A New OFDM System for IIR Channels

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***Abstract*—In this letter, we propose a new orthogonal frequency division multiplexing (OFDM) system for an infinite impulse response (IIR) channel with the form of $B(z)/A(z)$ for two polynomials $A(z)$ and $B(z)$. Different from the conventional OFDM transmission over a finite impulse response (FIR) channel, a guard interval of an OFDM symbol is added such that the corresponding part at receiver is the cyclic prefix (CP) of the received OFDM symbol. The guard interval and CP lengths are the same and not smaller than the orders of polynomials $A(z)$ and $B(z)$. The OFDM symbol without the guard interval is the same as the conventional OFDM symbol without the CP. At the receiver, the IIR channel is then converted to N intersymbol interference (ISI) free subchannels, where N is the number of subcarriers of an OFDM symbol.**

***Index Terms*—Infinite impulse response (IIR) channel, finite impulse response (FIR) channel, intersymbol interference (ISI), orthogonal frequency division multiplexing (OFDM), resonant chamber.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) has been well understood and applied in broadband communications systems these days, such as WiFi and cellular systems. It works well when a channel has finite impulse responses (FIR) and the cyclic prefix (CP) length is not shorter than the FIR channel length. When a channel is too long or infinite impulse responses (IIR), either channel shortened OFDM, see, for example, [2], [4], or time domain equalization, see, for example, [1], [3], [5], has been commonly proposed in the past.

Although most wireless and wireline channels can be well approximated by FIR channels with a reasonable length, some wireless channels may not be done so. Such an example is a resonant chamber or an autonomous factory with well reflected walls [6], where the channel is usually IIR and cannot be well approximated by a short FIR channel. Also, a long FIR channel, which may occur when a channel bandwidth is very wide, may be well approximated by an IIR channel of a small order of the recursive part.

In this letter, we propose a new OFDM system for an IIR channel of the form $B(z)/A(z)$ for two polynomials $A(z)$ and $B(z)$. We first consider the case of pure IIR channel, i.e., when $B(z) = 1$. In this case, if the input and the output are reversed, it would be an FIR channel and the conventional OFDM would work. To do so, the guard interval of an OFDM symbol at the transmitter over the IIR channel is designed such that the corresponding received OFDM symbol block includes a CP, i.e., the CP is automatically formed in the received signal at the receiver. Then, the guard interval of an OFDM symbol at the transmitter is determined by its two neighboring OFDM symbols and the coefficients of $A(z)$ of the IIR channel. Thus, when the coefficients of $A(z)$ of the channel are known at the transmitter, the guard interval of an OFDM symbol can be designed at the transmitter. Assume that the guard interval and CP lengths are the same and not smaller than the order of polynomial $A(z)$. At the receiver, the IIR channel can then be converted to N ISI free subchannels, where N is the number of subcarriers in the OFDM. In this letter we will show how this design is done and how signal can be recovered at the receiver.

Note that in our newly proposed method, the OFDM symbol without the guard interval is the same as a conventional OFDM symbol without the CP and it does not need to convolve with $A(z)$ to compensate the IIR channel at the transmitter as one may usually do when the IIR channel is known at the transmitter. This may be preferred in application scenarios where some users may not have severe intersymbol interference (ISI). Furthermore, it is still an OFDM system.

After a pure IIR channel is studied, we then generalize it to a general mixed FIR and IIR channel of the form $B(z)/A(z)$ without too much difficulty. In this case, the guard interval and CP length is not smaller than the orders of polynomials $A(z)$ and $B(z)$.

This letter is organized as follows. In Section II, we consider a pure IIR channel. In Section III, we consider a mixed FIR and IIR channel. In Section IV, we conclude this letter.

II. OFDM Transmission Over a Pure IIR Channel

We first consider a pure IIR channel $H(z)$, i.e., $H(z) = 1/A(z)$ for a polynomial

$$A(z) = \sum_{k=0}^{G} a(k) z^{-k}, \tag{1}$$

where $a(k)$, $0 \le k \le G$, are constants.

Let x_n and y_n be a transmitted and the corresponding received sequences with their z-transforms $X(z)$ and $Y(z)$, respectively. Then, we have $X(z) = A(z)Y(z)$. If we reverse the transmitter and the receiver, i.e., treat y_n as the transmitted sequence and x_n as the received sequence, then it is an FIR channel and the conventional OFDM transmission works. For this FIR channel $A(z)$, without loss of generality (WLOG), we choose the CP length as G, and consider N subcarrier OFDM with $N > G$.

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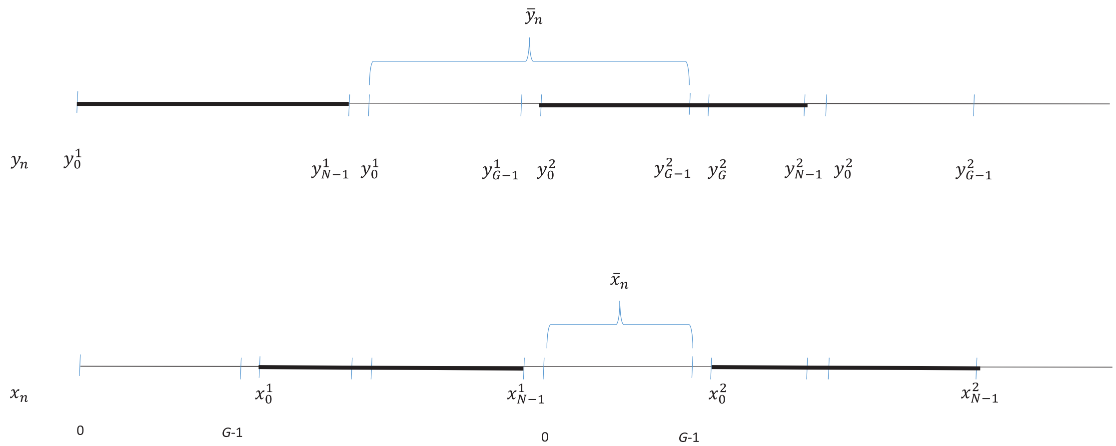

Fig. 1. OFDM signal structure for a pure IIR channel.

Assume that two blocks of sequence y_n, namely $y_0^1, \ldots, y_{N-1}^1$ and $y_0^2, \ldots, y_{N-1}^2$, of the same length N correspond to two blocks $x_0^1, \ldots, x_{N-1}^1$ and $x_0^2, \ldots, x_{N-1}^2$ of sequence x_n, respectively. The sequence y_n after the CP insertion becomes

$$\ldots, \\ y_0^1, \ldots, y_{N-1}^1, y_0^1, \ldots, y_{G-1}^1, y_0^2, \ldots, y_{N-1}^2, y_0^2, \ldots, y_{G-1}^2, \\ \cdots \quad (2)$$

Corresponding to the above sequence y_n, we set sequence x_n as

$$\ldots, x_0^1, \ldots, x_{N-1}^1, \bar{x}_0, \ldots, \bar{x}_{G-1}, x_0^2, \ldots, x_{N-1}^2, \ldots \quad (3)$$

as shown in Fig. 1, where $\bar{x}_n$ is the inserted guard interval that has length G and is to be determined below.

Since $X(z) = A(z)Y(z)$, we have

$$x_n = \sum_{k=0}^{G} a(k) y_{n-k}, \text{ for all integers } n. \quad (4)$$

Then, due to the CP insertion in sequence y_n, we have the following cyclic convolutions as in the conventional OFDM systems

$$x_n^i = a(n) \otimes y_n^i, \ i = 1, 2, \text{ and } 0 \le n \le N-1, \quad (5)$$

where $\otimes$ stands for the N-point cyclic convolution. Also, the $\bar{x}_n$ part shown in Fig. 1 is generated by the $\bar{y}_n$ part, which includes the CP part of y_n^1 and the tail part of y_n^2 of length G, shown in Fig. 1 as follows.

For $0 \le n \le G-1$,

$$\bar{x}_n = \sum_{k=0}^{n} a(k) y_{n-k}^2 + \sum_{k=n+1}^{G} a(k) y_{n+G-k}^1. \quad (6)$$

With the above designs, it is not hard to check that the linear convolution of the channel, (4), indeed holds.

Let

$$X_k^i = \mathrm{FFT}(x_n^i) \text{ and } Y_k^i = \mathrm{FFT}(y_n^i), \ 0 \le k \le N-1, \quad (7)$$

for $i = 1, 2$, where FFT stands for the N-point fast Fourier transform in terms of n. Let

$$A_k = \sum_{n=0}^{G} a(n) W_N^{nk}, \text{ for } 0 \le k \le N-1, \text{ and } W_N = e^{-j\frac{2\pi}{N}} \quad (8)$$

and assume $A_k \neq 0$ for $0 \le k \le N-1$. In practice, this assumption holds almost surely. Then, from (5), we have

$$X_k^i = A_k Y_k^i \text{ and } Y_k^i = \frac{1}{A_k} X_k^i, \ 0 \le k \le N-1, \text{ for } i = 1, 2, \quad (9)$$

which are ISI free, i.e., the original IIR channel $1/A(z)$ is converted to N ISI free subchannels.

Let X_k^i, $0 \le k \le N-1$ and $i = 1, 2$, be $2N$ information symbols to be transmitted. Then, y_n^i, $0 \le n \le N-1$ and $i = 1, 2$, can be solved from (9) and (7) for given X_k^i. With these solved y_n^i, $0 \le n \le G-1$ and $i = 1, 2$, and the channel parameters $a(n)$, $0 \le n \le G$, the guard interval $\bar{x}_n$, $0 \le n \le G-1$, as shown in Fig. 1, of an OFDM symbol can be obtained from (6). With the guard intervals solved above, we obtain a transmitted sequence x_n as shown in Fig. 1, where x_n^i are the blocks each of which includes N information symbols X_k^i, $0 \le k \le N-1$, to send.

At the receiver, after removing the CP parts from the received signal y_n, we obtain blocks y_n^i of block length N. From (9) and (7), the information symbols X_k^i can be solved/demodulated. In practice, when there is additive noise in the channel, more sophisticated and standard receivers can be used for the demodulation of the OFDM signals as before. Another remark is that the guard interval length of $\bar{x}_n$ and the CP length in y_n are the same and only required not smaller than the order G of $A(z)$.

From the above solution of the guard intervals $\bar{x}_n$, we see that it depends on the channel parameters $a(n)$. One might ask if the transmitter knows the channel parameters $a(n)$, why the IIR channel $1/A(z)$ is not compensated by passing the information sequence through the FIR filter $A(z)$ and then at the receiver it becomes the ideal channel. The answer is the following. First, different from compensating the channel at the transmitter, the above proposed method transmits

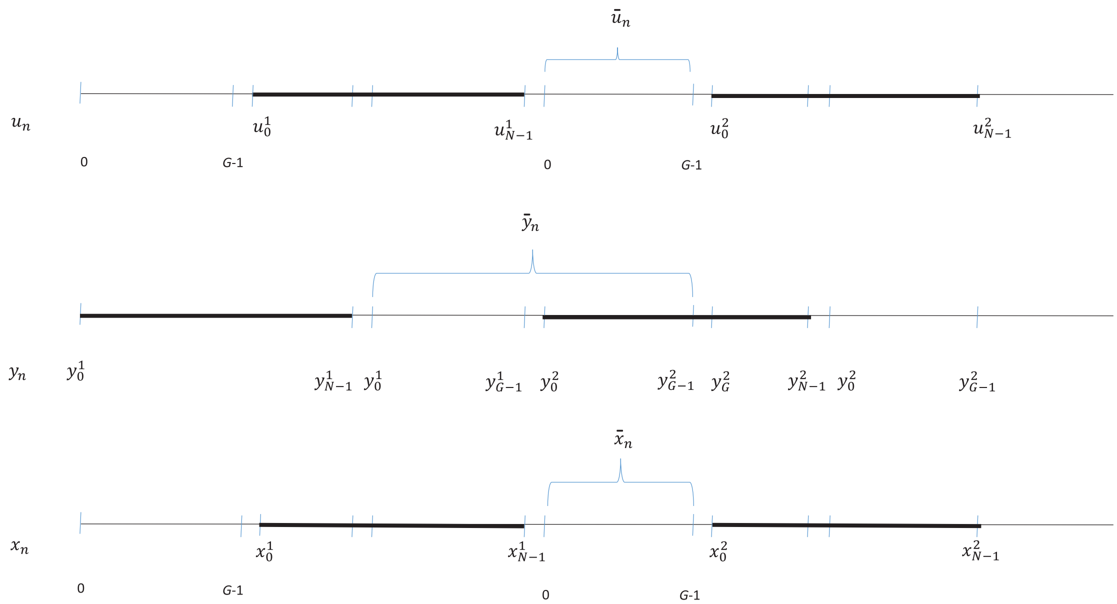

Fig. 2. OFDM signal structure for a mixed IIR channel.

information symbols directly without any intentional distortion in each OFDM block before the guard interval insertion. This may be suitable better for some applications where an IIR channel may not always exist and some users may not even have severe ISI. Second, a channel may be aged at the transmitter and/or the channel information may not be accurate enough at the transmitter, which may directly hurt the information symbols if they are filtered before transmission. Last, this letter proposes a different solution of an OFDM transmission to deal with an IIR channel, even when the IIR channel is known at the transmitter.

III. OFDM Transmission Over a Mixed IIR Channel

After a pure IIR channel was studied in the last section, it is much easier to study a mixed IIR channel in this section. Consider a general mixed FIR and IIR channel of the form:

$$U(z) = \frac{B(z)}{A(z)} X(z) = B(z) Y(z), \text{ with } Y(z) = \frac{1}{A(z)} X(z), \tag{10}$$

where the channel from $X(z)$ to $Y(z)$ is the pure IIR channel studied in the last section. WLOG, assume

$$B(z) = \sum_{k=0}^{G} b(n) z^{-k} \text{ and } A(z) = \sum_{k=0}^{G} a(n) z^{-k}. \tag{11}$$

Corresponding to the sequences x_n composed of x_n^i and $\bar{x}_n$, and y_n composed of y_n^i and $\bar{y}_n$ presented in the last section, the channel output sequence is u_n that is composed of u_n^i and $\bar{u}_n$, respectively. They are shown in Fig. 2 by adding sequence u_n to the top of the sequences y_n and x_n in Fig. 1.

Similar to $\bar{x}_n$ in (6) from the IIR channel part $A(z)$ in the last section shown in Fig. 1, we have $\bar{u}_n$ shown in Fig. 2 from the FIR channel part $B(z)$ as follows.

For $0 \le n \le G-1$,

$$\bar{u}_n = \sum_{k=0}^{n} b(k) y_{n-k}^2 + \sum_{k=n+1}^{G} b(k) y_{n+G-k}^1, \tag{12}$$

which will be removed the same as the CP removal at the receiver in the conventional OFDM systems. After this CP removal, we obtain

$$u_n^i = b(n) \otimes y_n^i, \ 0 \le n \le N-1, \text{ for } i = 1, 2, \tag{13}$$

where $\otimes$ is the N-point cyclic convolution as before. Let

$$U_k^i = \text{FFT}(u_n^i) \text{ and } B_k = \sum_{n=0}^{G} b(n) W_N^{nk}, \ 0 \le k \le N-1, \tag{14}$$

for $i = 1, 2$. Similar to the assumption of $A_k \neq 0$ made in the last section, we assume $B_k \neq 0$, $0 \le k \le N-1$, as well.

Then, from (9) and (13) we have

$$U_k^i = B_k Y_k^i = \frac{B_k}{A_k} X_k^i, \ 0 \le k \le N-1, \text{ for } i = 1, 2, \tag{15}$$

and thus, the information symbols X_k^i can be solved as

$$X_k^i = \frac{A_k}{B_k} U_k^i, \ 0 \le k \le N-1, \text{ for } i = 1, 2, \tag{16}$$

which are ISI free, i.e., the original IIR channel $B(z)/A(z)$ is converted to N ISI free subchannels (15) as in the last section for a pure IIR channel. Similar to what was mentioned in the last section, when there is additive noise in the channel, more sophisticated receivers can be used for the demodulation.

Note that since the guard interval $\bar{x}_n$ in this section is the same as that for a pure IIR channel in the last section, it only depends on the information signals x_n^i to send and the coefficients $a(n)$ in $A(z)$ of the channel, but does not depend on $B(z)$ of the channel. So, the transmitter does not need to know $B(z)$. Also note that the guard interval length of $\bar{x}_n$ and the CP length in y_n are the same and not smaller than the orders of $A(z)$ and $B(z)$.

As a final note, from the above study, it is clear that when the channel is only FIR, i.e., $A(z) = 1$, then, from (9) we have $y_n = x_n$ and the proposed OFDM system returns to the conventional OFDM system.

IV. Conclusion

In this letter, a new OFDM system is proposed for an IIR channel. The key idea is to design a guard interval of an OFDM symbol at the transmitter such that the received OFDM block has a CP structure. With this property, one can reverse the IIR channel to be an FIR channel and the OFDM then works the same as the conventional OFDM. At the receiver, the IIR channel can be then converted to N ISI free subchannels, where N is the number of subcarriers of the OFDM. This letter only presents the main OFDM system design but has not tested it in various situations, such as the case when the coefficients in $A(z)$ of the channel are not accurately known but have errors at the transmitter. More of such studies will follow.

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