DIFFUSION OF CARRIERS

Diffusion currents are present in semiconductor devices which generate a spatially non-uniform distribution of carriers. The most important examples are the p-n junction and the bipolar transistor, whose functions are based upon the diffusion of minority carriers within bulk regions of the opposite type.

To be specific, let us consider a non-uniform distribution of excess holes, $\delta p(x) = p(x) - p_0$, within an n-type region as illustrated in Fig. 4-17. Where $\delta p(x)$ is positive, the recombination rate for holes will be larger than the constant thermal generation rate. If no additional source of holes were present, the excess population $\delta p(x)$ would thus decay by recombination until $p(x) = p_0$ everywhere at equilibrium. At equilibrium, the lower recombination rate just balances thermal generation. The nonequilibrium distribution $\delta p(x)$ shown in the figure is maintained constant in time, however, by injecting more holes at a uniform rate through the boundary of the n-region at x=0. The way in which this is achieved provides the operating basis for p-n junctions. For now, we simply want to consider the consequences of having this sort of non-uniform minority carrier distribution, regardless of how it is created.

Whenever p(x) depends upon spatial location, the random thermal motion of the carriers will tend to even out the distribtion. The holes near the top of the valence band behave exactly like independent classical particles with an effective mass m_p^*. The average thermal velocity of these holes in thus given by equating their mean kinetic energy $\frac{1}{2} m_p^* \overline{V}_{th}^{\,2}$ to $\frac{1}{2} kT$, so that:

$$\begin{aligned}\overline{V}_{th} &= c\left(\frac{kT}{m_p^* c^2}\right)^{\frac{1}{2}} \\ &= 3\times10^{10}\ \mathrm{cm\,s^{-1}} \times \left(\frac{0.0259\mathrm{eV}}{0.511\times10^{6}\mathrm{eV}}\right)^{\frac{1}{2}}\left(\frac{m_e}{m_p^*}\right)^{\frac{1}{2}} \qquad (1)\\ &= 6.7\times10^{6}\ \mathrm{cm\,s^{-1}} \times \left(\frac{m_e}{m_p^*}\right)^{\frac{1}{2}}\end{aligned}$$

Although the holes are moving very rapidly with $\overline{V}_{th} \approx 10^7$ cm s^{-1} through the crystal, they don't go very far in any direction before being scattered. At room temperature, collisions with lattice vibrations limit their mean-free paths to distances of order:

$$\ell \sim 100\ \text{Å to } 1000\ \text{Å} \qquad (2)$$

Another way to characterize these velocity-changing collisions is by a collision time τ_c defined according to:

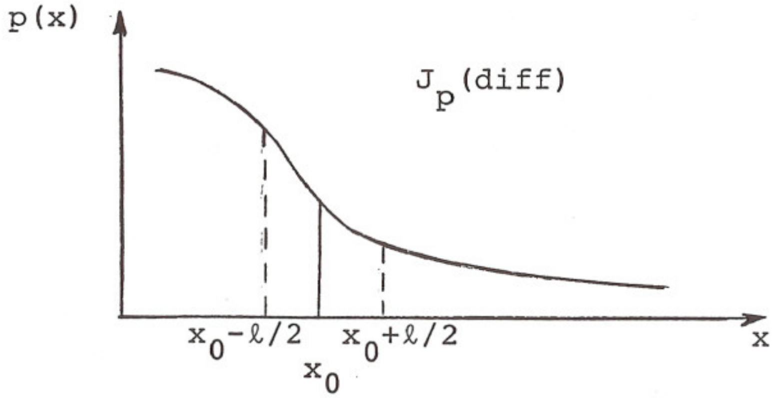
$$\begin{aligned}\tau_c &= \ell/\overline{V}_{th} \\ &\sim 10^{-12}\mathrm{s} \text{ to } 10^{-13}\mathrm{s}\end{aligned} \qquad (3)$$

Each hole undergoes random motion at an average velocity $\overline{V}_{th}$ between collisions. An example of the path traced out by one such carrier is shown in Fig. 3-20. If the spatial distribution of holes is uniform, it will not be altered by this random motion. If the distribution is non-uniform, however, the random motion of individual particles will result in a net flow from regions of high concentration toward regions of lower concentration.

The magnitude of the diffusion current is proportional to the concentration gradient:

$$J_p(\text{diff}) = -q\,D_p \frac{dp(x)}{dx} \tag{4}$$

and D_p is known as the diffusion coefficient. We can understand this result on the basis of our simple model of random thermal motion by the following argument. Consider the point $x = x_0$ in the non-uniform hole distribution illustrated here.



The average hole crossing x_0 in the +x direction had its last collision at $(x_0 - \ell/2)$, whereas the average hole crossing x_0 in the -x direction had <u>its</u> last collision at $(x_0 + \ell/2)$. In effect, one-half of the holes at $x = (x_0 - \ell/2)$ take their next step to the right, while one-half of the holes at $x = (x_0 + \ell/2)$ go left. Since all the holes have the same average thermal velocity $\overline{V}_{th}$, the net particle current at x_0 is given by:

$$\frac{J_p(\text{diff})}{q} = \frac{1}{2}\, p\,(x_0 - \ell/2)\overline{V}_{th} - \frac{1}{2}\, p\,(x_0 + \ell/2)\overline{V}_{th}$$

$$= -\frac{1}{2}\,\overline{V}_{th}\ell\left(\frac{p(x_0+\ell/2) - p(x_0-\ell/2)}{\ell}\right) \qquad (5)$$

The quantity in brackets is just $dp(x)/dx$, since the mean-free path ℓ is a very small distance. Comparing this result with Eq. (4), diffusion coefficient is found to be:

$$D_p = \frac{1}{2}\overline{V}_{th}\ell = \frac{1}{2}\,\frac{\ell^2}{\tau_c} \qquad (6)$$

The units of D_p are $cm^2\ s^{-1}$, and using the above expressions for $\overline{V}_{th}$ and ℓ yields an estimate for typical values of the diffusion coefficient:

$$D_p \sim 10\ cm^2\, s^{-1} \text{ to } 100\ cm^2\, s^{-1} \qquad (7)$$

Note that $J_p(\text{diff})$ here represents an electric current density, equal to the particle density $-\,D_p\, dp/dx$ multiplied by the charge $+q$ for each hole. The same arguments apply to electrons as well as holes, of course, except that the charge is $-q$ per carrier:

$$J_n(\text{diff}) = (-q)\left(-D_n\,\frac{dn(x)}{dx}\right)$$

$$= +q\,D_n\,\frac{dn(x)}{dx} \qquad (8)$$

The mean-free path for electrons will in general be different from that for holes, so that D_n is not identical to D_p.

There is a direct relationship between mobility and diffusion coefficient, called the Einstein relation, that can be illustrated within the context of our simple model. Acceleration of holes by an electric field is characterized by the equation of motion:

$$m_p^* \frac{dv}{dt} = q\,E \tag{9}$$

But the velocity of each hole is increased in the direction of the field only over an interval of order τ_c between collisions. After each collision, the velocity distribution is randomized and the acceleration along the field must start over again. This yields an average drift velocity super-imposed on the random thermal motion:

$$\delta V_{drift} = \frac{qE}{m_p^*}\,\tau_c \tag{10}$$

The current density that results from this drift of holes in an electric field is:

$$\begin{aligned} J_p(\text{drift}) &= q\,p(x)\,\delta V_{drift} \\ &= q\,p(x)\,\mu_p E(x) \end{aligned} \tag{11}$$

where the mobility is defined as:

$$\mu_p = \frac{\delta V_{drift}}{E} = \frac{q\tau_c}{m_p^*} \tag{12}$$

The Einstein relation may be found by combining Eqs. (6) and (12):

$$\frac{D_p}{\mu_p} = \frac{1}{q}\,(1/2\; m_p^* \overline{V}_{th}^{\,2}) = \frac{kT}{q} \tag{13}$$

This relation is more general than our simple model, and it allows the diffusion coefficient to be inferred from a measurement of the mobility.

The Haynes-Shockley experiment provides a direct illustration of the effects of diffusion. A light pulse creates ΔP excess holes in n-type material at x=0 in Fig. 4-18. If there were no electric field to produce an average drift velocity, the distribution of excess holes would remain centered at x=0 and spread out with time according to the solution of the diffusion equation (4-43) given in Eq. (4-44):

$$\delta p(x,t) = \frac{\Delta P}{2\sqrt{\pi D_p t}} e^{-x^2/4 D_p t} \tag{14}$$

We can understand the form of this result in the following way. Diffusion is fundamentally a random-walk process. Each carrier takes a succession of steps whose average length is ℓ, the mean-free path, and each of these steps is equally likely to be in either the +x or -x direction. After N steps, the mean-square displacement of a particle from its initial position is given by:

$$\Delta x_{rms}(t) \approx \ell \sqrt{N} \tag{15}$$

The individual particle is equally likely to be displaced in either the +x or -x direction, and all displacements between $\Delta x = 0$ and $\Delta x = N\ell$ are possible, so that $\Delta x_{rms}(t)$ represents the width of a distribution for a large number of holes that all start off from x=0 at t=0. This mean-square displacement increases as the square root of t, since the number of steps is given by $N \approx t/\tau_c$ where τ_c is the interval between collisions. Therefore:

$$\Delta x_{rms}(t) \approx \ell\sqrt{t/\tau_c} \approx \sqrt{2D_p t} \tag{16}$$

and the proportionality constant here is seen to be the diffusion coefficient defined in Eq. (6). The exact solution of Eq. (4-43) has precisely the form we expect. It is a Gaussian distribution given by Eq. (14) whose width spreads out with time according to the expression for $\Delta x_{rms}(t)$ derived here (within a factor of 2 or so).

In the Haynes-Shockley experiment, an electric field is applied along the bar in Fig. 4-18 so that the distribution created by diffusion is caused to drift underneath the detector at x=L. The effect of the electric field may be included by letting $x \rightarrow (x - \mu_p \mathcal{E} t)$ in Eq. (4-44) or Eq. (14) to give a distribution centered about $x_0(t) = \delta V_{drift}\, t$. Recombination of the excess holes can be taken into account by simply multiplying the entire distribution by the factor e^{-t/τ_p}, where τ_p is the lifetime for minority holes in the n-region. We note that the lifetime can vary greatly with material and impurity concentration, so that typical values can lie within a very wide range $10^{-3}s \gtrsim \tau_p \gtrsim 10^{-9}s$, but τ_p will always be long compared to the collision time τ_c.

Finally, we return to the non-equilibrium steady-state distribution of Fig. 4-17 in which holes are injected into the n-region at x=0. The solution to the diffusion equation which describes this case is given in Eq. (4-36):

$$\delta p(x) = \Delta p(x=0)\ e^{-x/L_p} \tag{17}$$

where $L_p = \sqrt{D_p \tau_p}$ is known as the diffusion length. The form of this result can also be understood very simply. An excess hole which begins to diffuse into the n-region from x=0 will, on the average, live for a minority carrier lifetime τ_p before being destroyed by recombination with an electron. The average number of "steps" taken by a hole within the n-region is therefore $\overline{N} \approx \tau_p/\tau_c$. The mean distance that these holes diffuse into the n-region can then be obtained approximately using the random-walk formula:

$$L_p = \sqrt{\overline{N}}\ \ell \approx \sqrt{D_p \tau_p} \tag{18}$$

Typical values are $L_p \sim 10\ \mu m$ to $100\ \mu m$, but the diffusion length can sometimes lie outside this range.

The exponential decay of $\delta p(x)$ over this distance L_p in Eq. (17) represents an average distribution created by large numbers of holes as they diffuse into the n-region and recombine. This diffusion process produces a current that may be calculated using Eq. (4):

$$J_p(\text{diff}) = q\ \frac{D_p}{L_p}\ \Delta p(x=0)\, e^{-x/L_p} \tag{19}$$

This hole current falls off exponentially over a diffusion length L_p as the excess holes recombine. In order to preserve a steady-state, electrons must be fed in from the right in Fig. 4-17 to replace those that are destroyed by recombination with the injected holes. At large $x >> L_p$, the total electron current required is the same as the number of holes crossing at x=0:

$$J_n(x >> L_p) = J_p(x=0)$$
$$= q \frac{D_p}{L_p} \Delta p(x=0) \qquad (20)$$

The concentration $n_0 = N_D$ of electrons is high in the bulk n-region, so that this electron current at large x can be produced by a very tiny elecric field imparting a very slow drift velocity to all of the electrons. The slow drift of electrons toward x=0 just balances the recombination, and keeps the total current density constant and equal to the value in Eq. (20) throughout the entire n-region. Note that the total current is carried entirely by diffusion of minority carriers at x=0, and then converted to drift of majority carriers in the bulk over several diffusion lengths. This example of minority carrier injection and diffusion should be studied carefully, since it provides the basis for understanding the detailed operation of a p-n junction.