1. (25 pts) Consider the AR process

$$x(n) = ax(n-1) + v(n)$$

where $v(n)$ is white with variance σ^2.

- (5 pts) What bounds on a will guarantee $x(n)$ to be stable?
- (10 pts) Derive a general expression for $r(k) = E\{x(n)x(n-k)\}$.
- (5 pts) Derive a general expression for the weight of a single tap predictor with lag k,

$$\hat{x}(n) = w_k x(n-k).$$

- (5 pts) Derive a general expression for the error of this predictor.

2. (25 pts) Prove the following:

- (13 pts) The LMS algorithm (weight update) is convergent in the Mean. Under what conditions does the convergence hold?
- (12 pts) Assuming that the desired signal is formed by the regression model,

$$d(n) = e_o(n) + \mathbf{w}_o^H \mathbf{x}(n)$$

where $e_o(n)$ is white with variance σ^2, show that the RLS algorithm is convergent in the Mean. Under what conditions does the convergence hold?

3. (25 pts) The transform–LMS algorithm operates on the transformed observation vector $\mathbf{z}(n) = \mathbf{Q}\mathbf{x}(n)$, where $\mathbf{Q}$ is a unitary matrix. Derive the following:

- (10 pts) The optimal transform domain (operating on $\mathbf{z}(n)$) Wiener filter, $\mathbf{h}_o$. Express the solution in terms of the Wiener filter, $\mathbf{w}_o$, (that operating on $\mathbf{x}(n)$).
- (10 pts) Determine the minimum MSE, $J_{\min}(\mathbf{h}_o)$ and $J_{\min}(\mathbf{w}_o)$, which occur when operating on $\mathbf{z}(n)$ and $\mathbf{x}(n)$, respectively.
- (5 pts) Determine the LMS update for the transform domain weight vector $\mathbf{h}(n)$. Derive a step size condition that guarantees convergence and which is based strictly on the eigenvalues of $\mathbf{R}_{\mathbf{x}}$.

4. (25 pts) A random variable is given by

$$x = \ln \Theta + \eta$$

where Θ is uniformly distributed on $[0, 1]$ and η is exponentially distributed, $f(\eta) = e^{-\eta}$, $\eta \geq 0$. Determine the Baye's estimate of Θ from a single observation for the following cases:

- (9 pts) The minimum means squared error estimate of Θ.
- (9 pts) The MAP estimate of Θ.
- (7 pts) Assume now the distribution of Θ is unknown. If N samples of x are observed, what is the ML estimate of Θ?