ELEG 309 Laboratory 0
Lab Instrumentation.
Frequency Response of simple RC circuits

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1 Objectives

The general purpose of this lab is to become familiar with the instrumentation available in the laboratory. Another objective is to refresh the knowledge about the behavior of simple passive circuits. These objectives are accomplished by analyzing in detail the response to various stimuli of five different RC circuits.

2 Theoretical analysis

2.1 Preparation

Figure 1 shows the arrangement for an ohmmeter using a current source and a digital voltmeter. To get the full 1.99 mV readout for 1.99 kΩ, we need a current source $I = 1.99\,\mathrm{mV}/1.99\,\mathrm{k\Omega} = \underline{1\,\mu\mathrm{A}}$. In the second case, a $\underline{0.1\,\mu\mathrm{A}}$ current source would produce 1.99 V across a 19.9 MΩ resistor, well within the 2.5 V limit listed.

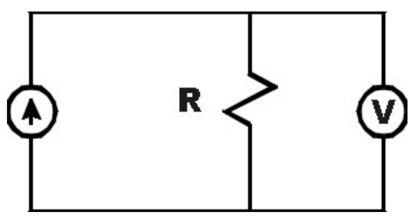

Figure 1: Ohmmeter circuit.

To connect ten resistors totally isolated from one another, we need <u>twenty</u> five-socket strips. To connect them in parallel, we need <u>six</u> strips. To connect them in series, we need <u>eleven</u> strips. To connect them as a series of two groups of five resistors in parallel, we need <u>eight</u> strips.

Given a sinusoidal 10 Vpp signal at f =100 kHz, the maximum rate of change is $2\pi f(10/2) \simeq \underline{3.14\,\mathrm{V}/\mu\mathrm{s}}$. For a trianugular wave of the same amplitude and frequency, the maximum rate of change is $10*2*f = \underline{2\,\mathrm{V}/\mu\mathrm{s}}$. The maximum rate of change in a square wave is limited by a bandwidth. Assuming that the bandwidth upper limit is a result of an STC response, the maximum rate of change can be calculated from $10\exp(-t/\tau)$ by taking its derivative at $t = 0$. $\tau = 1/2\pi f_0$, where $f_0 = 20$ MHz. Hence, the maximum rate of change in this case is $10/\tau = 10*2*\pi*f_0 \simeq \underline{1.26\,\mathrm{kV}/\mu\mathrm{s}}$.

2.2 Explored circuits

To be explored in this lab are simple RC circuits. For each, certain range of frequencies is of interest.

1. A high-pass STC circuit. Frequency response

$$\frac{v_o}{v_i} = \frac{j\,\omega}{j\,\omega + \omega_o}, \quad \omega_0 = \frac{1}{RC}.$$

   Time constant $\tau = RC = 1*10^3*0.1*10^{-6}\,\mathrm{s} = 0.1\,\mathrm{ms}$. Break (or 3 dB) frequency $f_0 = 1/2\pi\tau \simeq 1.6$ kHz.

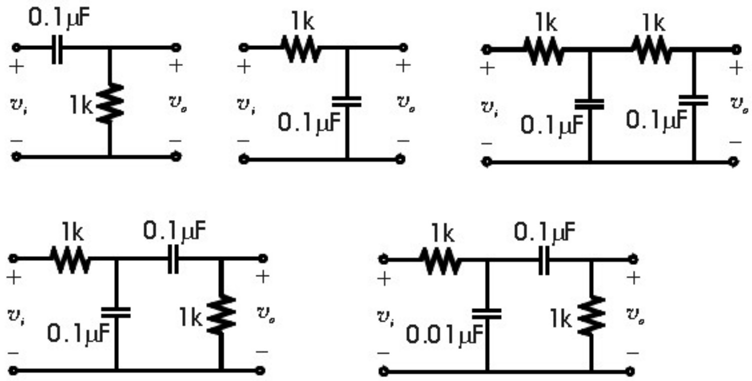

Figure 2: Simple RC cirucuits.

2. A low pas STC circuit. Frequency response
$$\frac{v_o}{v_i} = \frac{1}{1 + j\,\omega/\omega_o}, \quad \omega_0 = \frac{1}{RC}.$$
Time constant $\tau = RC = 1{*}10^3{*}0.1{*}10^{-6}\,\mathrm{s} = 0.1\,\mathrm{ms}$. Break (or 3 dB) frequency $f_0 = \omega_0/2\pi = 1/2\pi\tau \simeq 1.6\,\mathrm{kHz}$.
3. Cascade of two low pass STC-s. The frequency response can be calculated directly from the circuit: it is an $R-Z$ voltage divider followed by an $R-C$ voltage divider, where $Z = 1/(sC)||(R + 1/(sC))$. After simple algebra
$$\frac{v_o}{v_i} = \frac{1}{1 + 3\,j\,\omega\,R\,C - \omega^2\,R^2\,C^2}.$$
The break frequency of each of the low pass STC-s is 1.6 kHz.
4. Low-pass followed by a high-pass STC of the same time constant. Frequency response can be calculated directly from the circuit as above
$$\frac{v_o}{v_i} = \frac{j\,\omega\,R\,C}{1 + 3\,j\,\omega\,R\,C - \omega^2\,R^2\,C^2}.$$
The break frequency of each of the low pass STC-s is 1.6 kHz. The circuit response goes to 0 for both very low frequencies (dc) and very high frequencies.
5. Low-pass followed by a high-pass STC of ten times higher time constant (the capacitor value $0.1\,\mu$F is ten times larger than the $0.01\,\mu$F in the first stage). Frequency response can be calculated directly from the circuit as above
$$\frac{v_o}{v_i} = \frac{j\,\omega\,R\,C}{1 + 2.1\,j\,\omega\,R\,C - 0.1\,\omega^2 R^2 C^2}.$$
Frequencies of interest are 1.6 kHz and 16 kHz. The circuit response goes to 0 for both very low frequencies (dc) and very high frequencies. Between 1.6 kHz and 16 kHz, the response of the circuit is relatively flat.

In each of the above formulas $R = 1\,\mathrm{k}\Omega$ and $C = 0.1\,\mu$F.

3 Procedure

Each of the circuits of Fig. 2 has been assembled in turn on a prototyping board. We connected the oscillator to the input. We also observed the input by connecting it to channel A of the digital oscilloscope. We used 10 Vpp for the input amplitude throughout this lab. The output has been connected to channel B of the oscilloscope, which we used to measure both the input and the output amplitude as well as the phase shift of the output with respect to the input sine-wave.

The amplitudes of the signals have been measured using the oscilloscope built-in function. We obtained the phase shift by measuring the time delay of the output as compared to the input waveform, which we did by using movable cursors on the oscilloscope display. Then the phase-shift is obtained from
$$\phi = 360 f \Delta t$$
where f is the frequency of the input sine-wave and Δt is the measured time delay.

4 Results and discussion

The results are presented in the tables 1 through 5 and in the figures 3 through 12. Figures 3 through 7 show the comparison of the measured frequency response of the circuits with their theoretical counterparts. The theoretical curves have been drawn based on the formulas presented in section 2.2. Good general agreement has been found between the theoretical and experimental values. Circuit 1, for which the amplitude and phase response is presented in Fig. 3 behaves like a high-pass STC circuit with a break frequency of ∼1.6 kHz since below this freqency the output signal is attenuated at a rate of about 20 dB/decade. Above the 1.6 kHz the response approaches a flat 0 dB. This is true for both the theoretical curve, obtained from the first of the formulas presented in section 2.2 and the experimental results. Fig. 4 shows a typical response of a low-pass STC circuit with a break frequency of ∼1.6 kHz. The "low-pass" refers to the fact that frequencies below the break frequency pass relatively unattenuated from the input to the output, whereas high frequencies are blocked. In both cases the theoretical curve closely traces the experimental points both for the amplitude and the phase response.

The frequency response of the third circuit is presented in Fig. 5. It is a low-pass filter, since it lets through low frequencies relatively unafected, but not of the form of an STC circuit. At high frequencies, beyond the break frequency, the slope of the curve tends to approach about –12 dB/octave (–10.6 dB difference between 8 kHz and 16 kHz—see Table 3). This is consistent with the theoretical expectations as the circuit consists of two "stages" of STC circuits, each providing a 6 dB/octave slope at high frequencies. The phase goes from about 0 to almost –180 degrees, which is no surprise as each of the low-pass STC circuit in the cascade provides a phase shift approaching –90 degrees at high frequency. Again, the experimental points line up well with the theoretical curve.

Circuit 4 is a band-pass filter with a center freqency of about 1.6 kHz. The amplitude response curve does not have any flat portion as the break frequency of the low- and high-pass filter constituting the band-pass filter is the same ∼1.6 kHz. The phase goes between –90 and +90 degrees. The general trend of the amplitude response obtained experimentally coincides with the theoretical curve. The signal gets attenuated when its frequency deviates from the center frequency in either direction, hence the name "band-pass" filter. There is, however, some discrepancy between theory and experiment especially at low frequencies and at the peak of the curve. The reason for the discrepancy at the peak might be the extra attenuation introduced by cables and connections that has not been incorporated into the theoretical analysis. The deviation at low and high frequencies will be discussed later. For low and high frequencies the slope of the curve is approximately +20 dB/decade and –20 dB/decade, respectively.

Figure 7 shows the frequency response of the fifth circuit. Very good agreement has been obtained for the amplitude response between theoretical predictions and experimental results. Inside of a certain frequency range (∼800 Hz–∼40 kHz is the 3 dB range) the response is relatively flat. Outside of this range the signal gets attenuated; hence the name: "band-pass" filter. The theoretical and experimental phase curves also coincide, showing, in particular, the slight flattening of the curve in the vicinity of the mid-point, around 5 kHz where the phase is approximately 0 degree.

It is worthwhile to note that in all cases experimental points exhibit a similar deviation from the theoretical curves. For all of the circuits, the theoretical phase curve is steeper than the trend exhibited by experimental points. Also for the amplitude curves, at the sloping portion, the theoretical curve is steeper than the experimental points suggest. The author does not entirely understand the reason for this behavior. One possibility, or rather speculation, is that the cables and connections introduce extra induction, which has not been accounted for in the theoretical analysis. This extra induction would cause flattening of the frequency response of the analyzed circuits.

Note that the deviation of the true resistance and capacitance values from the assumed ones would only shift the position of the break frequency. It cannot account for the deviation in slope observed in this experiment.

Figures 8 through 12 show the response of the circuits to square-wave input. The frequency of the square wave has been selected individually for each circuit to show the most interesting behavior.

Figure 8 shows a typical response of a high-pass STC circuit as described in Appendix F of the textbook. The time constant, or equivalently the break frequency, can be obtained from this curve by fitting $A\exp(-t/\tau)$ to it. Alternatively, one could proceed by identifying the time, at which the magnitude drops to $1/\mathrm{e} \approx 0.37$ of its highest value ~ 2 V. This will give $t = \tau$, which can be read from the graph to be $\sim 100\,\mu$s. It yields $f_0 = 1/(2\pi\tau) \approx 1.6$ kHz, similar to the value that can be read from Figure 3 for the 3 dB drop.

Figure 9 shows a similar curve for a low-pass STC circuit. As before, the time constant can be found by identifying the point of the curve where the value reaches (1–1/e) of the maximum.

Finding specific parameters of the circuits 3, 4, and 5 from the shape of the square response curve is not straightforward. However, their qualitative behavior can be described. Fig. 10 shows a typical response of a low-pass filter (similar to the capacitor charging curve). The steep edges of the square wave, which produce the high harmonics in the frequency spectrum of the waveform, have been rounded. Figures 11 and 12 exhibit a mixed behavior: there is the typical overshot and "discharging" of the high-pass filter and also the rounding of the corners usually atributed to a low-pass filter. Hence, it can be argued that they are a combination of both.

To summarize, the lab served its purpose of refreshing the knowlege about simple passive circuits. Familiarity with

Table 1: Frequency response of the first circuit.

f [Hz]	In [V]	Out [V]	Out/In [dB]	time lag [ms]	phase [deg]
100	10	0.77	-22.2701855	2.24	80.64
200	10	1.422	-16.94200807	1.08	77.76
400	9.9	2.6	-11.61323693	0.5	72
800	9.8	4.5	-6.760271238	0.2	57.6
1000	9.8	5.2	-5.504454641	0.148	53.28
1200	9.8	5.8	-4.555961643	0.112	48.384
1400	9.75	6.3	-3.793281325	0.09	45.36
1600	9.75	6.7	-3.25859626	0.072	41.472
1800	9.7	7	-2.833473885	0.06	38.88
2000	9.7	7.3	-2.468977483	0.049	35.28
3000	9.625	8.19	-1.402336728	0.024	25.92
4000	9.625	8.625	-0.952832689	0.0148	21.312
5000	9.625	8.875	-0.704647529	0.0096	17.28
8000	9.625	9.2	-0.392258217	0.004	11.52
12000	9.625	9.312	-0.287155417	0.0017	7.344
16000	9.562	9.375	-0.171549259	0.00088	5.0688

Table 2: Frequency response of the second circuit.

f [Hz]	In [V]	Out [V]	Out/In [dB]	time lag [ms]	phase [deg]
100	9.938	9.75	-0.165887534	-0.16	-5.76
200	9.875	9.625	-0.222727322	-0.12	-8.64
400	9.812	9.25	-0.512316136	-0.116	-16.704
800	9.812	8.25	-1.50607182	-0.098	-28.224
1000	9.812	7.75	-2.049116741	-0.094	-33.84
1200	9.688	7.25	-2.517922471	-0.09	-38.88
1400	9.688	6.75	-3.138607146	-0.084	-42.336
1600	9.688	6.375	-3.635078821	-0.078	-44.928
1800	9.688	6	-4.161657595	-0.075	-48.6
2000	9.688	5.7	-4.607185489	-0.072	-51.84
3000	9.625	4.375	-6.848453616	-0.056	-60.48
4000	9.625	3.5	-8.786653877	-0.045	-64.8
5000	9.625	2.938	-10.30697893	-0.039	-70.2
8000	9.562	1.953	-13.79692992	-0.026	-74.88
12000	9.562	1.35	-17.00429942	-0.018	-77.76
16000	9.562	1.031	-19.34580148	-0.0138	-79.488

the available instruments has also been improved. The agreement between the experimental values and theoretical predictions has been good.

Table 3: Frequency response of the third circuit.

f [Hz]	In [V]	Out [V]	Out/In [dB]	time lag [ms]	phase [deg]
100	9.938	9.5	-0.391507742	-0.36	-12.96
200	9.812	8.812	-0.933661021	-0.324	-23.328
400	9.75	7.375	-2.424851821	-0.284	-40.896
800	9.688	5.188	-5.424683254	-0.22	-63.36
1000	9.688	4.438	-6.780936645	-0.2	-72
1200	9.625	3.844	-7.972347184	-0.178	-76.896
1400	9.625	3.375	-9.10253922	-0.164	-82.656
1600	9.625	3	-10.12558967	-0.152	-87.552
1800	9.625	2.672	-11.13128569	-0.142	-92.016
2000	9.625	2.422	-11.98453199	-0.133	-95.76
2400	9.625	1.984	-13.71718141	-0.12	-103.68
3000	9.625	1.531	-15.96851095	-0.103	-111.24
4000	9.625	1.056	-19.1947364	-0.082	-118.08
5000	9.625	0.769	-21.94948797	-0.071	-127.8
8000	9.562	0.368	-28.29401841	-0.05	-144
12000	9.562	0.182	-34.40954703	-0.0356	-153.792
16000	9.562	0.108	-38.94249968	-0.0272	-156.672

Table 4: Frequency response of the fourth circuit.

f [Hz]	In [V]	Out [V]	Out/In [dB]	time lag [ms]	phase [deg]
100	9.938	0.725	-22.73921972	2	72
200	9.812	1.306	-17.51628725	0.84	60.48
400	9.75	2.094	-13.36055877	0.31	44.64
800	9.688	2.797	-10.79083328	0.078	22.464
1000	9.625	2.938	-10.30697893	0.045	16.2
1200	9.625	3.016	-10.07938802	0.023	9.936
1400	9.625	3.047	-9.990565679	0.009	4.536
1600	9.625	3.047	-9.990565679	0	0
1800	9.625	3.047	-9.990565679	-0.008	-5.184
2000	9.625	3.031	-10.03629604	-0.01	-7.2
2400	9.625	2.953	-10.26274582	-0.0176	-15.2064
3000	9.625	2.812	-10.68770844	-0.0212	-22.896
4000	9.625	2.547	-11.54743586	-0.0228	-32.832
5000	9.625	2.328	-12.32835524	-0.022	-39.6
8000	9.562	1.719	-14.90545725	-0.0192	-55.296
12000	9.562	1.269	-17.54174235	-0.0144	-62.208
16000	9.562	0.988	-19.7158359	-0.0118	-67.968

Table 5: Frequency response of the fifth circuit.

f [Hz]	In [V]	Out [V]	Out/In [dB]	time lag [ms]	phase [deg]
100	9.938	0.75	-22.44475458	2.1	75.6
200	9.812	1.369	-17.10708183	0.94	67.68
400	9.812	2.328	-12.49549127	0.38	54.72
800	9.812	3.422	-9.149550687	0.128	36.864
1000	9.75	3.703	-8.409018073	0.09	32.4
1200	9.688	3.906	-7.890037824	0.066	28.512
1400	9.688	4.062	-7.549884221	0.049	24.696
1600	9.688	4.156	-7.351171825	0.038	21.888
1800	9.75	4.25	-7.212313713	0.03	19.44
2000	9.75	4.281	-7.149187757	0.0232	16.704
2400	9.75	4.438	-6.836346356	0.016	13.824
3000	9.812	4.5	-6.770900515	0.0092	9.936
4000	9.812	4.56	-6.655853937	0.0032	4.608
5000	9.812	4.562	-6.652045172	0	0
8000	9.812	4.5	-6.770900515	-0.0022	-6.336
12000	9.812	4.406	-6.954260934	-0.0035	-15.12
16000	9.688	4.188	-7.284549143	-0.0035	-20.16
20000	9.688	4	-7.683482776	-0.0035	-25.2
24000	9.688	3.812	-8.101624763	-0.0035	-30.24
30000	9.688	3.5	-8.843321716	-0.0035	-37.8
40000	9.625	3.062	-9.947911036	-0.0032	-46.08
80000	9.625	1.875	-14.20798932	-0.0022	-63.36
160000	9.688	1.037	-19.40910747	-0.0013	-74.88

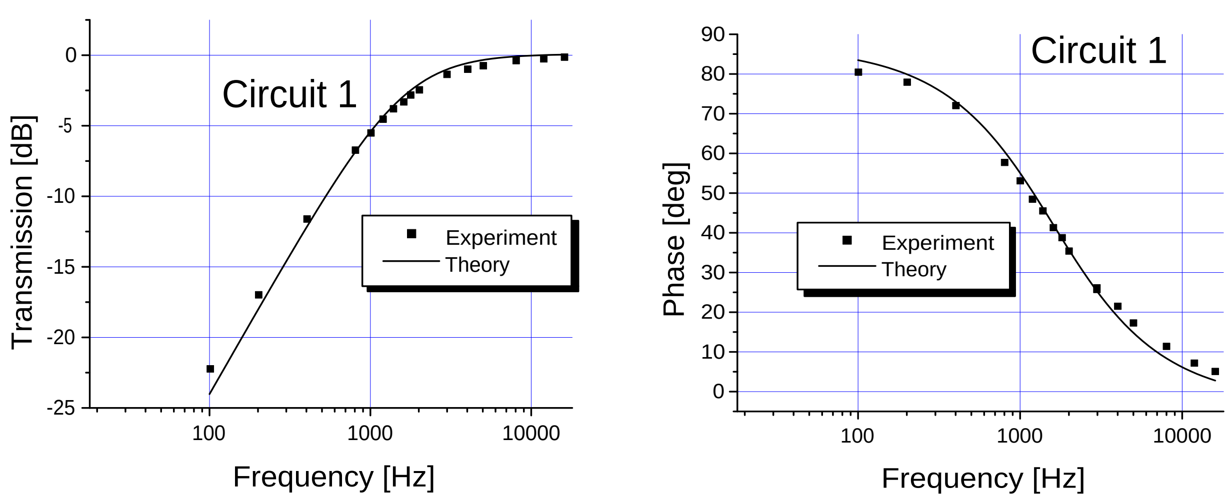

Figure 3: Transmission and phase versus frequency (log scale) for the first circuit (high-pass).

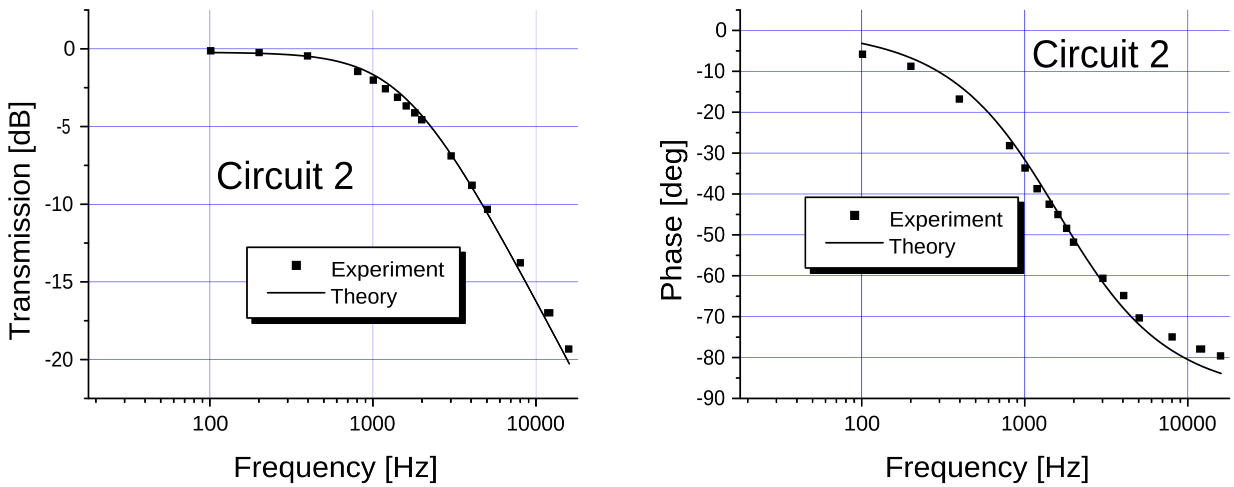

Figure 4: Transmission and phase versus frequency (log scale) for the second circuit (low-pass).

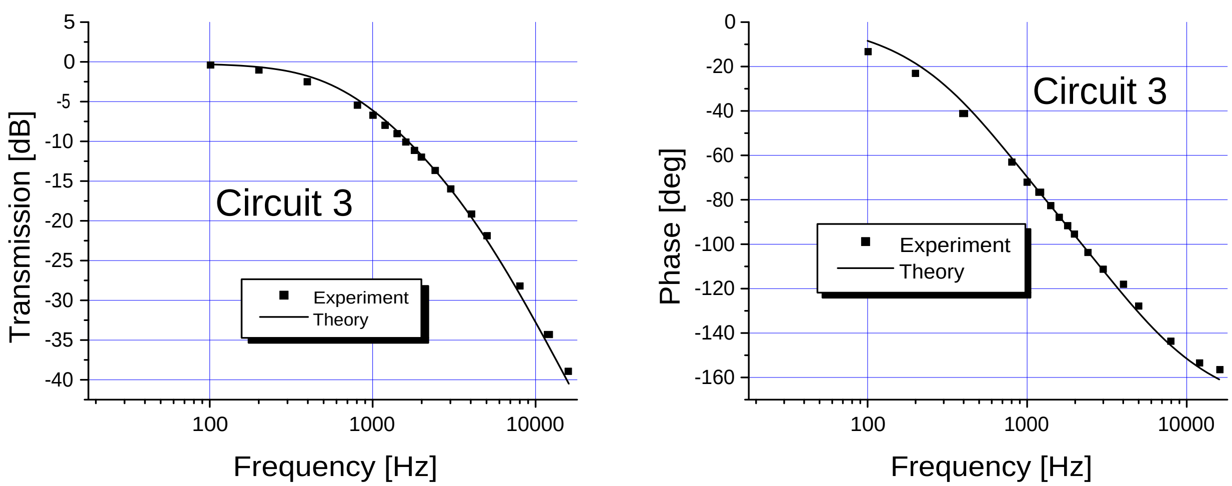

Figure 5: Transmission and phase versus frequency (log scale) for the third circuit (low-pass).

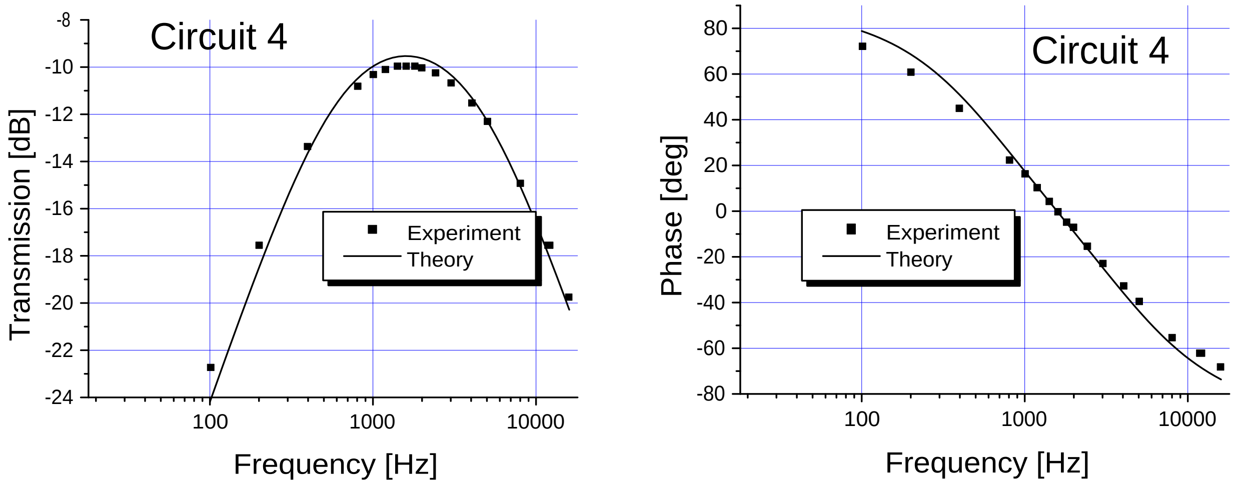

Figure 6: Transmission and phase versus frequency (log scale) for the fourth circuit (band-pass).

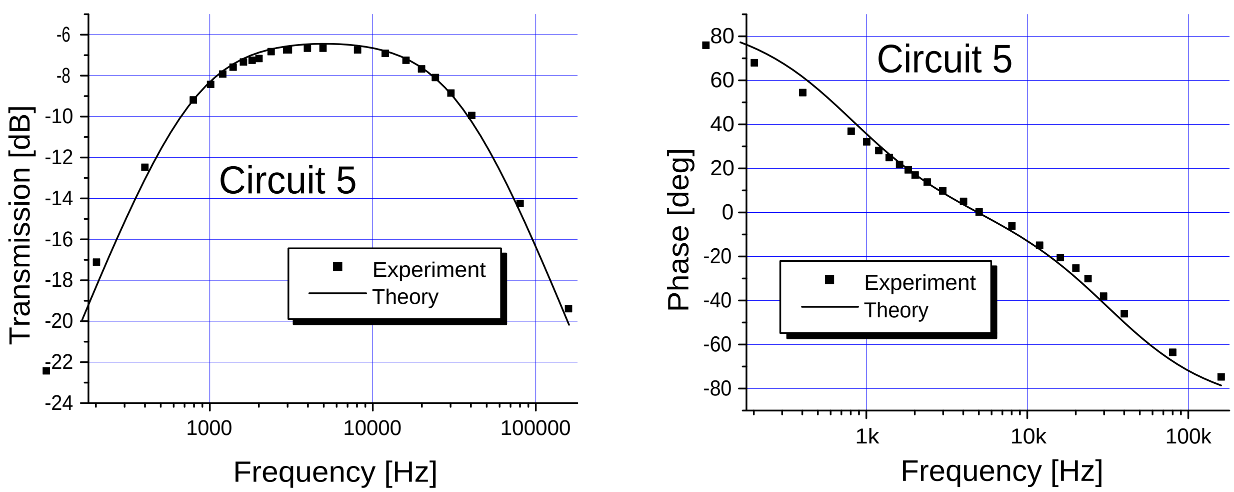

Figure 7: Transmission and phase versus frequency (log scale) for the fifth circuit (band-pass).

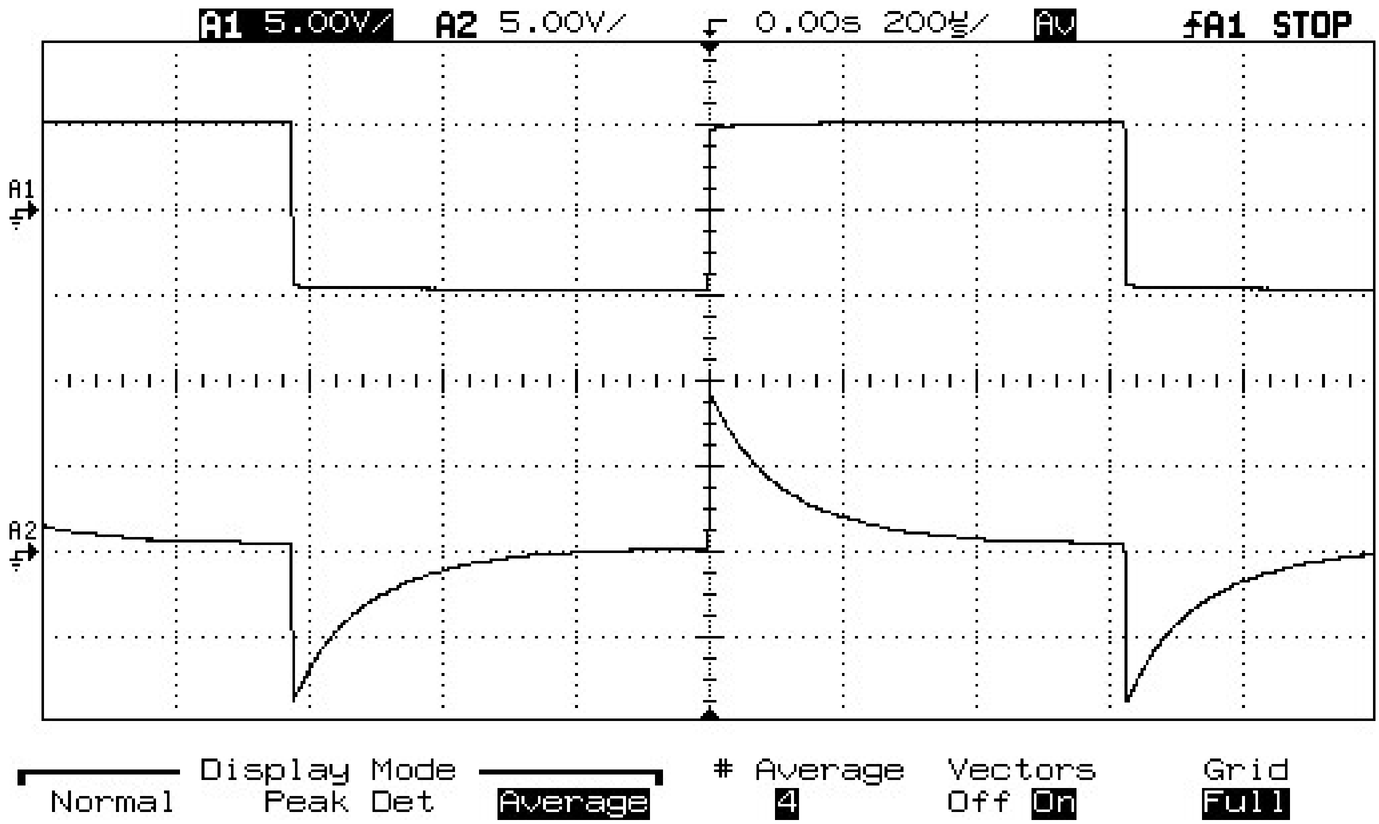

Figure 8: Response of the first circuit to the square wave input.

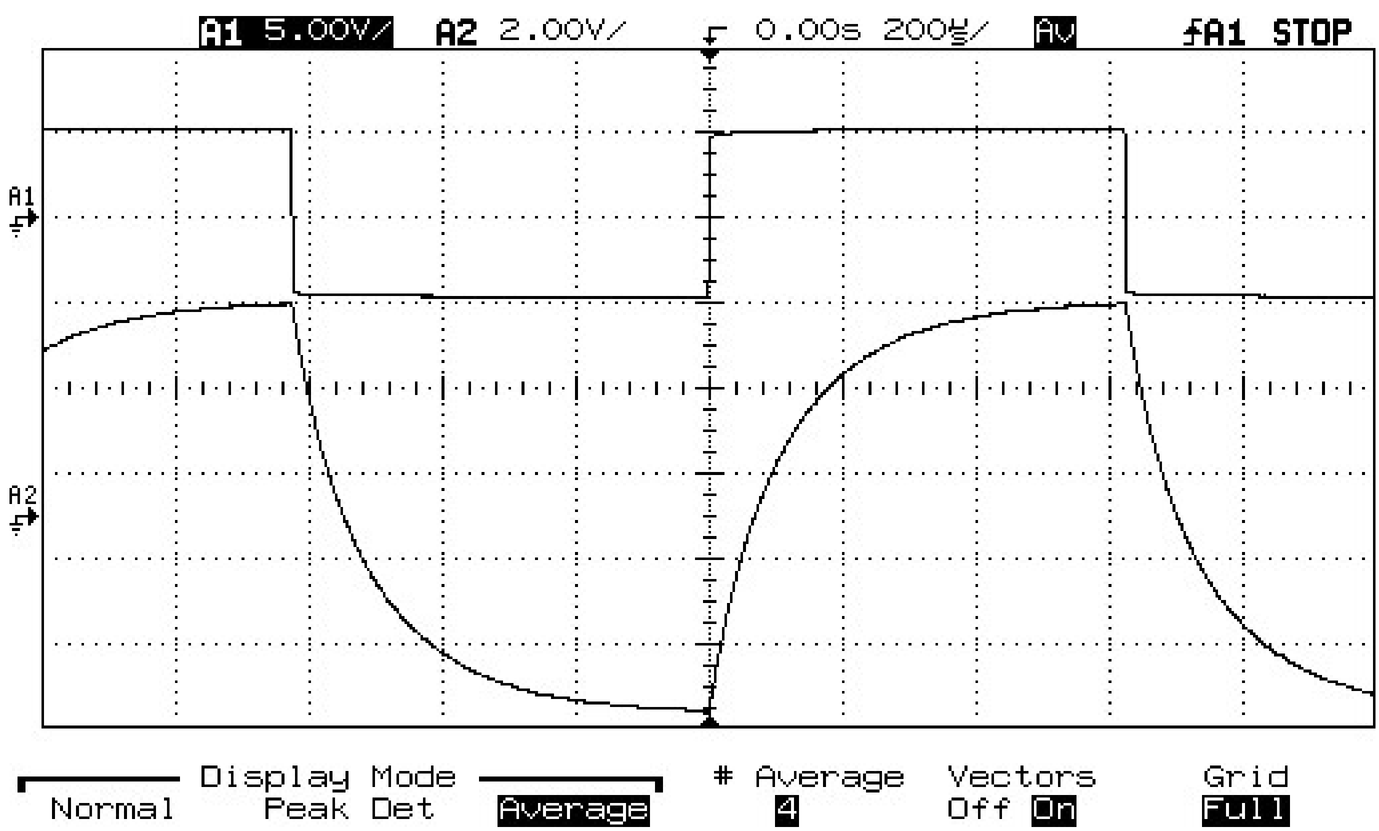

Figure 9: Response of the second circuit to the square wave input.

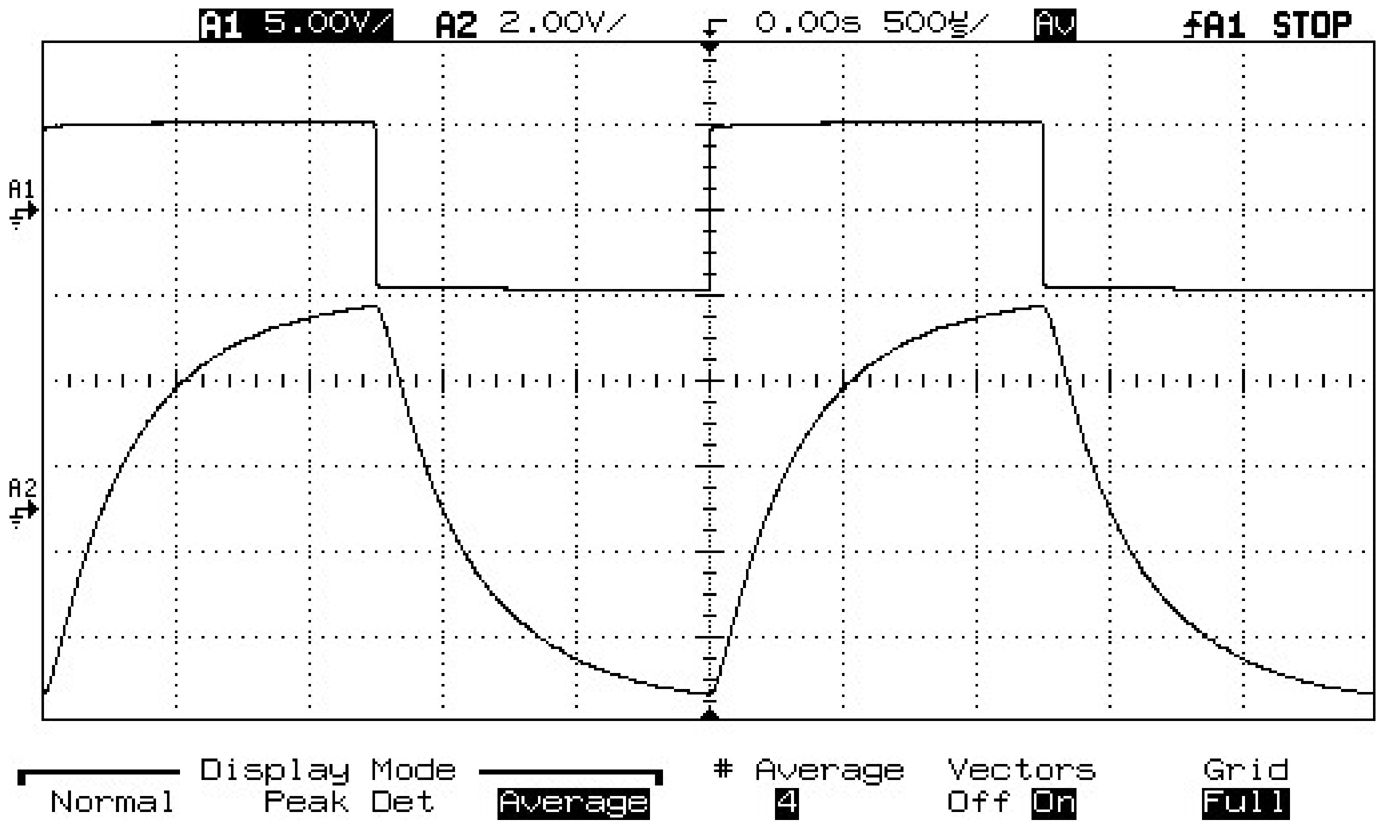

Figure 10: Response of the third circuit to the square wave input.

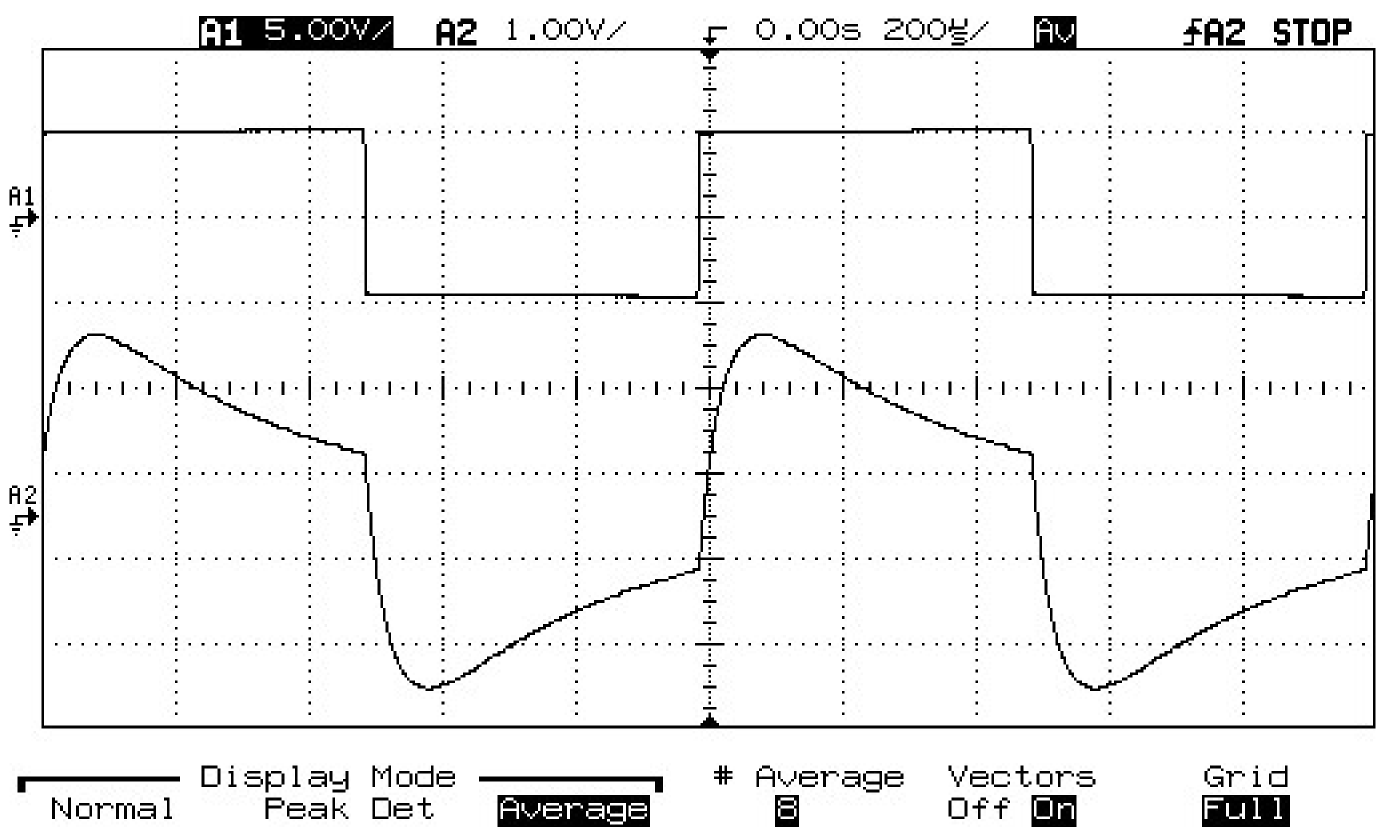

Figure 11: Response of the fourth circuit to the square wave input.

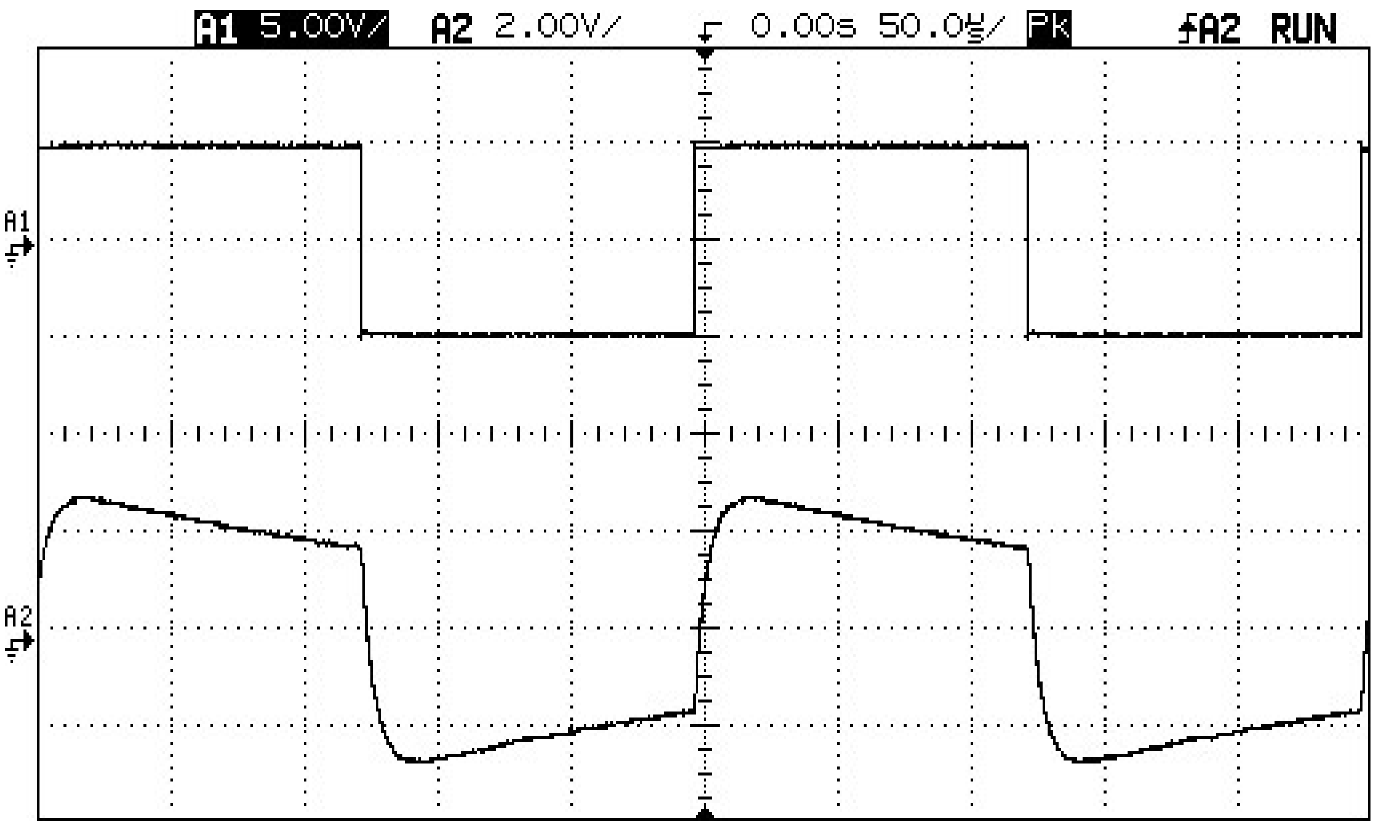

Figure 12: Response of the fifth circuit to the square wave input.