Derivation of the Ideal Diode Equation for Photovoltaics
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Basic Equations:

1. **Poisson's equation:**

$$\frac{\partial \vec{E}}{dx} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}\left(p(x) - n(x) - N_A^- + N_D^+\right)$$

2. **Transport equations:**

$$J_n = q\mu_n n(x)\vec{E} + qD_n \frac{dn(x)}{dx}$$

(first term is drift, second is diffusion)

$$J_p = q\mu_p p(x)\vec{E} - qD_p \frac{dp(x)}{dx}$$

3. Continuity equations:

General conditions

$$\frac{dn}{dt} = \frac{1}{q}\frac{\partial J_n}{\partial x} - (U - G)$$

$$\frac{dp}{dt} = \frac{1}{q}\frac{\partial J_p}{\partial x} + (U - G)$$

Under thermal equilibrium and steady state conditions

$$\frac{1}{q}\frac{\partial J_n}{\partial x} = (U - G)$$

$$\frac{1}{q}\frac{\partial J_p}{\partial x} = -(U - G)$$

where U and G are the recombination and generation rates in the particular material and depend on the details of the device and may also depend on distance.

Applying above equations to a pn junction: Depletion approximation

The above equations can be difficult to solve in closed form unless several simplifying assumptions are made. However, the equations are readily solved using numerical approaches, and there are many device simulators that perform this task. In addition to the assumption of a one-dimensional device, the most central simplifying assumption in determining a closed form solution to the above equations is the depletion approximation. The depletion approximation assumes that the electric field in the device is confined to some region of the device. According to this assumption, the device can then be broken up into regions that have an electric field and those that do not. This is shown below for a pn junction, where Regions I and III do not have an electric field (called quasi-neutral regions or QNR) and Region II has an electric field (which is called space-charge or depletion region).

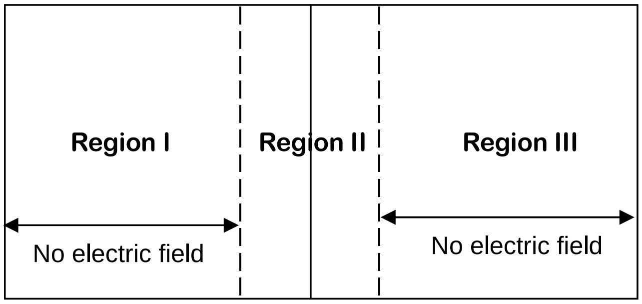


Figure 1: Schematic showing the regions with and without the electric field according to the depletion approximation.

The depletion approximation greatly simplifies the solution to the basic semiconductor equations since in the regions with no electric field, Poisson's Equation and the drift transport term in the transport equations "disappear". In this case in the regions I and III, the equations reduce from 3 to 2 equations which have *n(x)* or *p(x)* as the only variable, and hence can be more readily solved in closed form.

General Procedure using the depletion approximation:

Divide the device into regions with an electric field and without an electric field.

1. Solve for electrostatic properties in the depletion region (Region II on the diagram). This solution depends on the doping profile assumed. Here we will restrict the calculations to constant doping.
2. Solve for the carrier concentration and current in the quasi-neutral regions (Regions I and III on the diagram) under steady-state conditions. The steps in this are:
   a. Determine the general solution for the particular device. The general solution will depend only on the types of recombination and generation in the device.
   b. Find the particular solution, which depends on the surfaces and the conditions at the edges of the depletion region.
3. Find the relationship between the currents on one side of the depletion region and the currents on the other side. This depends on the recombination/generation mechanisms in the depletion region.

Part 1: Solving for static properties in region with electric field

Assumptions are:

(1) Depletion approximation: the electric field is confined to a particular region.

(2) No free carriers (*n(x), p(x) = 0*) in depletion region.

We can assume no free carriers since the electric field sweeps them out of the depletion region quickly. No free carriers means (1) transport equations drop out and (2) no recombination or generation, so the continuity equation becomes $\frac{1}{q}\frac{\partial J_n}{\partial x} = \left(U - G\right) = 0$. This means that J_n is constant across the depletion region. Similarly, J_p is also constant across the depletion region**.**

(3) Abrupt or step doping profile (N_A^+, N_D^+ are constant).

(4) All dopants are ionised ($N_A^+ = N_A$, $N_D^+ = N_D$).

(5) One-dimensional device.

The only equation left to solve is Poisson's Equation, with *n(x)* and *p(x) =0*, abrupt doping profile and ionized dopant atoms. Poisson's equation then becomes:

$$\frac{\partial \bar{E}}{dx} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}\left(-N_A + N_D\right) \qquad \text{or} \qquad \frac{d\hat{E}}{dx} = \frac{\rho}{\varepsilon_0 \varepsilon_s} \text{ where } \rho = \begin{cases} \dfrac{-qN_A}{\varepsilon_0 \varepsilon_s} \text{ when } -x_p \le x < 0 \\ \dfrac{qN_D}{\varepsilon_0 \varepsilon_s} \text{ when } 0 \le x < x_n \end{cases}$$

ε_0 is the permittivity in free space, and ε_s is the permittivity in the semiconductor and x_p and x_n are the edges of the depletion region in the p- and n-type side respectively, measured from the physical junction between the two materials.

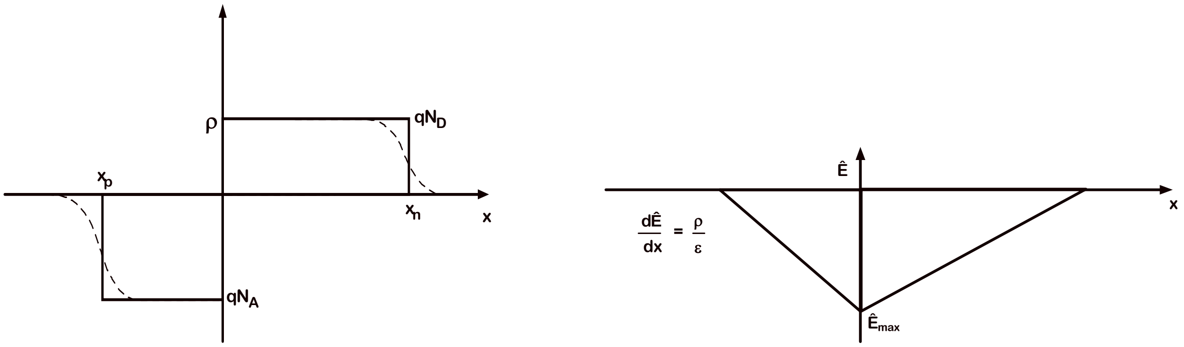
The electric field then becomes $E = \begin{cases} \displaystyle\int \frac{-qN_A}{\varepsilon_s} dx = \frac{-qN_A}{\varepsilon} x + C_1 & \text{for } -x_p \le x < 0 \\ \displaystyle\int \frac{qN_D}{\varepsilon_s} dx = \frac{qN_D}{\varepsilon} x + C_2 & \text{for } 0 \le x < x_n \end{cases}$

The integration constants C_1 and C_2 can be determined by using the depletion approximation, which states that the electric field must go to zero at the boundary of the depletion regions. This gives:

$$E(x = -x_p) = 0 \qquad \Rightarrow C_1 = \frac{-qN_A}{\varepsilon} x_p \qquad \text{and} \qquad E(x = x_n) = 0 \qquad \Rightarrow C_2 = \frac{-qN_D}{\varepsilon_S}$$

$$E=\begin{cases}\dfrac{-qN_A}{\varepsilon_s}\left(x+x_p\right) & \text{when } -x_p \le x < 0\\[2ex] \dfrac{-qN_D}{\varepsilon_s}\left(x_n-x\right) & \text{when } 0 \le x < x_n\end{cases}$$

The maximum electric field occurs at the junction between the p- and n-type material. Further, we know that the electric field lines must be continuous across the interface, such the electric field in the p-type side and the n-type side must equal each other at the interface or when $x = 0$. Putting $x = 0$ in the above equation for electric field and setting the two values of ***E*** equal to each other gives: $N_A x_p = N_D x_n$. This equation makes physical sense since it states that the total charge on one side of the junction must be the same as the total charge on the other. In other words, if the electric field is confined to the depletion region, then the net charge in Region II must be zero, and hence the negative charge and the positive charge must be equal. $N_A x_p A$ is the total negative charge, since N_A is the charge density and $x_p A$ is the volume of the depletion region (A is the cross-sectional area and x_p is the depth). Similarly, $N_D x_n A$ is the positive charge. The cross sectional area (A) is the same and cancels out.



(a) (b)

Figure 2: (a) Doping concentration in a pn junction. The dotted lines are the actual net charge density (the tails are exaggerated) and the solid line represents the assumed charge density in the depletion approximation. (b) The electric field in a pn junction.

To find the voltage as a function of distance, we integrate the equation for the electric field.

$$V(x)=\begin{cases}\displaystyle\int -E(x)dx=\int\frac{qN_A}{\varepsilon}\left(x+x_p\right)dx=\frac{qN_A}{\varepsilon}\left(\frac{x}{2}+x_p\right)x+C_3 & \text{for } -x_p \le x < 0\\[2ex] \displaystyle\int -E(x)dx=\int\frac{qN_D}{\varepsilon}\left(x_n-x\right)dx=\frac{qN_D}{\varepsilon}\left(x_n-\frac{x}{2}\right)x+C_4 & \text{for } 0 \le x < x_n\end{cases}$$

We are usually interested in the potential ***difference*** across the junction and can arbitrarily set one side to zero. Here we define the voltage on the p-type side as zero, such that at $x=-x_p$, $V=0$. This gives the constant C_3 as:

$$C_3=\frac{qN_A}{2\varepsilon}x_p^2 \text{ which gives } V(x)=\frac{qN_A}{2\varepsilon}\left(x+x_p\right)^2 \text{ for } -x_p \le x < 0.$$

We can find C_4 by using the fact that the potential on the n-type side and p-type side are identical at the interface, such that:

$$V_p(x=0)=\frac{qN_A}{2\varepsilon}x_p^2=V_n(x=0)=\frac{qN_A}{2\varepsilon}\left(x_n-\frac{x}{2}\right)x+C_4 \text{ or } C_4=\frac{qN_A}{2\varepsilon}x_p^2.$$ Overall, $V(x)$ is:

$$V(x)=\begin{cases}\dfrac{qN_A}{2\varepsilon}\left(x+x_p\right)^2 & \text{for } -x_p \le x < 0\\ \dfrac{qN_D}{\varepsilon}\left(x_n-\dfrac{x}{2}\right)x+\dfrac{qN_A}{2\varepsilon}x_p^2 & \text{for } 0 \le x < x_n\end{cases}$$

The total voltage is plotted below.

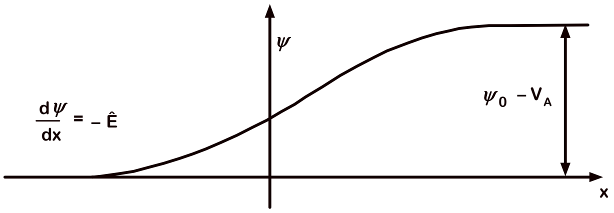


Figure 3: Plot of the voltage across a pn junction, assuming that the voltage on the p-type side is zero.

The maximum voltage across the junction is a $x= x_n$, which is:

$$V(x=x_n)=\frac{q}{2\varepsilon}\left(N_D x_n^2+N_A x_p^2\right).$$

This voltage is also equal to the built-in voltage across the pn junction, V_0, (which we can find from the difference in Fermi-levels between the n and p-type material), giving

$$V_0=\frac{q}{2\varepsilon}\left(N_D x_n^2+N_A x_p^2\right)$$

Using $N_A x_p = N_D x_n$ in the above equation and rearranging allows x_p and x_n to be determined. They are:

$$x_n=\left[\frac{2\varepsilon\left(V_0\right)}{q}\left(\frac{N_A}{N_D\left(N_A+N_D\right)}\right)\right]^{1/2} \qquad \text{and} \qquad x_p=\left[\frac{2\varepsilon\left(V_0\right)}{q}\left(\frac{N_D}{N_A\left(N_A+N_D\right)}\right)\right]^{1/2}$$

From these equations, the maximum electric field and total depletion width can be determined.

$$\hat{E}_{\max}=-\sqrt{\frac{2q}{\varepsilon}\frac{V_o}{\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$$

$$W=x_p+x_n=\sqrt{\frac{2\varepsilon}{q}V_o\left(\frac{1}{N_A}+\frac{1}{N_D}\right)^{-1}}$$

$$x_p=W\frac{N_D}{N_A+N_D} \quad \text{and} \quad x_n=W\frac{N_A}{N_A+N_D}$$

where $V_o=\dfrac{kT}{q}\ln\left(\dfrac{N_A N_D}{n_i^2}\right)$ is the built-in voltage and is calculated separately.

Part 2: Solving for Regions I and III: no electric field

In Regions I and III, the depletion approximation states that there is no electric field. Therefore, Poisson's Equation becomes zero, and the drift term in the transport equation ($q\mu_n nE$ or $q\mu_p pE$) also becomes zero.

The transport equations then become:

$$J_n = qD_n \frac{dn(x)}{dx} \qquad (1a)$$

$$J_p = -qD_p \frac{dp(x)}{dx} \qquad (1b)$$

The continuity equations remain:

$$\frac{1}{q}\frac{\partial J_n}{\partial x} = (U - G) \qquad (2a)$$

$$\frac{1}{q}\frac{\partial J_p}{\partial x} = -(U - G) \qquad (2b)$$

For electrons, differentiating (1a) above and substituting into (2a) above gives:

$$\frac{dJ_n}{dx} = qD_n \frac{d^2 n(x)}{dx^2} = q\big(U(x) - G(x)\big), \text{ or:}$$

$$\boxed{\frac{d^2 n(x)}{dx^2} = \frac{U(x) - G(x)}{D_n}} \qquad (3a)$$

For holes, differentiating (1b) above and substituting into (2b) above gives:

$$\frac{dJ_p}{dx} = -qD_p \frac{d^2 p(x)}{dx^2} = -q\big(U(x) - G(x)\big) \text{ or:}$$

$$\boxed{\frac{d^2 p(x)}{dx^2} = \frac{U(x) - G(x)}{D_p}} \qquad (3b)$$

Equations (3a) and (3b) are general under steady state conditions as long as the depletion approximation holds (which assumes low injection) and as long as drift and diffusion are the only transport mechanisms. For steady state and low injection, these are the equations that must be solved to determine the electrical characteristics, of the IV equation, of a device. The solution to these second order differential equations involves firstly finding equations for U and G to determine the general solution to the differential equation, and secondly determining boundary conditions to find the particular solution.

Dominance of the minority carrier currents

In either Region I or Region III, the total current consists of two current components, J_n (the current composed of electrons) and J_p (the current composed of holes). However, we will solve only for the minority carrier current in each material, since the recombination of minority carrier control the current flow. Later, we will also calculate the majority carrier currents J_p (in p-type material) based on the fact that the total current is constant.

2a: (i) Finding The Recombination Rate: Assumption of Low Injection

The general form of the recombination rate for SRH recombination is:

$$U = \frac{\left(np - n_i^2\right)}{\tau_{p0}\left(n + n_1\right) + \tau_{n0}\left(p + p_1\right)}$$

where n_1, p_1 are the number of carriers in recombination sites, n_o, p_o are the electron and hole concentration in equilibrium and τ_{p0} and τ_{n0} are the minority carrier lifetimes for holes and electrons.

In p-type material and under low injection (low bias) conditions, $p >> n$ also $p \approx p_o$. Further, we will assume that $n >> n_1$ and $p >> p_1$ and that the lifetimes do not vary dramatically in n- and p-type material (i.e., that $\tau_{n0}\, p >> \tau_{p0}\, n$). The above equation then reduces to the recombination rate for electrons in p-type material as:

$$U = \frac{\left(np - n_i^2\right)}{\tau_{n0} p} = \frac{\left(np - n_0 p_0\right)}{\tau_{n0} p} = \frac{\left(np_0 - n_0 p_0\right)}{\tau_{n0} p_0} = \frac{\left(n - n_0\right)}{\tau_{n0}} = \frac{\Delta n}{\tau_n}$$

This is the low injection form of the recombination rate and it is commonly used in closed form solutions of pn junctions as it greatly simplifies the mathematics.

For holes in n-type material, $U = \dfrac{\Delta p}{\tau_p}$.

2a: (ii) Finding the Generation Rate:

In general G will be given by the equation:

$$G = \alpha N_S \exp(-\alpha x)$$

Note that N_S may vary with time and may be change with wavelength, such that in general N_S will be a function of time and wavelength denoted by $N_S(\lambda,t)$, which also makes the generation rate G a function of wavelength and time. However, here we are solving only for steady state and ignore any time dependence. Wavelength dependence is often included by summing the result obtained at one wavelength over all wavelengths of interest. The exponential form of the generation rate generally makes the differential equation to be solved a nonhomogeneous second order differential equation. Therefore, approximation to the generation rate are often that the generation is constant (valid when the dimensions of interest are small compared to α^{-1}), that there is an impulse generation at the surface, or that the generation is zero.

2b: Finding the general solution

The general solution to the differential equation in (3a) and (3b) will depend on the equation for *U* and *G*. There are several common general solutions. These are shown below, where $\zeta(x)$ is any function dependent only on *x*, and for the pn junction equations $\zeta(x)$ will usually correspond to one of $n(x)$, $p(x)$, $\Delta n(x)$ or $\Delta p(x)$. *A* and *B* are constants that need to be determined by the boundary conditions. *C* and *K* are semiconductor or device constants. *C* for pn junction equations is usually L_n or L_p (the minority carrier diffusions length) and *K* is often a constant generation term, *G*.

Differential equation of the form	**General Solution**	**When Used**
$\frac{d^2\zeta(x)}{dx^2} = \frac{\zeta(x)}{C^2}$	$\zeta(x) = Ae^{-x/C} + Be^{x/C}$	Bulk recombination, no generation
$\frac{d^2\zeta(x)}{dx^2} = \frac{\zeta(x)}{C^2} + \frac{K}{C^2}$	$\zeta(x) = Ae^{-x/C} + Be^{x/C} + K$	Bulk recombination, constant gen.
$\frac{d^2\zeta(x)}{dx^2} = 0$	$\zeta(x) = Ax + B$	Zero recombination and generation
$\frac{d^2\zeta(x)}{dx^2} = K$	$\zeta(x) = \frac{K}{2}x^2 + Ax + B$	Zero recombination, constant gen.

2c: Finding the particular solution (all equations will be for electrons in p-type material)

The particular solution depends on the conditions of each region at the edges or the regions. For many semiconductor devices, at least one edge will be a pn junction, and hence Boundary Condition #1 below applies to many semiconductor devices.

1. Boundary condition at the edge of a depletion region of a pn junction:

In p-type material: $n_p(x=0) = n_{p0}e^{qV/kT}$ or $\Delta n_p(x=0) = n_{p0}\left(e^{qV/kT} - 1\right)$

In n-type material $p_n(x=0) = p_{n0}e^{qV/kT}$ or $\Delta p_n(x=0) = p_{n0}\left(e^{qV/kT} - 1\right)$

This boundary condition holds as long as the device is in low injection.

2. Possible boundary conditions at a semiconductor surface

The other boundary condition may depend on the surface of the device. The surface recombination velocity, S_r, determines the conditions at the surface. Some common boundary conditions for surfaces are listed below for p-type material.

Location of surface	**Boundary Description**	**Equation**
Surface is far away from the junction ($W \gg L_n$)	The minority carrier concentration must be finite as $x \to \infty$	$n(x \to \infty) = \text{finite}$
Surface is within a few	Surface recombination is "infinitely"	$\Delta n(x=W)=0$

diffusion lengths of junction	fast ($S_r = \infty$) All carriers that reach the surface recombine.	
Surface is within a few diffusion lengths of junction	Surface recombination is finite.	$qD_n \frac{dn}{dx} = qS_r \Delta n$
Surface next to a light generation source	Impulse light generation at surface with no surface recombination	$J(x=0) = qG$

2d: Finding diffusion currents in Regions I and III.

Once we have a form of *n(x)* and *p(x)* from the procedures described in 2(b) and 2(c), we can readily find the minority carrier currents by using the general equations:

$$J_p = -qD_p \frac{dp(x)}{dx} \quad \text{and} \quad J_n = qD_n \frac{dn(x)}{dx}.$$

Part 3: Finding total current

To find the total current, we note that the TOTAL current in the device must be constant, independent of distance as long as there is not a contact that can extract or inject carriers and as long as the device is under steady state conditions. This can be shown by:

$$\frac{dJ_T}{dx} = \frac{d\left(J_p + J_n\right)}{dx} = \frac{dJ_p}{dx} + \frac{dJ_n}{dx} = -q\left(U_p + G_p\right) + q\left(U_n + G_n\right) = q\left(G_n - G_p\right) - q\left(U_n - U_p\right)$$

Since each electron generates a holes and each recombining electron also uses up a hole, $U_n = U_p$ and $G_n = G_p$ so that the derivative of J_T is 0 and J_T is a constant. Physically, the continuity equation is stating that the total number of electrons and holes cannot change in the semiconductor (in steady state), and hence the total current also cannot change. Therefore if we find J_T anywhere in the device, we have found it everywhere in the device. It is most convenient to find the total current at the edges of the depletion regions. Since we know the currents in Regions I and III, to calculate the total current, we need to do two things. (1) Account for the fact that the distance variable x is not the same in Region I and Region III and (2) find the current at the depletion region edges.

1. Making the distance variable the same.

In our solutions, the distance variable, x, in the above equations is usually not the same for the different regions of the device. Typically, we define x' in Region I (here p-type region with an electron minority carrier current) as the distance from depletion region edge and increasing further into Region I. The other distance variable x, is defined as zero at the other depletion region edge and is increasing into Region III (here n-type material with a hole minority carrier current). Using these definitions, the transport equations are:

$$J_p = -qD_p \frac{dp(x)}{dx} \quad \text{and} \quad J_n = qD_n \frac{dn(x')}{dx'}.$$

Since $x = -x' + W$, $\frac{dn(x')}{dx'}\frac{dx'}{dx} = -\frac{dn(x')}{dx}$ and then the current becomes $J_n(x) = -qD_n \frac{dn(-x+W)}{dx}$ or $J_n(x) = -J_n(x') = -J_n(-x+W)$.

2. Current across the depletion region.

Previously, we stated that the generation was zero and the number of free carriers was small, so the recombination could also be neglected the depletion region. As previously stated, under these conditions, the change in current across the depletion region is zero and we can find the total current just as the sum of the currents at the edges of Region I and III, as shown below:

$$J_{total} = J_n(x=0) + J_p(-x=0)$$

A more accurate solution includes the change in J_n and J_p across the depletion region, and we find the total current by: $J_{total} = (x=0) + J_p(x=0) + \Delta J_{p-dep}$ where $\Delta J_{p\text{-}dep}$ is the change in J_p across the depletion region. We could solve for $\Delta J_{n\text{-}dep}$ via the continuity equation. The continuity equations, repeated below, give current dependence on recombination and generation. $\frac{1}{q}\frac{\partial J_p}{\partial x} = -(U - G)$ and in the depletion region this becomes $J_{p-depletion} = \Delta J_p = -\int_{x=x_n}^{x_p}\left(U_{depletion} - G_{depletion}\right)$. Often, the recombination term is ignored and G is assumed to be a constant, such that $\Delta J_p = G\left(x_n + x_p\right) = GW$.

Example 1: General solution for low injection recombination and constant generation, wide base pn junction.

A wide base junction is one in which the surface are far away enough from the junction edges such that they to not impact the recombination properties of carriers injected into the QNR under forward bias.

Step 1: Solve for properties in depletion region.

As in most devices, the solution for the electrostatic properties in the depletion region does not change, and so is not repeated.

Step 2: Solve for carrier concentrations and currents in quasi-neutral regions

The solution below is shown in detail only for the n-type material (in which there is a hole current).

Step 2a: Find U and G.

We will set *G* equal to a constant and in the n-type material $U = \frac{\Delta p}{\tau_p}$ (in p-type material, $U = \frac{\Delta n}{\tau_n}$).

Step 2b: Find general solution

Using low injection recombination and constant generation gives the equation:

$$\frac{d^2 p}{dx^2} = \frac{(U-G)}{D_p} = \frac{\Delta p}{D_p \tau_p} - \frac{G}{D_p}$$

Note that $\frac{d^2 p}{dx^2} = \frac{d^2 \Delta p}{dx^2}$ since $\Delta p = p_n - p_{n0}$ (where p_{n0} is a constant), so the derivative (and second derivative) of $\Delta p(x)$ is the same as the derivative of $p(x)$. In addition for simplicity, we introduce a variable change using: $L_p = \sqrt{D_p \tau_p}$.

The overall differential equation now becomes:

$$\frac{d^2 p}{dx^2} = \frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{D_p \tau_p} - \frac{G}{D_p} = \frac{\Delta p}{L_p^2} - \frac{G\tau_p}{L_p^2} \quad \text{or} \quad \boxed{\frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{L_p^2} - \frac{G\tau_p}{L_p^2}}$$

, which has a general solution:

$$\Delta p(x) = Ae^{-x/L_p} + Be^{x/L_p} + G\tau_p$$

For electrons (p-type material), the differential equations and solutions are:

$$\frac{d^2 \Delta n}{dx'^2} = \frac{\Delta n}{D_n \tau_n} - \frac{G}{D_n} = \frac{\Delta n}{L_n^2} - \frac{G\tau_n}{L_n^2} \qquad \text{and} \quad \Delta n(x') = Ae^{-x'/L_n} + Be^{x'/L_n} + G\tau_n$$

Step 2c: Particular solution for wide base diode

We need two boundary conditions these are:

(1) At the edge of the depletion region, $p_n(x=0) = p_{n0}e^{qV/kT}$

(2) The minority carrier concentration must be finite even as x tends to infinity. This can only be achieved if B = 0.

Since B = 0, the general solution for holes then becomes

$$\Delta p(x) = p(x) - p_{n0} = Ae^{-x/L_p} + G\tau_p$$

at x=0, $\Delta p_{(x=0)} = p_n(x=0) - p_{n0} = p_{n0}e^{qV/kT} - p_{n0} = A + G\tau_p$

Rearranging gives $A = p_{n0}\left(e^{qV/kT} - 1\right) - G\tau_p$

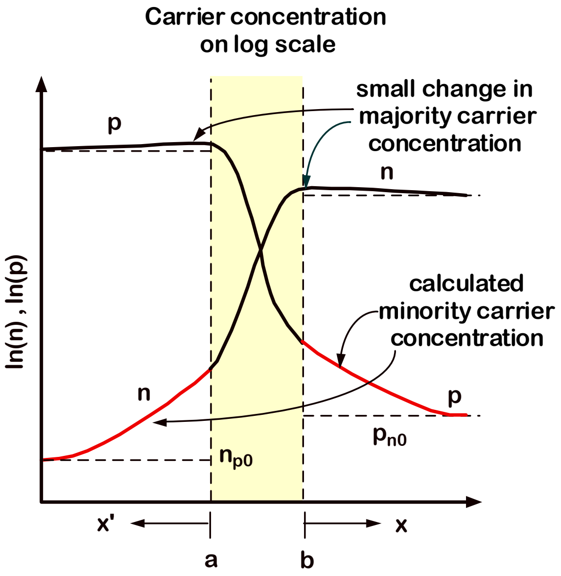
Plugging A back in gives:

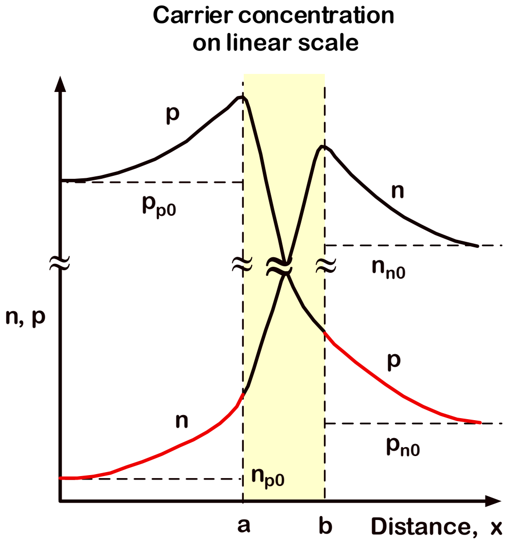
$$\Delta p = \left[p_{n0}\left(e^{qV/kT} - 1\right) - G\tau_p\right]e^{-x/L_p} + G\tau_p \quad \text{or} \qquad p(x) = \left[p_{n0}\left(e^{qV/kT} - 1\right) - G\tau_p\right]e^{-x/L_p} + G\tau_p + p_{n0}$$

The equation for electrons in p-type material, *$\Delta n(x')$*, can be similarly derived as:

$$\Delta n(x') = \left[n_{p0}\left(e^{qV/kT} - 1\right) - G\tau_n\right]e^{-x'/L_n} + G\tau_n$$

This is plotted below for G=0.





Differentiating and plugging into equation for current gives:

$$J_p(x) = -qD_p\frac{dp(x)}{dx} = \frac{qD_p}{L_p}\left[p_{n0}\left(e^{qV/kT} - 1\right) - G\tau_p\right]e^{-x/L_p}$$

$$J_n(x') = qD_n\frac{dn(x')}{dx'} = -\frac{qD_n}{L_n}\left[n_{p0}\left(e^{qV/kT} - 1\right) - G\tau_n\right]e^{-x'/L_n}$$

Making the change from x to x' gives

$$J_n(x) = -J_n(x') = \frac{qD_n}{L_n}\left[n_{p0}\left(e^{qV/kT} - 1\right) - G\tau_n\right]e^{(-x+w)/L_n}$$

Step 3: Find total current

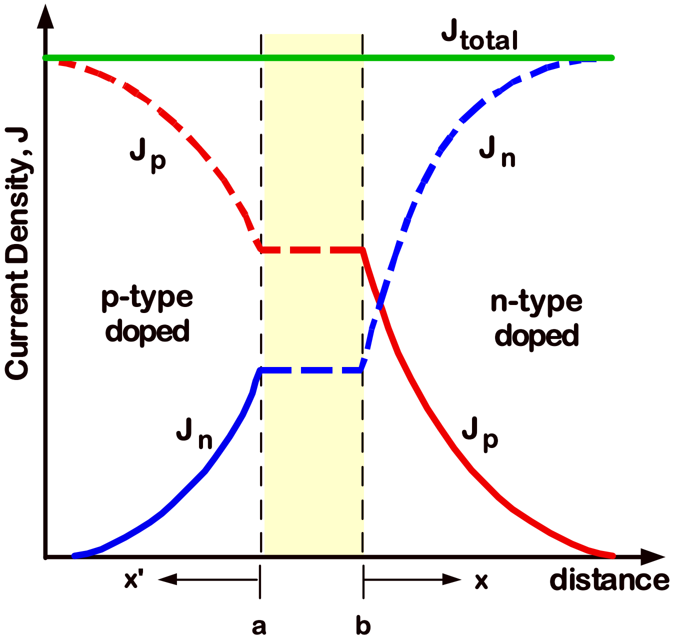
The change in the current across the depletion region is:

$$\frac{\partial J_n}{\partial x} = \Delta J_n = q\left(U_{n-depletion} - G_{n-depletionn}\right)$$

Assuming that there is no generation and recombination, then $\Delta J_n = 0$ and

$$J_{total} = J_n(x=0) + J_p(x=-W)$$

This case is shown in the graph below.



If there is a constant generation across the depletion region, then $\Delta J_{n-dep} = qGx_n$, where x_n is the depletion width in the p-type material and $x_n + x_p = W$.

J_n at the edge of the depletion region in the p-type material is:

$$J_n(x=0) = -\frac{qD_n n_{p0}}{L_n}\left(e^{qV/kT}-1\right) + \frac{qD_n}{L_n}G\tau_n = -\frac{qD_n n_{p0}}{L_n}\left(e^{qV/kT}-1\right) + qGL_n$$

J_n at the edge of the depletion region in the n-type material is:

$$J_n(x'=0) = J_n(x=0) + \Delta J_{n-dep} = -\frac{qD_n n_{p0}}{L_n}\left(e^{qV/kT}-1\right) + qG(L_n + x_n + x_p)$$

An analogous equations exists for Jp, and the total current is:

$$\begin{aligned} J_{total} &= \left[q\frac{D_n}{L_n}n_{p0} + q\frac{D_p}{L_p}p_{n0}\right]\left[e^{qV/kT}-1\right] - qG\left(L_n + L_p + W\right) \\ &= \left[q\frac{D_n n_i^2}{L_n N_A} + q\frac{D_p n_i^2}{L_p N_D}\right]\left[e^{qV/kT}-1\right] - qG\left(L_n\tau_n + L_p\tau_p\right) \end{aligned}$$

Typically, we write the equation in the form:

$J = J_0\left[e^{qV/kT}-1\right] - J_{SC}$ or $I = I_0\left[e^{qV/kT}-1\right] - I_{SC}$

where

$$J_0 = \left[q\frac{D_n n_i^2}{L_n N_A} + q\frac{D_p n_i^2}{L_p N_D}\right]$$

Example 2: General solution for narrow base diode, assumption no recombination in base.

Step 1: Solve for properties in depletion region.

As in most devices, the solution for the electrostatic properties in the depletion region does not change, and so is given here.

Step 2: Solve for carrier concentration and current in quasi-neutral regions

Step 2a: Find U and G.

We will set G equal to a constant and $U = 0$.

Step 2b: Find general solution

We still start out with the same equation derived from the continuity equations. However, in this case the recombination is zero, so the equation becomes:

$$\frac{d^2 \Delta n}{dx^2} = \frac{(U_n - G_n)}{D_n} = -\frac{G}{D_n} = -\frac{G}{D_n}$$

The general solution is:

$$\Delta n(x) = -\frac{G}{2D_n} x^2 + Ax + B$$

Step 2c: Particular solution for narrow base diode with high recombination at edges.

We need boundary conditions and these are:

(1) At the edge of the depletion region, $\Delta n_p (x = 0) = n_{p0} \left(e^{qV/kT} - 1 \right)$

(2) The excess minority carrier concentration Δn must be zero at x = W, or Δn(x = W)=0.

The first boundary condition gives $B = n_{p0} \left(e^{qV/kT} - 1 \right)$

The second boundary conditions gives

$0 = -\frac{G}{2D_n} W^2 + AW + n_{p0} \left(e^{qV/kT} - 1 \right)$, which simplifies to $A = \frac{G}{2D_n} W - \frac{n_{p0}}{W} \left(e^{qV/kT} - 1 \right)$ or

$$A = \frac{G}{2D_n} W - \frac{n_{p0}}{W} \left(e^{qV/kT} - 1 \right)$$

Substituting these equations into the general solutions gives the equation for the carrier concentration:

$$\Delta n(x) = \frac{G}{2D_n} x^2 + \left[\frac{G}{2D_n} W - \frac{n_{p0}}{W} \left(e^{qV/kT} - 1 \right) \right] x + n_{p0} \left(e^{qV/kT} - 1 \right)$$

The current is found by differentiating the carrier concentration:

$$J_n(x) = qD_n \frac{d\Delta n(x)}{dx} = qD_n \left[\frac{G}{D_n} x + \frac{G}{2D_n} W - \frac{n_{p0}}{W} \left(e^{qV/kT} - 1 \right) \right]$$

Simplifying this gives:

$$J_n(x) = qGx + q\frac{G}{2} W - q\frac{D_n n_{p0}}{W} \left(e^{qV/kT} - 1 \right)$$

Step 3: Find total current

The change in the current across the depletion region is given by the general equation:

$$\frac{\partial J_n}{\partial x} = \Delta J_n = q\left(U_{n-depletion} - G_{n-depletionn}\right)$$

If there is a constant generation across the depletion region and no recombination, then $\Delta J_{n-dep} = qGx_n$, where x_n is the depletion width in the p-type material.

J_n at the edge of the depletion region in the p-type material is:

$$J_n(x=0) = q\frac{G}{2}W - q\frac{D_n n_{p0}}{W}\left(e^{qV/kT} - 1\right)$$

J_n at the 0edge of the depletion region in the n-type material is:

$$J_n(x'=0) = J_n(x=0) + \Delta J_{n-dep} = q\frac{G}{2}W - q\frac{D_n n_{p0}}{W}\left(e^{qV/kT} - 1\right) + qGx_n$$

An analogous equations exists for Jp, and the total current is:

$$\begin{aligned} J_{total} &= \left[q\frac{D_n}{W_{emitter}}n_{p0} + q\frac{D_p}{W_{base}}p_{n0}\right]\left[e^{qV/kT} - 1\right] - qG\left(\frac{W_{emitter}}{2} + \frac{W_{base}}{2} + W_{dep}\right) \\ &= \left[q\frac{D_n n_i^2}{W_{emitter}N_A} + q\frac{D_p n_i^2}{W_{base}N_D}\right]\left[e^{qV/kT} - 1\right] - qG\left(\frac{W_{emitter}}{2} + \frac{W_{base}}{2} + W_{dep}\right) \end{aligned}$$

Summary:

	Surface Condition	U (for n-type material)	J_0
Wide-base diode	Far away	$U = \frac{\Delta p}{\tau_p}$	$J_0 = \left[q \frac{D_n n_i^2}{L_n N_A} + q \frac{D_p n_i^2}{L_p N_D} \right]$
Narrow Base diode	Infinite Δp = 0	U = 0	$J_0 = \left[q \frac{D_n n_i^2}{W_{emitter} N_A} + q \frac{D_p n_i^2}{W_{base} N_D} \right]$
	Infinite Δp = 0	$U = \frac{\Delta p}{\tau_p}$	$J_0 = \left(\frac{q D_n n_i^2}{L_n N_A} \cdot \coth\left(\frac{W_P}{L_n}\right) + \frac{q D_p n_i^2}{L_p N_D} \coth\left(\frac{W_N}{L_p}\right) \right)$
General case	$q D_n \frac{dn}{dx} = q S_r \Delta n$	$U = \frac{\Delta p}{\tau_p}$	$J_0 = \left(\frac{q D_n n_i^2}{L_n N_A} \cdot \frac{\cosh\left(\frac{W_P}{L_n}\right) + \frac{D_n}{S_n L_n} \sinh\left(\frac{W_P}{L_n}\right)}{\frac{D_n}{S_n L_n} \cosh\left(\frac{W_P}{L_n}\right) + \sinh\left(\frac{W_P}{L_n}\right)} + \frac{q D_p n_i^2}{L_p N_D} \frac{\cosh\left(\frac{W_N}{L_p}\right) + \frac{D_p}{S_p L_p} \sinh\left(\frac{W_N}{L_n}\right)}{\frac{D_p}{S_p L_p} \cosh\left(\frac{W_N}{L_p}\right) + \sinh\left(\frac{W_N}{L_p}\right)} \cdot \right)$