Omnidirectional Quasi-Orthogonal Space–Time Block Coded Massive MIMO Systems

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***Abstract*— Common signals in public channels of cellular systems are usually transmitted omnidirectionally from the base station (BS). In recent years, both discrete and consecutive omnidirectional space–time block codings (STBC) have been proposed for massive multiple-input multiple-output (MIMO) systems with a uniform linear array (ULA) configuration to ensure cell-wide coverage. In these systems, constant received signal power at discrete angles, or constant received signal sum power in a few consecutive time slots at any angle is achieved. In addition, an equal-power transmission per antenna and a full spatial diversity can be achieved as well. In this letter, by utilizing the property of orthogonal complementary codes (OCCs), a new consecutive omnidirectional quasi-orthogonal STBC (QOSTBC) design is proposed, in which constant received sum power at any angle can be realized with the equal-power transmission per antenna through one STBC transmission, and a higher diversity order of four can be achieved. Moreover, the proposed design can be further extended to the uniform planar array (UPA) configuration with the two-dimensional OCCs.**

***Index Terms*— Massive MIMO, QOSTBC, omnidirectional transmission, orthogonal complementary codes.**

I. Introduction

To MEET the challenging capacity requirement of the fifth generation (5G), massive multiple-input multiple-output (MIMO) system with tens to hundreds of antennas deployed at base station (BS) has attracted substantial attentions [1]. Public channels play important roles since many essential common signals are provided to users from BS through public channels.

In order to broadcast common information from BS, discrete and consecutive omnidirectional space-time codings have been recently proposed in [2]–[4] for massive MIMO systems under a uniform linear array (ULA) configuration. In [2], [3], by utilizing the Zadoff-Chu (ZC) sequences, equal-power transmission per antenna and constant received signal power at finite discrete angles are satisfied, where the number of discrete angles is the same as the number of transmit antennas. In [4], a design of consecutive omnidirectional space-time coding is proposed, where the orthogonal space-time block code (OSTBC), Alamouti code (AC), is used for 2 data streams, the sum of received signal powers at 2 consecutive time slots is constant at any angle, and equal-power transmission per antenna and diversity order of 2 are achieved as well. However, the design in [4] can only be applied to AC. Although quasi-OSTBCs (QOSTBC) for 4 data streams of diversity order 4 are designed in [3], constant received signal power can be achieved only at finite discrete angles.

In this letter, to further increase the diversity order over the AC coding in [4], by utilizing the orthogonal complementary codes (OCCs) [5], a new consecutive omnidirectional QOSTBC design is proposed, where the received signal sum power in 4 consecutive time slots at any angle is constant, and equal-power transmission per antenna at any time and full diversity order of 4 are achieved as well. We want to emphasize that these three properties are new and additional to all the existing properties of the QOSTBC studies in, for example, [6]– [13]. Unlike the discrete omnidirectional STBCs, the proposed design is insensitive to the number of BS antennas. Moreover, constructed with binary OCCs, high-resolution phase shifters are not necessary at the BS when employing the proposed STBCs, which will significantly reduce the energy consumption and the BS deployment expense. In addition, by utilizing the two-dimensional OCCs (2D-OCCs) [15], similar omnidirectional STBCs can be designed for massive MIMO with uniform planar arrays (UPAs) equipped in BSs.

II. Problem Description

A. System Model

In this letter, we consider STBC transmission for common information broadcasting. For simplicity, here we consider that a BS is equipped with a ULA of M antennas and serves K users each with a single antenna. The common information is mapped to an STBC $\mathbf{S} \in \mathbb{C}^{M\times T}$ with $M \geq T$, and the received signal of user k can be written as

$$[y_{k,1}, y_{k,2}, \ldots, y_{k,T}] = \sqrt{P_t}\mathbf{h}_k^{\mathbb{T}}\mathbf{S} + [z_{k,1}, z_{k,2}, \ldots, z_{k,T}] \quad (1)$$

where P_t denotes the total transmit power, the channel $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ is assumed to keep constant within T time slots, $(\cdot)^{\mathbb{T}}$ stands for the transpose, and $z_{k,t} \sim \mathcal{CN}\left(0, \sigma_n^2\right)$ is the additive white Gaussian noise (AWGN) at time slot t $(t = 1, 2, \ldots, T)$. Note that, since the BS transmits the same common signals to all the users in public channels, there is no interference between different users. Therefore, K can be arbitrary here.

To decode the transmitted information symbols in codeword $\mathbf{S}$, the instantaneous CSI $\mathbf{h}_k$ must be known at the user

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side. However, in massive MIMO systems, since the number of BS antennas is very large and the number of downlink resources needed for pilots is proportional to the number of BS antennas, the downlink channel estimation becomes challenging. In order to reduce the pilot overhead, a dimensional-reduced STBC is utilized in, for example, [2]–[4], where a high dimensional STBC is composed by a precoding matrix $\mathbf{W}$ and a low dimensional STBC $\mathbf{X}$, i.e., $\mathbf{S} = \mathbf{W}\mathbf{X}$, where $\mathbf{W} \in \mathbb{C}^{M\times N}$ is a tall precoding matrix independent of the channel or the information data (since users may be inactive and no feedback is available), and $\mathbf{X} \in \mathbb{C}^{N\times T}$ is a low dimensional STBC modulated by the common information data. To decode the common information data, users only need to estimate the effective channel $\mathbf{W}^{\mathbb{T}}\mathbf{h}_k$ of dimension $N \times 1$ with $N \ll M$ instead. To normalize the total average transmission power at the BS side, we assume that $\mathbb{E}\left(\mathbf{X}\mathbf{X}^H\right) = T/N \cdot \mathbf{I}_N$ and $\mathrm{tr}(\mathbf{W}\mathbf{W}^H) = N$ where $\mathbb{E}(\cdot)$ denotes the expectation, $(\cdot)^H$ donotes the Hermitian operation, $\mathbf{I}_N$ is the $N\times N$ identity matrix and $\mathrm{tr}(\cdot)$ stands for the trace of a matrix.

B. Criteria of Consecutive Omnidirectional STBC

For consecutive omnidirectional STBC design, the following three criteria should be guaranteed [4].

1. Criterion of Constant Instant Transmission Power at Each Antenna at Any Time Slot t: Assume $\mathbf{x}_t$ is the t-th column vector of the low dimensional STBC $\mathbf{X}$, then $\mathbf{W}\mathbf{x}_t$ is the transmitted signal in BS at time slot t. To sufficiently utilize all the power amplifier (PA) capacities of BS antennas, the precoding design should satisfy

$$|[\mathbf{W}\mathbf{x}_t]_m| = 1/\sqrt{M}, \quad m = 1, \ldots M \tag{2}$$

at any time slot t, where $[\mathbf{W}\mathbf{x}_t]_m$ denotes the transmitted signal on the $m-$th antenna at time slot t.

2. Criterion of Full Diversity Order T: The STBC $\mathbf{S} = \mathbf{W}\mathbf{X}$ satisfies the full column rank, i.e., for any two distinct STBC codewords $\mathbf{S}_1$ and $\mathbf{S}_2$ of $\mathbf{S}$, the difference $\mathbf{S}_1 - \mathbf{S}_2$ has full column rank T.

3. Criterion of Constant Received Signal Sum Power at Any Angle: The corresponding transmitted signal in the angle domain under a ULA configuration can be written as

$$S_t(\omega) = \mathbf{a}(\omega) \cdot [\mathbf{W}\mathbf{x}_t], \tag{3}$$

where $\mathbf{a}(\omega) = \left[1, e^{-\mathrm{j}\omega}, \cdots, e^{-\mathrm{j}(M-1)\omega}\right]$ is the antenna array response vector under the ULA setup and $\omega = 2\pi\frac{d}{\lambda}\sin(\theta)$ with carrier wavelength λ, antenna spacing d and azimuth angle of departure (AoD) θ. The criterion is

$$\sum\nolimits_{t=1}^{T} |S_t(\omega)|^2 = c, \quad \forall\theta \in (-\pi, \pi] \tag{4}$$

for a positive constant c. For a UPA configuration in BSs, the criterion can be rewritten as

$$\sum\nolimits_{t=1}^{T} |S_t(\omega, \upsilon)|^2 = c, \quad \forall\theta \in (-\pi, \pi],\ \phi \in [0, \pi] \tag{5}$$

for a positive constant c. Here, $\omega = 2\pi\frac{d_a}{\lambda}\sin(\theta)\sin(\phi)$, and $\upsilon = 2\pi\frac{d_e}{\lambda}\cos(\phi)$ with antenna spacings d_a and d_e, and AoDs θ and ϕ in azimuth and elevation, respectively.

From (3), one can see that, $S_t(\omega)$ is the Fourier transform (FT) of $\mathbf{W}\mathbf{x}_t$. Note that the received signal power of $S_t(\omega)$ at time slot t cannot be constant for any angle θ as mentioned in [4]. It is the motivation in [4] to consider a sum of a few consecutive signal powers as (4).

In this letter, we design precoded QOSTBC for $N = 4$ data streams, i.e., symbol rate 1, with spatial diversity order of 4. Note that OSTBCs for $T = N = 4$ can accommodate at most 3 data streams [14], i.e., their symbol rates are at most 3/4, although they can achieve full diversity order.

C. Orthogonal Complementary Codes

Some useful mathematical results are reviewed here to help the omnidirectional STBC design. Let $\{\mathbf{c}_{i,j}, 1 \le i \le p, 1 \le j \le q\}$ be a set of orthogonal complementary codes, where each code $\mathbf{c}_{i,j}$ is a sequence[1] of length L. Every i-th subset $\{\mathbf{c}_{i,1}, \mathbf{c}_{i,2}, \ldots, \mathbf{c}_{i,q}\}$ is a complementary set of q sequences, and for $i \ne i'$, the i-th and i'-th subsets are mutually orthogonal complementary [5], [15]. The OCC has the following properties.

$$\begin{cases} \sum\nolimits_{j=1}^{q} R_{\mathbf{c}_{i,j}}(\tau) = 0, & \forall\tau \ne 0, 1 \le i \le p \\ \sum\nolimits_{j=1}^{q} R_{\mathbf{c}_{i,j}\mathbf{c}_{i',j}}(\tau) = 0, & \forall\tau, 1 \le i \ne i' \le p \end{cases} \tag{6}$$

with

$$\begin{cases} R_{\mathbf{c}_{i,j}}(\tau) = \sum\nolimits_{l=1}^{L} [\mathbf{c}_{i,j}]_l [\mathbf{c}_{i,j}]^*_{l+\tau} \\ R_{\mathbf{c}_{i,j}\mathbf{c}_{i',j}}(\tau) = \sum\nolimits_{l=1}^{L} [\mathbf{c}_{i,j}]_l [\mathbf{c}_{i',j}]^*_{l+\tau} \end{cases} \tag{7}$$

where $R_{\mathbf{c}_{i,j}}(\tau)$ and $R_{\mathbf{c}_{i,j}\mathbf{c}_{i',j}}(\tau)$ are the aperiodic autocorrelation function (AACF) and the aperiodic crosscorrelation function (ACCF) for shift τ, respectively, $(\cdot)^*$ denotes the complex conjugate, and $[\mathbf{c}_{i,j}]_l$ is the l-th element of $\mathbf{c}_{i,j}$ if $1 \le l \le L$, and 0 otherwise.

III. Omnidirectional QOSTBC Design

OSTBCs have both advantages of complex symbol-wise maximum-likelihood (ML) decoding and full diversity. However, their symbol rates are upper bounded by 3/4 for more than two antennas for complex symbols [14] as mentioned earlier. Therefore, QOSTBCs are proposed in [6], [7] where the orthogonality is relaxed to achieve high symbol transmission rate. By rotating the constellations of the complex symbols, the QOSTBCs can further achieve full diversity [8]–[12]. We next want to study the consecutive omnidirectional STBC design based on the QOSTBCs and OCCs.

Consider the QOSTBC of Tirkkonen, Boariu, and Hottinen (TBH) scheme [7] as an example, and constellation rotations [12] are applied to achieve the diversity order of $T = 4$,

$$\mathbf{X} = \mathbf{X}_Q \triangleq \begin{bmatrix} x_1 & x_2^* & x_3 & x_4^* \\ x_2 & -x_1^* & x_4 & -x_3^* \\ x_3 & x_4^* & x_1 & x_2^* \\ x_4 & -x_3^* & x_2 & -x_1^* \end{bmatrix} \tag{8}$$

where x_1, x_2 are taken from a symbol constellation $\mathcal{S}$, and x_3, x_4 are taken from the rotated symbol constellation $e^{\mathrm{j}\varphi}\mathcal{S}$. The correlation matrix of $\mathbf{X}_Q$ in (8) is

$$\mathbf{X}_Q\mathbf{X}_Q^H = \alpha\mathbf{I}_4 + \beta\mathbf{\Pi}_2 \tag{9}$$

[1] In [4], complementary pair is introduced to construct the precoding matrix $\mathbf{W}$ for AC. However, it does not work for QOSTBC due to the nonorthogonality. Therefore, OCC is introduced here to overcome this problem, and more details are given in Section III.

where $\alpha = \sum_{i=1}^{4}|x_i|^2$, $\beta = 2\mathrm{Re}\left(x_1x_3^* + x_2x_4^*\right)$, $\mathrm{Re}\left(\cdot\right)$ denotes the real part of a complex number, and $\mathbf{\Pi}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{I}_2 \\ \mathbf{I}_2 & \mathbf{0} \end{bmatrix}$. Note that while x_1, x_2, x_3, x_4 are independent with each other, β is not always possible to be 0. Then, for a ULA configuration, (4) can be rewritten as

$$\begin{aligned}\sum\nolimits_{t=1}^{4}&|S_t(\omega)|^2 \\ &= \mathbf{a}(\omega)\mathbf{W}\mathbf{X}_Q\mathbf{X}_Q^H\mathbf{W}^H\mathbf{a}^H(\omega) \\ &= \alpha\mathbf{a}(\omega)\mathbf{W}\mathbf{W}^H\mathbf{a}^H(\omega) \\ &\quad + \beta\mathbf{a}(\omega)\mathbf{W}\mathbf{\Pi}_2\mathbf{W}^H\mathbf{a}^H(\omega) \\ &= \alpha\sum\nolimits_{n=1}^{4}|W_n(\omega)|^2 \\ &\quad + 2\beta\mathrm{Re}\left(\sum\nolimits_{n=1}^{2}W_n(\omega)W_{n+2}^*(\omega)\right) \end{aligned} \tag{10}$$

where $W_n(\omega)$ is the FT of $\mathbf{w}_n$, and $\mathbf{w}_n$ is the n-th column vector of $\mathbf{W}$ for $n = 1, \cdots, 4$.

Theorem 1: If the sequence sets $\{\mathbf{w}_1, \mathbf{w}_2\}$ *and* $\{\mathbf{w}_3, \mathbf{w}_4\}$ *are mutually orthogonal complementary, and* α *is constant for signal constellation* $\mathcal{S}$*, then, the criterion 3 for a ULA configuration, i.e.,* (4)*, of constant received signal sum power with* $T = 4$ *holds for all* $\theta \in (-\pi, \pi]$.

Proof: See Appendix A. □

In addition, the QOSTBC design should satisfy the criterion of equal power at each antenna as well. Therefore, we propose the precoding matrix $\mathbf{W} = \mathbf{W}_Q$ under ULAs as follows.

Assume that the number of BS antennas M is an integer multiple of 4, and $\{\mathbf{c}_1, \ldots, \mathbf{c}_4\}$ is a set of binary OCCs of length $L = M/4$, in which $\{\mathbf{c}_1, \mathbf{c}_2\}$ and $\{\mathbf{c}_3, \mathbf{c}_4\}$ are two sets of complementary pairs of components either 1 or -1 and mutually orthogonal complementary. $\mathbf{W}_Q$ can be designed as

$$\mathbf{W}_Q = \sqrt{4/M}\left[\mathbf{c}_1 \otimes \mathbf{u}_1, \mathbf{c}_2 \otimes \mathbf{u}_2, \mathbf{c}_3 \otimes \mathbf{u}_3, \mathbf{c}_4 \otimes \mathbf{u}_4\right] \tag{11}$$

where $\mathbf{u}_i$ is the i-th column vector of the 4×4 identity matrix $\mathbf{I}_4$, and $\otimes$ denotes the Kronecker product. The signal constellation $\mathcal{S}$ is selected to be a phase shift keying (PSK), i.e., $x_i \in \mathcal{S}_{PSK} = \frac{1}{2}\left\{1, e^{\mathrm{j}2\pi/\vartheta}, \ldots, e^{\mathrm{j}2\pi(\vartheta-1)/\vartheta}\right\}$ for some positive integer ϑ. The optimal rotation angle for PSK signal x_3 and x_4 is π/ϑ when ϑ is even and $\pi/(2\vartheta)$ or $3\pi/(2\vartheta)$ when ϑ is odd [12]. In this way, the precoding design will satisfy all the three criteria as we shall see below.

First, let $c_{n,i}$ be the i-th element of $\mathbf{c}_n$, we have

$$\begin{aligned}\mathbf{S} &= \mathbf{W}_Q\mathbf{X}_Q \\ &= \sqrt{\frac{4}{M}}\begin{bmatrix} c_{1,1}x_1 & c_{1,1}x_2^* & c_{1,1}x_3 & c_{1,1}x_4^* \\ c_{2,1}x_2 & -c_{2,1}x_1^* & c_{2,1}x_4 & -c_{2,1}x_3^* \\ \vdots & \vdots & \vdots & \vdots \\ c_{3,L}x_3 & c_{3,L}x_4^* & c_{3,L}x_1 & c_{3,L}x_2^* \\ c_{4,L}x_4 & -c_{4,L}x_3^* & c_{4,L}x_2 & -c_{4,L}x_1^* \end{bmatrix}. \end{aligned} \tag{12}$$

Since $\mathbf{c}_n$ is a sequence of 1's and -1's, and x_i is constant even with the rotations, it is easy to see that, all the elements in $\mathbf{S}$ have the same amplitude, so the criterion 1 of equal-power transmission per antenna at any time holds.

Since the low dimensional QOSTBC $\mathbf{X}_Q$ in (8) has diversity order of 4 after the optimal angle rotation, and the precoding matrix $\mathbf{W}_Q$ is a constant full column rank matrix, it is clear that the high dimensional QOSTBC $\mathbf{S} = \mathbf{W}_Q\mathbf{X}_Q$ has diversity order $T = 4$, i.e., the criterion 2 of full diversity order holds.

Then, let us prove that it satisfies the constant received signal sum power criterion 3. When PSK signals are adopted, we have $\alpha = \sum_{i=1}^{4}|x_i|^2 = 1$. Let $C_n(\omega)$ be the FT of $\mathbf{c}_n$, so $W_n\left(\omega\right) = \sqrt{4/M}e^{-\mathrm{j}(n-1)\omega}C_n(4\omega)$. According to (10) and Theorem 1, we have

$$\begin{aligned}\sum\nolimits_{t=1}^{4}&|S_t(\omega)|^2 \\ &= \frac{4\alpha}{M}\sum\nolimits_{n=1}^{4}|C_n(4\omega)|^2 \\ &\quad + \frac{8\beta}{M}\mathrm{Re}\left(e^{2\mathrm{j}\omega}\sum\nolimits_{n=1}^{2}C_n(4\omega)C_{n+2}^*(4\omega)\right) = 4 \end{aligned} \tag{13}$$

which is constant for $\theta \in (-\pi, \pi]$ under a ULA configurations.

In summary, the above precoded QOSTBC satisfies all the three criteria presented in Section II. Similarly, the omnidirectional QOSTBC based on Jafarkhani scheme [6] or other schemes can be done and we omit the details here. In addition, to decrease the ML decoding complexity, we also find that the proposed scheme can work for the QOSTBC with minimum decoding complexity (MDC) [10], [13] when QPSK modulation ($x_i \in \frac{1}{2}\{\pm 1, \pm j\}$) is used (refer to the Case 2 and (42) in [13]). Instead of complex symbol pairwise ML decoding, QOSTBC with MDC has real symbol pairwise ML decoding which has the same complexity as complex symbol-wise decoding.

Moreover, for massive MIMO systems under a UPA configuration, similar omnidirectional QOSTBC designs can be done by utilizing the property of 2D-OCCs [15], in which all the three criteria, including (5), can be satisfied. Limited to the space, we omit the details here.

Note that the length of a binary OCC may not be arbitrary. The construction of binary OCC is possible with any length $L = 2^a \cdot 10^b \cdot 26^c$ for all integers a, b, $c \geq 0$, and there is no binary OCC containing two sequences whose length cannot be expressed as a sum of two squares [5], [15]. Therefore, the number of BS antennas may not be arbitrary either for our proposed designs in this letter.

IV. Numerical Results

In this section, we will evaluate the performance of the proposed omnidirectional QOSTBC design under both ULA and UPA configurations. We consider that the BS is a ULA of $M = 64$ antennas or a 8×8 UPA which has 8 antennas in each row and column, respectively. The antenna space is $d_a = d_e = \lambda/\sqrt{3}$, and $K = 300$ users each with a single antenna are served by the BS. The channel model here refers to the model in [3]. To represent that users have different azimuth and elevation angles with respect to the BS, the mean AoDs θ_0 and ϕ_0 follow uniform distribution, and the angle spread (AS) $\sigma = 5°$. The signal constellation in all the schemes is QPSK, while for either the discrete [3] or the preposed precoded QOSTBC, the optimal rotation angle of $\pi/4$ is adopted [12]. The binary OCC, which is shown in Table I [5], [15], is used to obtain the proposed precoding matrix under ULAs.

First, we have the average bit error rate (BER) performances under both ULA and UPA configurations with respect to the signal-to-noise ratio (SNR), i.e., P_t/σ_n^2, as shown in Fig. 1.

TABLE I
BINARY OCC OF LENGTH 16

$\mathbf{c}_1$	1	1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1	1	-1
$\mathbf{c}_2$	1	1	1	-1	1	1	-1	1	-1	-1	-1	1	1	1	-1	1
$\mathbf{c}_3$	1	-1	1	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	1
$\mathbf{c}_4$	1	-1	1	1	1	-1	-1	-1	-1	1	-1	-1	1	-1	-1	-1

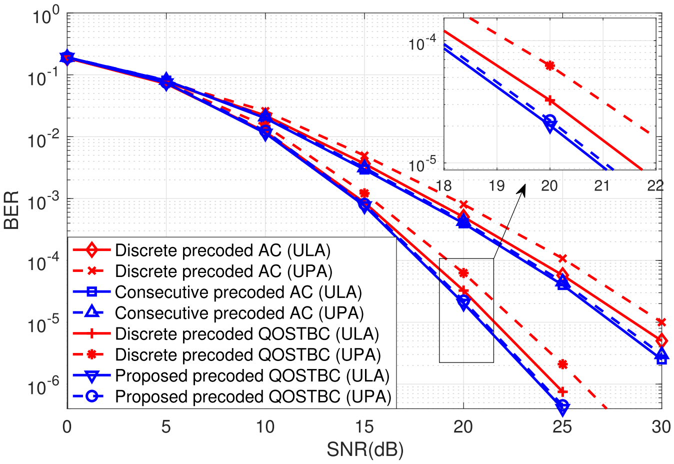

Fig. 1. BER performance versus SNR for 2 bps/Hz.

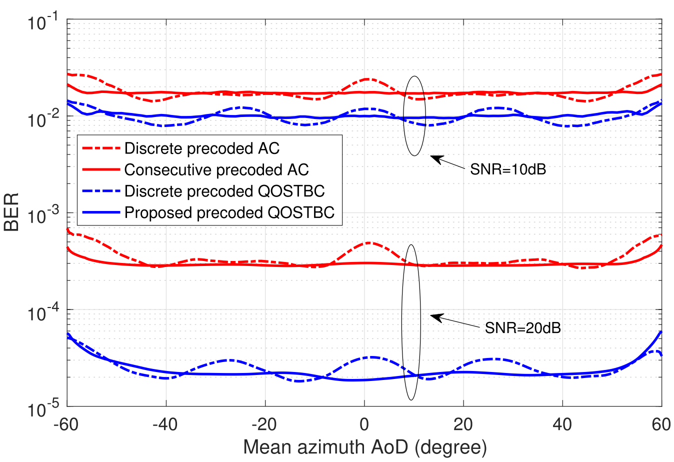

Fig. 2. BER performance versus mean AoD under a ULA configuration.

We can see that all the schemes can achieve full diversity order of 2 or 4, and consecutive precoded STBCs, whether AC [4] or preposed QOSTBC, have almost identical performance under ULAs or UPAs, but outperform the discrete ones obtained in [3] with the same low dimensional STBCs. This is because when the antenna number is limited, the angle resolution of the discrete precoded STBCs is not enough.

Then, we evaluate the omnidirectional transmission performance of proposed precoded QOSTBC. Fig. 2 shows the BER performance with respect to the mean azimuth AoD θ_0 under a ULA configuration. We can see that compared with discrete precoded STBCs, consecutive precoded STBCs have flatter BER performance for different values of mean AoDs, since the sum of the received signal powers in the consecutive precoded STBCs is constant at any angle, rather than just at finite discrete angles. Fig. 3 shows the power radiation pattern,

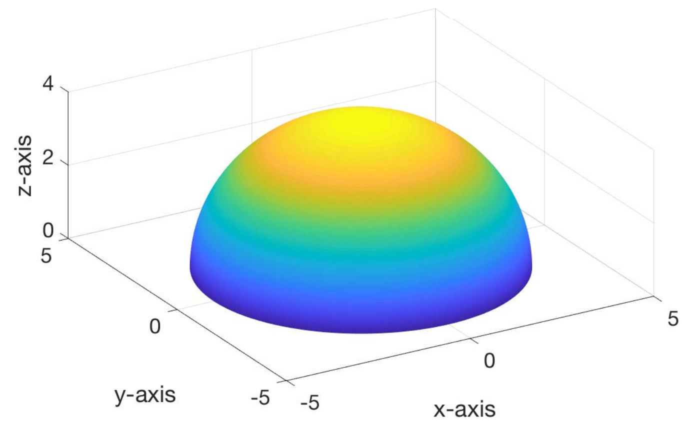

Fig. 3. Power radiation pattern $\sum_{t=1}^{4}|S_t(\omega,\upsilon)|^2$ of the proposed precoded QOSTBC under a 8×8 UPA configuration.

i.e., $\sum_{t=1}^{4}|S_t(\omega,\upsilon)|^2$, of the proposed precoded QOSTBC under a 8×8 UPA, which is perfectly omnidirectional.

V. CONCLUSION

In this letter, a consecutive omnidirectional QOSTBC design is proposed to guarantee omnidirectional transmission, i.e., the sum of $T=4$ consecutive received signal power is constant at any angle, equal instantaneous power on each transmit antenna, and achieve the full diversity of the low-dimensional QOSTBCs. Compared with the discrete omnidirectional precoding design, our proposed QOSTBCs has better omnidirectional transmission performance under either ULA or UPA configurations.

APPENDIX
PROOF OF THEOREM 1

Let $\mathbf{w}_n = (w_{n,m})_{1\le m\le M}$ be the n-th column vector of precoding matrix $\mathbf{W}$, the Fourier transform of $\mathbf{w}_n$ is

$$W_n(\omega)=\sum_{m=1}^{M} w_{n,m}e^{-\mathrm{j}(m-1)\omega}. \tag{14}$$

Thus, $W_n(\omega)W_k^*(\omega)$ can be written as

$$\begin{aligned} W_n(\omega)W_k^*(\omega) &= \sum_{m=1}^{M}\sum_{l=1}^{M} w_{n,m}w_{k,l}^*e^{-\mathrm{j}(m-l)\omega} \\ &\overset{(a)}{=} \sum_{a=0}^{M-1}\sum_{m=1}^{M-a} w_{n,m}w_{k,m+a}^*e^{\mathrm{j}a\omega} \\ &\quad + \sum_{a=1-M}^{-1}\sum_{m=1-a}^{M} w_{n,m}w_{k,m+a}^*e^{\mathrm{j}a\omega} \\ &\overset{(b)}{=} \sum_{a=1-M}^{M-1} R_{\mathbf{w}_n\mathbf{w}_k}(a)\,e^{\mathrm{j}a\omega} \end{aligned} \tag{15}$$

where (a) follows by letting $a=l-m$, and (b) is due to the definition of AACF and ACCF in (7). Letting $\{\mathbf{w}_1,\mathbf{w}_2\}$ and $\{\mathbf{w}_3,\mathbf{w}_4\}$ be mutually orthogonal complementary, with the properties (II-C) of OCC, it is easy to know that

$$\sum_{n=1}^{4}|W_n(\omega)|^2$$

$$= \sum_{a=1-M}^{M-1} \sum_{n=1}^{4} R_{\mathbf{w}_n}(a) e^{\mathrm{j}a\omega} = \sum_{n=1}^{4} R_{\mathbf{w}_n}(0),$$

$$\sum_{n=1}^{2} W_n(\omega) W_{n+2}^*(\omega) = \sum_{a=1-M}^{M-1} \sum_{n=1}^{2} R_{\mathbf{w}_n \mathbf{w}_{n+2}}(a) e^{\mathrm{j}a\omega} = 0.$$

Therefore, (10) can be rewritten as

$$\sum_{t=1}^{4} |S_t(\omega)|^2 = \alpha \sum_{n=1}^{4} R_{\mathbf{w}_n}(0) = \alpha \cdot \mathrm{tr}\left(\mathbf{W}\mathbf{W}^{\mathrm{H}}\right) = 4\alpha \tag{16}$$

which is constant for all $\theta \in (-\pi, \pi]$, and proves the theorem.

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