\zeta}_{2,1}), \quad \left(r_5 e^{j\frac{2\pi}{16}}, \hat{\zeta}_{1,1}, \hat{\zeta}_{2,1}\right)$$
$$\left(r_6 e^{j\frac{\pi}{16}}, \hat{\zeta}_{1,2}, \hat{\zeta}_{2,0}\right), \quad \left(r_7 e^{j\frac{3\pi}{16}}, \hat{\zeta}_{1,3}, \hat{\zeta}_{2,0}\right).$$

The final step selects the one out of these eight candidates that maximizes (47).

In the algorithm, the decoding of APSK-UA codes needs $2p$ multiples of L_0-PSK demodulation and the detection of a_1 from set $\mathbb{A}$ after obtaining g_1 and g_2. The decoding complexity comparison with PSK-UA codes is shown in Table III. The decoding of PSK-UA codes has a complexity in the order of $O(N)$. The decoding of APSK-UA codes needs another $2(p-1)$ multiples of PSK demodulation and a k element search. For all APSK-UA codes, $1 \leq p \leq 4$ and $k \leq 8$. As an example, for the (8, 2, 1) code, the decoding needs the demodulation of two 8-PSK signals and the detection of a_1 from a two-element set with obtained g_1 and g_2. The complexity is about the same as that of the PSK-UA code. However, the diversity product of the (8, 2, 1) code, with respect to that of the same rate PSK-UA code, has an increase from 0.1379 to 0.2083. The worst case in terms of complexity is the decoding of the (8, 8, 4) code. Its decoding requires eight multiples of 8-PSK demodulation and the detection of a_1 from an eight-element set $\mathbb{A}$. The decoding of the 4.5 b/s/Hz PSK-UA code, configured as in Table I, needs the demodulation of one 8-PSK signal and one 16-PSK signal after obtaining g_1 and g_2. When $N = 1$, the decoding complexity of (8, 8, 4) is about four times that of the 4.5 b/s/Hz PSK-UA code. When N is large, the calculation of g_1 and g_2 starts to dominate the complexity of the decoding, and therefore, the difference between their complexities becomes insignificant.

TABLE IV
DIVERSITY PRODUCT ξ OF SOME 2 BY 2 UNITARY SPACE-TIME CODES.

R	L	ξ	Codes
1.5	8	0.5000	PSK-UA code
		0.5946	cyclic code
		0.7071	quaternion code, parametric code (2, 2, 2) APSK-UA code
2.29	24	0.5000	FPF group code
2.5	32	0.1951	quaternion code
		0.2494	cyclic code
		0.2706	PSK-UA code
		0.4082	(4, 2, 2) APSK-UA code
		0.4461	parametric code
2.79	48	0.3868	FPF group code
3	64	0.0980	quaternion code
		0.1985	cyclic code
		0.2706	PSK-UA code
		0.3362	(4, 4, 3) APSK-UA code
		0.3535	parametric code
3.45	120	0.3090	FPF group code
3.5	128	0.0491	quaternion code
		0.1379	PSK-UA code
		0.1498	cyclic code
		0.2646	(4, 8, 4) APSK-UA code
		0.2869	parametric code
3.95	240	0.2257	FPF group code
4	256	0.1379	PSK-UA code
		0.1991	(8, 4, 4) APSK-UA code
		0.2152	parametric code
4.5	512	0.0693	PSK-UA code
		0.1584	(8, 8, 4) APSK-UA code

G. Comparison With Other Codes

In Table IV, APSK-UA codes are compared with some known unitary space–time codes for two transmit antennas. The diversity products of PSK-UA codes are obtained from (11). At $R = 1.5$ b/s/Hz, the APSK-UA code, along with the quaternion code, and the parametric code, has the largest known diversity product. At $R = 4.5$ b/s/Hz, the APSK-UA code offers a diversity product larger than that of the 4 b/s/Hz PSK-UA code. For other rates, APSK-UA codes have larger diversity products than PSK-UA codes, cyclic codes, and quaternion codes in the table. APSK-UA codes have the second largest diversity products in the table, only inferior but close to parametric codes. However, parametric codes are general unitary space–time codes, whose ML decoding is performed through an exhaustive search in (5).

Hamiltonian codes in [5] are constructed from a unitary matrix

$$\mathcal{H}(a_1, a_2, a_3, a_4) = \begin{bmatrix} a_1 + ja_2 & -a_3 + ja_4 \\ a_3 + ja_4 & a_1 - ja_2, \end{bmatrix} \tag{48}$$

where $\sum_{i=1}^{4} a_i^2 = 1$ and $a_i \in \mathbf{R}, i = 1$ to 4. Therefore, Hamiltonian codes can be built from four-dimensional spherical codes by mapping $\mathbf{a} = [a_1\ a_2\ a_3\ a_4]$ to a point in $\mathbf{R}^4$. In this sense, APSK-UA codes, along with PSK-UA codes, are a subset of Hamiltonian codes. Furthermore, Hamiltonian codes can have fast decoding algorithm by using bucketing techniques [14]. Such decoding algorithm first finds a point A on a four-dimensional sphere to maximize (5). Then, the final estimation is the Hamiltonian codeword whose correspondent point in $\mathbf{R}^4$ is nearest to A in terms of Euclidean distance. Therefore, such decoding algorithm is not ML because the nearest codeword might not be the one that maximizes (5). Furthermore, the maximal search time in buckets depends on the structure of the employed spherical codes.

In short, APSK-UA codes offer a tradeoff solution between the optimum performance with a high decoding complexity and the lowest ML decoding complexity.

IV. A Two-Level Block-Mean Power Differential Modulation Using APSK-UA Codes

In Section III, the block-mean power is constant over time because all space–time codewords are unitary. In [9], a two-level block-mean power differential modulation using PSK-UA codes is proposed to increase data throughput in noncoherent communications. An extra information bit is carried by varying the block-mean power of the transmitted signal matrix. Furthermore, the two-level block-mean power differential modulation offers a separate decoding algorithm for this extra information bit. In this section, we combine the two-level block-mean power differential modulation proposed in [9] with APSK-UA codes in Section III. This scheme is named *combined scheme* in what follows. By using the (8, 4, 4) code, the two-level block-mean power differential modulation provides a 5 b/s/Hz transmission scheme.

A. Encoding Algorithm

As in [9], the transmitted signal matrix $\mathbf{s}_\tau$ is the product of an differentially encoded amplitude (block-mean power), $d_\tau \in \mathbb{A}_R = \{r_L, r_H\}$, and a differentially encoded matrix $\mathbf{p}_\tau$. In the real element set $\mathbb{A}_R$, $(1/2)(r_H^2 + r_L^2) = 1$ with $r_L \leq r_H$. The ratio of r_H to r_L is defined as β, i.e., $\beta \triangleq r_H/r_L \geq 1$. The binary information sequence is grouped into blocks of $2R$ bits: at the τth block

$$I_1(\tau), I_2(\tau), \ldots, I_{2R}(\tau)$$

where R is the rate defined as before. The first bit $I_1(\tau)$, along with the previous block-mean power level $d_{\tau-1}$, decides the amplitude value d_τ. The remaining $2R-1$ bits are mapped to an APSK-UA codeword $\mathbf{c}_\tau$ in (12). Matrix $\mathbf{p}_\tau$ is obtained by differentially encoding $\mathbf{c}_\tau$. The detailed encoding algorithm is as follows:

$$\begin{aligned} d_0 &= d_L \\ d_\tau &= d_{\tau-1} b_\tau \end{aligned} \tag{49}$$

where

$$b_\tau = \begin{cases} 1 & \text{if } I_1(\tau) = 0 \\ \beta & \text{if } I_1(\tau) = 1 \quad \text{and } d_{\tau-1} = r_L \\ 1/\beta & \text{if } I_1(\tau) = 1 \text{ and } d_{\tau-1} = r_H \end{cases} \tag{50}$$

$$\mathbf{p}_0 = \mathbf{I}_2 \tag{51}$$

$$\mathbf{p}_\tau = \mathbf{c}_\tau \mathbf{p}_{\tau-1}, \quad \text{where } \mathbf{c}_\tau = \begin{bmatrix} a_1\zeta_1 & -a_2^*\zeta_2^* \\ a_2\zeta_2 & a_1^*\zeta_1^* \end{bmatrix} \in \mathcal{C}_{\text{APSK}} \tag{52}$$

$$\mathbf{s}_\tau = d_\tau \mathbf{p}_\tau \tag{53}$$

where $\tau = 1, 2, \ldots$.

Via this encoding scheme, the average transmission power on each transmit antenna is still (1/2). But the peak power increases by a factor r_H^2 with respect to that of DSTM. Therefore, the peak-to-average power ratio of combined scheme is $\gamma_{\text{CS}} = 10\log_{10} r_H^2 + \gamma_{\text{NC}}$. If $\beta = 1.5$, as used in the simulation, $\gamma_{\text{CS}} = 4.4$ dB. However, such a disadvantage can be justified by the increase of throughput and the improvement of performance.

B. Decoding Algorithm

When channel coefficient matrix $\mathbf{h}_\tau$ is constant within a frame

$$\begin{aligned} \mathbf{x}_\tau &= \sqrt{\rho}\mathbf{s}_\tau \mathbf{h}_\tau + \mathbf{w}_\tau \\ &= b_\tau \mathbf{c}_\tau \mathbf{x}_{\tau-1} + \hat{\mathbf{w}}_\tau \end{aligned} \tag{54}$$

where $\hat{\mathbf{w}}_\tau = \mathbf{w}_\tau - b_\tau \mathbf{c}_\tau \mathbf{w}_{\tau-1}$. Thus, the differential decoding can be implemented in the following two steps [9].

Step 1) Detect b_τ from the metric

$$\hat{b}_\tau = \arg \min_{b \in \{1, \beta, 1/\beta\}} \left|\|\mathbf{x}_\tau\| - b\|\mathbf{x}_{\tau-1}\|\right| \tag{55}$$

which gives the bit $I_1(\tau)$ according to (50).

Step 2) Detect a_1, ζ_1, and ζ_2 of the τth block with the metric

$$\hat{\mathbf{c}}_\tau = \arg \min_{\mathbf{c} \in \mathcal{C}_{\text{APSK}}} \|\mathbf{x}_\tau - \hat{b}_\tau \mathbf{c} \mathbf{x}_{\tau-1}\|^2 \tag{56}$$

which gives the remaining $2R-1$ bits. After simple manipulation, the decoder (56) becomes

$$\hat{\mathbf{c}}_\tau = \arg \max_{\mathbf{c} \in \mathcal{C}_{\text{APSK}}} \Re\{\text{tr}\{\mathbf{x}_\tau^H \mathbf{c} \mathbf{x}_{\tau-1}\}\} \tag{57}$$

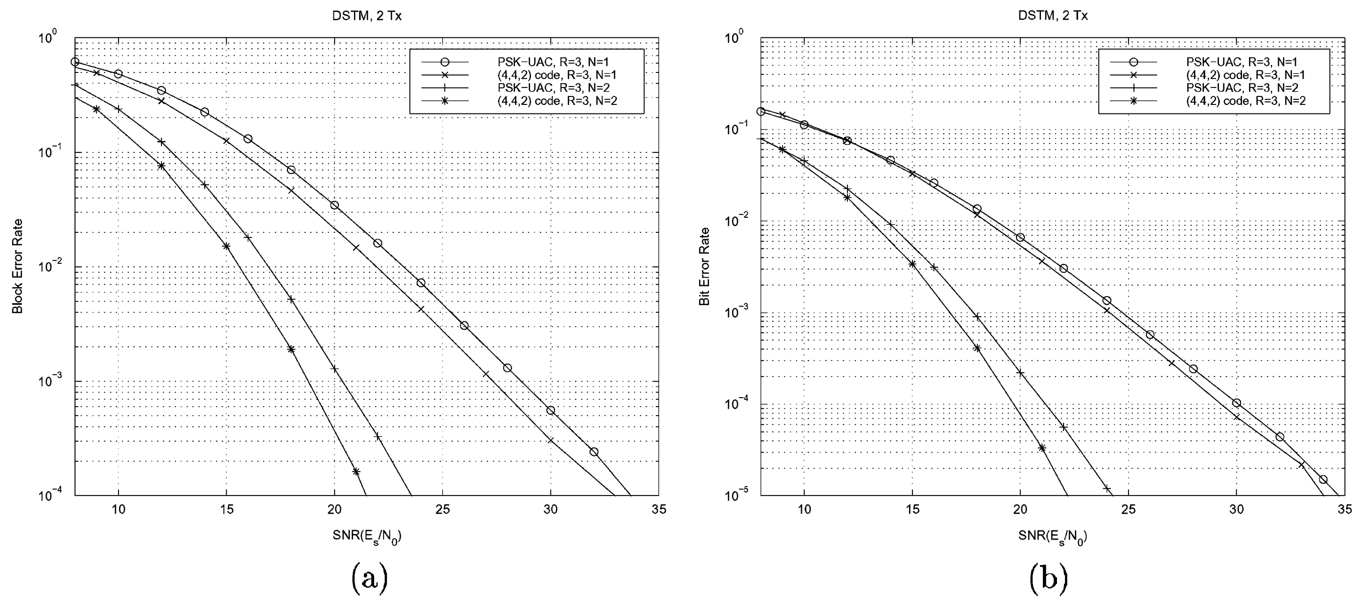

Fig. 2. The performance comparison between the PSK-UA code and the APSK-UA code at $R = 3$ b/s/Hz: (a) BLER and (b) BER.

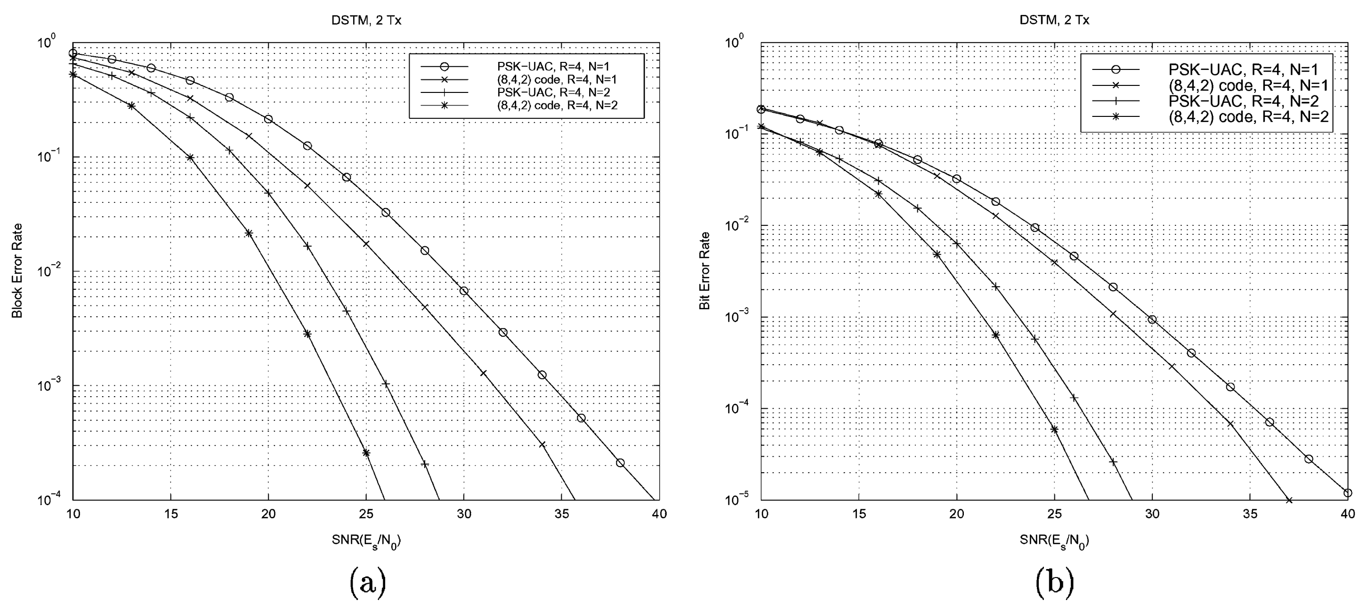

Fig. 3. The performance comparison between the PSK-UA code and the APSK-UA code at $R = 4$ b/s/Hz: (a) BLER and (b) BER.

which is equivalent to (41). Therefore, the decoding algorithm developed in Section III-F can be applied to decode the remaining $2R-1$ bits by the estimation of a_1, ζ_1, and ζ_2.

In the above algorithm, only a search over set $\{1, \beta, 1/\beta\}$ is added with respect to the decoding of APSK-UA codes in DSTM in Section III-D. If the (8, 8, 4) code is used in the 5 b/s/Hz combined scheme, the decoding algorithm needs to calculate g_1, g_2, demodulate eight 8-PSK signals, and search over the eight-element set $\mathbb{A}$ and over set $\{1, \beta, 1/\beta\}$.

V. Simulation Results

In this section, the performance of PSK-UA codes in DSTM and the combined scheme is shown. Simulations confirm the performance results indicated by the diversity products of the codes. The channels in the following simulations are quasi-static. Channel coefficients are constant within a frame of block length 200. Gray mapping is used for the two PSK constellations $\mathbb{S}_1$ and $\mathbb{S}_2$ in PSK-UA codes. For APSK-UA codes, Gray mapping is used for both $\mathbb{S}_0$ and $\mathbb{A}$. BER and BLER are averaged over 10 000 frames. On the x-axis in the following figures, E_s stands for the energy per symbol and E_b stands for the energy per bit, at each receive antenna.

Figs. 2 and 3 show the performance comparison between PSK-UA codes and APSK-UA codes at 3, 4 b/s/Hz in DSTM. Both one and two receive antennas are considered. At $R = 3$ b/s/Hz, the (4, 4, 2) code is 1 dB better than the PSK-UA code at BLER of 10^{-3} with one receive antenna. In terms of BER, the (4, 4, 2) code outperforms the PSK-UA code by 0.5 dB with one receive antenna. However, the BER performance gap is extended to more than 1 dB at BER of 10^{-3} when two receive antennas are used for both codes. At $R = 4$ b/s/Hz, the (8, 4,

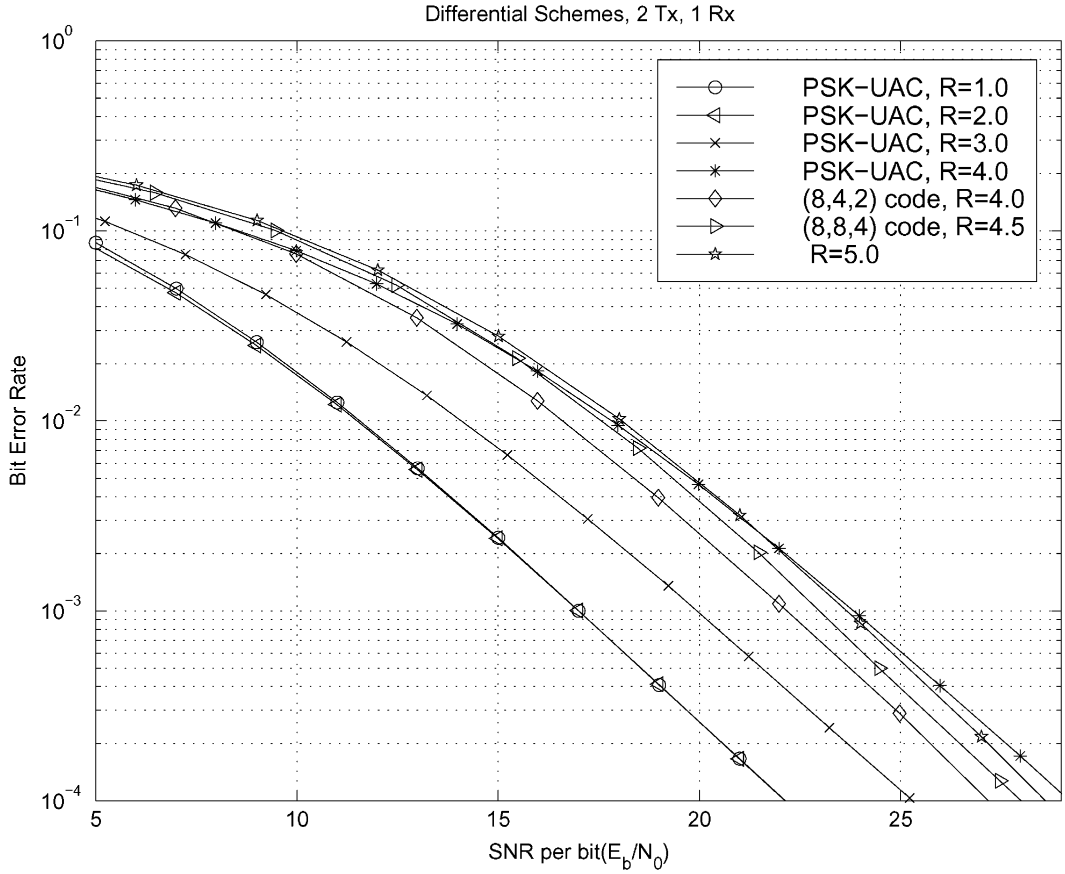

Fig. 4. The performance comparison between PSK-UA codes and APSK-UA codes of different rates with one receive antenna. The $R = 5$ b/s/Hz curve is that of 5 b/s/Hz combined scheme.

2) code is about 2 dB better than the PSK-UA code at BER of 10^{-3} with one receive antenna.

In order to compare BER performance of communication systems at different rates, Fig. 4 shows BER versus SNR per bit. The 5 b/s/Hz combined scheme is also shown in the figure. In the combined scheme, the (8, 8, 4) code is used, as is $\beta = 1.5$. Fig. 4 shows that the 4 b/s/Hz APSK-UA code is only 2 dB worse than the 3 b/s/Hz PSK-UA code while the 4 b/s/Hz PSK-UA code is 4 dB worse. The 4.5 b/s/Hz APSK-UA code is 1 dB better than the 4 b/s/Hz PSK-UA code. The 5 b/s/Hz combined scheme has the same BER performance as the 4 b/s/Hz PSK-UA code.

VI. Conclusion

In this paper, we propose unitary space–time codes from Alamouti's scheme with APSK signals. The resultant unitary codes have larger diversity products than unitary orthogonal space–time codes with PSK signals at 1.5, 2.5, 3, 3.5, 4, and 4.5 b/s/Hz. These codes have also been combined with the two-level block-mean power differential modulation in [9]. In the combined scheme, both the individual information symbol power in Alamouti's scheme and the block-mean power have multiple levels. Such a combined scheme provides a 5 b/s/Hz noncoherent transmission scheme.

Interestingly, at 4 b/s/Hz, our proposed code outperforms the unitary orthogonal space–time codes by 2 dB at BER of 10^{-3} with one receive antenna. The 5 b/s/Hz combined scheme has the same performance as the 4 b/s/Hz unitary orthogonal space–time code in DSTM. Additionally, our codes and scheme both have fast decoding algorithm, whose complexity is comparable to that of unitary orthogonal space–time codes.

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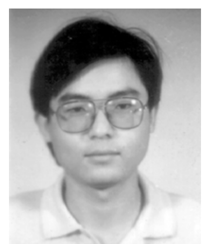

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