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## Basic Equations

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Density of States in Conduction and Valence Band

$$N_C(E) = \frac{(m_n^*)^{3/2} \sqrt{2}}{\pi^2 \hbar^3} \sqrt{E - E_C} \quad N(E) = \frac{(m_p^*)^{3/2} \sqrt{2}}{\pi^2 \hbar^3} \sqrt{E_V - E}$$

Fermi function:  $f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$

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## Carrier Concentrations in Equilibrium

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$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right) = n_i \exp\left(\frac{E_F - E_i}{kT}\right) \text{ cm}^{-3}$$

$$p = N_V \exp\left(\frac{E_V - E_F}{kT}\right) = n_i \exp\left(\frac{E_i - E_F}{kT}\right) \text{ cm}^{-3}$$

Law of Mass Action:  $np = n_i^2$

Carrier concentrations:

n-type material:  $n \approx N_D \quad p \approx \frac{n_i^2}{N_D} \text{ cm}^{-3}$

p-type material:  $p \approx N_A \quad n \approx \frac{n_i^2}{N_A} \text{ cm}^{-3}$

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## Carrier concentration. under bias

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$$n = n_i \exp\left(\frac{F_N - E_i}{kT}\right) \quad p = n_i \exp\left(\frac{E_i - F_P}{kT}\right) \text{ cm}^{-3}$$

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## Generation

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Number of photons:  $N_{ph} = N_S \exp(-\alpha x) \text{ cm}^{-2} \cdot \text{sec}^{-1}$

Generation rate:  $G = \alpha N_S \exp(-\alpha x) \text{ carriers} \cdot \text{sec}^{-1} \cdot \text{cm}^{-3}$

Generation, homogeneous semiconductor, G = constant:

P-type:  $\Delta n = G\tau_n$     N-type:  $\Delta p = G\tau_p \text{ cm}^{-3}$

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## Recombination:

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General SRH rec. rate:

$$U = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} \text{ carriers} \cdot \text{sec}^{-1} \cdot \text{cm}^{-3}$$

Under low injection conditions:

For electrons  $U_n = \frac{\Delta n}{\tau_n}$     For holes  $U_p = \frac{\Delta p}{\tau_p}$

$$\frac{1}{\tau_{TOT}} = \frac{1}{\tau_{AUG}} + \frac{1}{\tau_{RAD}} + \frac{1}{\tau_{SRH}} \text{ where } \tau \text{ is in sec}$$

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## Basic Pn Junction Equation Set

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1. Poison's Eq.  $\frac{\partial \hat{E}}{\partial x} = \frac{\rho}{\epsilon} = p + N_D^+ - n - N_A^-$

2. Transport Eq.

$$J_n = q\mu_n n \hat{E} + qD_n \frac{dn}{dx} \quad J_p = q\mu_p p \hat{E} - qD_p \frac{dp}{dx}$$

3. Continuity or State Equation

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} + G_n - U_n \quad \frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} + G_p - U_p$$

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General solution for no E field, constant G

$$\Delta n(x) = Ae^{-x/L_n} + Be^{x/L_n} + G\tau_n$$

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## Equations for Pn Junctions

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Built-in voltage pn homojunction:  $V_0 = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$

General Ideal Diode Equation

$$I = I_0 (\exp(qV/kT) - 1)$$

$I_0$  for wide base diode

$$I_0 = qA \left( \frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right)$$

Depletion region recombination

$$I = I_{rec} \exp(qV/2kT)$$

$$I = I_0 \left[ \exp\left(\frac{q(V - IR_{series})}{kT}\right) - 1 \right]$$

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## Solar Cell Equations

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$$J = J_0 (\exp(qV/nkT) - 1) - J_{sc} \quad J_{sc} \approx J_L$$

$$V_{oc} = \frac{nkT}{q} \ln\left(\frac{I_{sc}}{I_0} + 1\right) \quad \eta = \frac{V_{oc} I_{sc} FF}{P_{in}} = \frac{V_{mpp} I_{mpp}}{P_{in}}$$

$$J_L = qG(L_n + L_p + W) \text{ for constant G, wide base.}$$

$$J_L = q \int_0^W G(x) CP(x) dx = q \int_0^W (\alpha(\lambda) H_0 e^{-\alpha(\lambda)x} d\lambda) CP(x) dx$$

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## Material Constants and common units

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Intrinsic carrier concentration

$$n_i = \sqrt{N_C N_V} \exp\left(\frac{-E_G/2}{kT}\right) \text{ cm}^{-3}$$

Effective Density of States

$$N_C = 2 \left[ \frac{m_n^* kT}{2\pi \hbar^2} \right]^{3/2} \quad N_V = 2 \left[ \frac{m_p^* kT}{2\pi \hbar^2} \right]^{3/2} \text{ cm}^{-3}$$

Intrinsic Energy Level

$$E_i = \frac{E_G}{2} + \frac{kT}{2} \ln\left(\frac{N_V}{N_C}\right) \approx \frac{E_G}{2} \text{ eV}$$

Diffusivity

$$D_n = \left(\frac{kT}{q}\right) \mu_n \quad D_p = \left(\frac{kT}{q}\right) \mu_p \text{ cm}^2/\text{s}$$

Minority Carrier Diffusion Length

$$L_n = \sqrt{D_n \tau_n} \quad L_p = \sqrt{D_p \tau_p} \text{ (length units)}$$

Resistivity and Conductivity

$$\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)} \text{ } \Omega \text{ cm}$$

Resistance, homogeneous:  $R = \frac{\rho l}{A} \text{ } \Omega$

Permittivity

$$\epsilon = \epsilon_s \epsilon_0 = K_s \epsilon_0 \text{ Farad/cm}$$

## Radiant Energy

Wavelength and energy of a photon:  $E = \frac{hc}{\lambda}$

If E is in eV and  $\lambda$  in  $\mu\text{m}$ :  $E(\text{eV}) = \frac{1.24}{\lambda(\mu\text{m})}$

Spectral Irradiance for black body:

$$F(\lambda) = \frac{2\pi hc^2}{\lambda^5 \left( \exp\left(\frac{hc}{\lambda kT}\right) - 1 \right)}$$

Power density of a non-ideal black body:

$$H\left(\frac{W}{m^2}\right) = \varepsilon\sigma T^4$$

Photon Flux and power density:

$$H\left(\frac{W}{m^2}\right) = \Phi \times \frac{E(J)}{\text{photon}} = q\Phi E(\text{eV}) = q\Phi \frac{1.24}{\lambda}$$

## Solar Radiation

Air Mass:  $AM = 1/\cos(\theta)$  Where  $\theta$  =zenith

Radiation:  $I = 1.1 \times 1.353 \times (0.7^{0.678})^{AM}$

Standard Light Intensity (AM1.5) =  $1\text{kW/m}^2$

Calculations for Solar Noon

Solar Angle Calculations

$$\text{Declination angle: } \delta = 23.45^\circ \sin\left(\frac{360}{365}(d-81)\right)$$

Elevation angle at solar noon:

Northern Hemisphere:  $\alpha = 90^\circ + \phi - \delta$

Southern Hemisphere:  $\alpha = 90^\circ - \phi + \delta$

Radiation of a tilted surface

$$S_{\text{horizontal}} = S_{\text{incident}} \sin(\alpha)$$

$$S_{\text{module}} = S_{\text{incident}} \sin(\alpha + \beta)$$

Calculations for any time of day (local solar time)

$$HRA = 15^\circ (LST - 12)$$

$$\text{Elevation} = \sin^{-1} [\sin(\delta) \sin(\phi) + \cos(\delta) \cos(\phi) \cos(HRA)]$$

$$\text{Azi} = \cos^{-1} \left[ \frac{\sin(\delta) \cos(\phi) - \cos(\delta) \sin(\phi) \cos(HRA)}{\cos(\alpha)} \right]$$

$$S_{\text{module}} = S_{\text{incident}} \cos(\alpha) \sin(\beta) \cos(\psi - \Theta) + \sin(\alpha) \cos(\beta)$$

Converting local time to local solar time

$$LSTM = 15^\circ \times \Delta T_{GMT}$$

$$T_{\text{ime}} C_{\text{orrection}} = 4(LSTM - \text{Longitude}) + EoT$$

$$EoT = 9.87 \sin(2B) - 7.53 \cos(B) - 1.5 \sin(B)$$

$$B = \frac{360}{365}(d - 81)$$

$$LST = LT + TC$$

## Constants and conversions

At 300K:  $kT/q = 25.9 \text{ mV}$   $kT = 25.9 \text{ meV}$

$k = 1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K}$

$m_0 = 9.11 \times 10^{-31} \text{ kg}$

$q = 1.6 \times 10^{-19} \text{ C}$

$h = 6.63 \times 10^{-34} \text{ J-sec} = 4.135 \times 10^{-15} \text{ eV-sec}$

$\hbar = h/2\pi = 1.0552 \times 10^{-34} \text{ J-sec}$

$\varepsilon_0 = 8.85 \times 10^{-14} \text{ F/cm}$

$c = 2.99 \times 10^{10} \text{ cm/sec}$

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$1 \mu\text{m} = 1 \times 10^{-4} \text{ cm} = 1 \times 10^{-6} \text{ m}$

Current =  $I = J \times A$ , where A is area

1 Joule = 1 Watt-sec

In Uniform material:  $E = V/L$

Material	$m_n^*/m_0$	$m_p^*/m_0$	$E_G$ (eV)	$n_i$ ( $\text{cm}^{-3}$ )	$\chi$ (eV)	$\varepsilon_s$
Si	1.18	0.81	1.12	$1.0 \times 10^{10}$	4.03	11.8
Ge	0.55	0.36	0.66	$2.0 \times 10^{13}$	4.13	16.0
GaAs	0.066	0.52	1.42	$1.8 \times 10^6$	4.07	13.1
InP	0.08	0.6	1.34	$1.3 \times 10^7$	4.38	12.5