Blind Deconvolution

Adaptive equalizers typically require a training period during which they operate on known signals/statistics

This known signal training is not always appropriate such as in mobile communications

- Cost is too high (time/bandwidth)
- Multipathing or other interference

In such cases, we must use blind equalization.

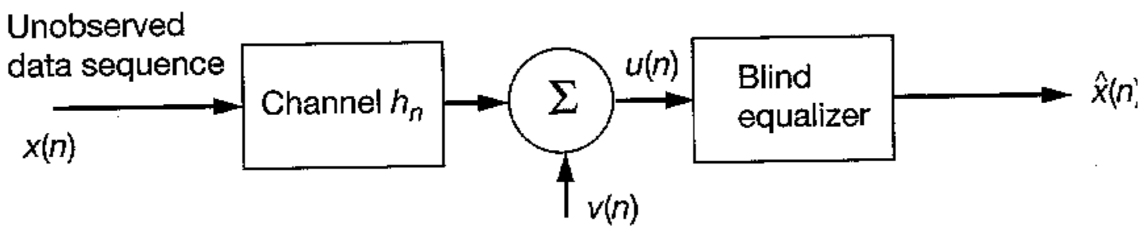


Figure 18.2 Cascade connection of an unknown channel and blind equalizer.

Assume a baseband model of communications.

- Multilevel pulse amplitude modulation (*M-ary PAM*)

Then

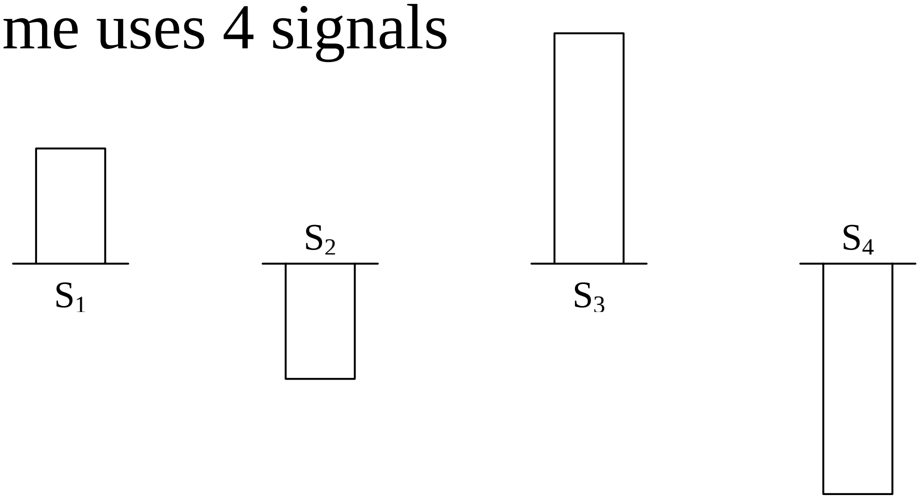
$$u(n) = \sum_k h_k x(n-k)$$

where the dominating interference is due to intersymbol interference (ISI) from channel distortion. Thus the noise is ignored.

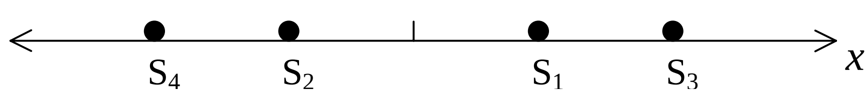
Also assume that

- $h_n \neq 0$ for $n < 0$ (noncausal)
- $\sum_k h_k^2 = 1$ to keep the variance of the output constant.

As an example, a 4-ary PAM modulation scheme uses 4 signals



Or



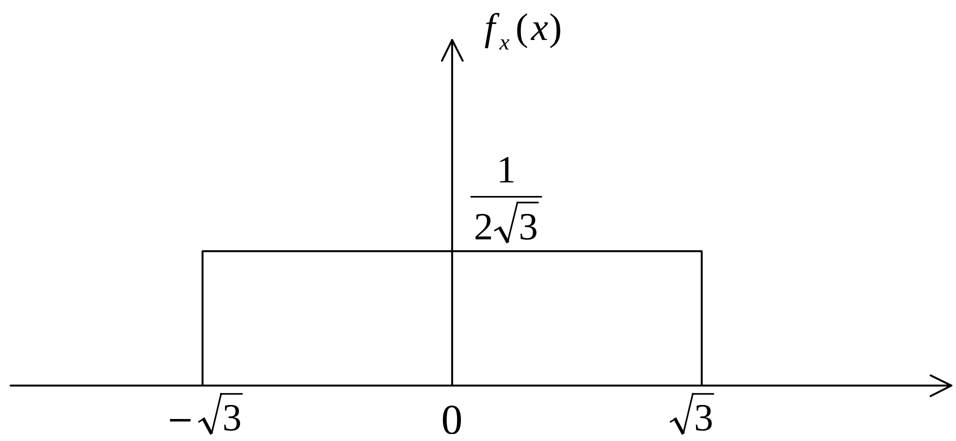
To solve the equalization problem, we need a statistical model of the data. Assume,

1) The data is white

$$E\{x(n)\} = 0$$

$$E\{x(n)x(k)\} = \begin{cases} 1 & k = n \\ 0 & else \end{cases}$$

2) The pdf of $x(n)$ is symmetric and uniform



Deconvolution objective : if $\{w_i\}$ are the coefficients of the ideal inverse filter, then

$$\sum_i w_i h_{l-i} = \delta_l = \begin{cases} 1 & l = 0 \\ 0 & else \end{cases}$$

If this is the case, the output of the equalizer is

$$y(n) = \sum_i w_i u(n-i) = \sum_i \sum_k w_i h_k x(n-i-k)$$

letting $k = l - i$

$$\begin{aligned} y(n) &= \sum_l x(n-l) \sum_i w_i h_{l-i} \\ &= \sum_l x(n-l) \delta_l \\ &= x(n) \end{aligned}$$

Since h_n is not known, the exact inverse can not be used.

We thus use an iterative procedure to find the filter.

At iteration *n*, let the output be given by

$$y(n) = \sum_{i=-L}^{L} \hat{w}_i(n) u(n-i)$$

where a 2*L*+1 tap filter is used.

If we set $\hat{w}_i(n) = 0$ for | *i* | > *L*, then

$$\begin{aligned} y(n) &= \sum_i \hat{w}_i(n) u(n-i) \\ &= \sum_i w_i u(n-i) + \sum_i [\hat{w}_i(n) - w_i] u(n-i) \\ &= x(n) + \underbrace{v(n)}_{\text{Residual ISI}} \end{aligned}$$

where $v(n) = \sum_i [\hat{w}_i(n) - w_i] u(n-i)$

and for the ideal inverse filter

$$x(n) = \sum_i w_i u(n-i)$$

$v(n)$ is the convolution noise containing the residual ISI since ideal filter was not used. The output $y(n)$ is applied to a zero memory nonlinear estimator

$$\hat{x}(n) = g(y(n))$$

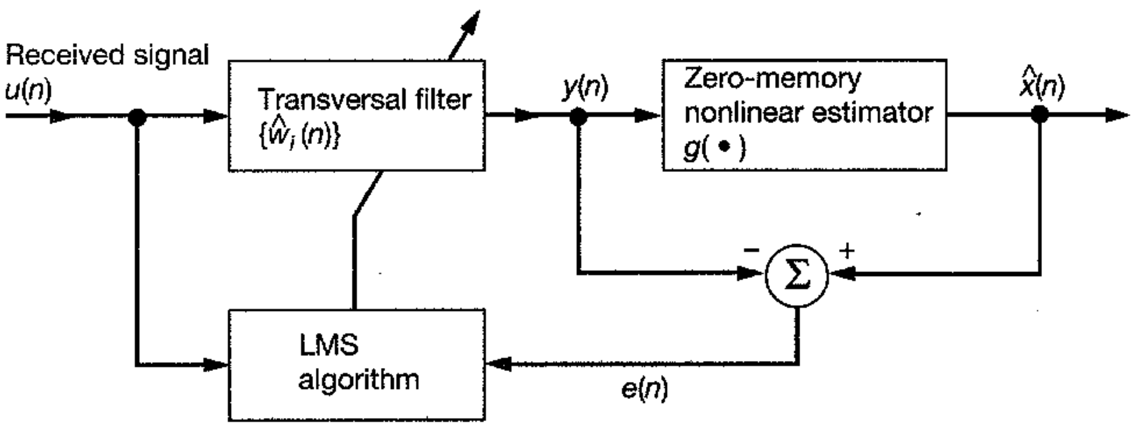


Figure 18.5 Block diagram of blind equalizer.

- The nonlinear estimate $g[y(n)]$ can be used to update the equalizer to produce a better estimate at time $n + 1$
- Consider the equalizer error to be

$$e(n) = \hat{x}(n) - y(n)$$

This error can be used in the LMS algorithm to update the equalizer weights

$$\hat{w}_i(n+1) = \hat{w}_i(n) + \mu u(n-i)e(n)$$

for $i = 0, \pm 1, \pm 2, \ldots \pm L$

Note that in this case, the cost function is

$$J(n) = E\{e^2(n)\} = E\{(\hat{x}(n) - y(n))^2\}$$
$$= E\{(g[y(n)] - y(n))^2\}$$

Note $J(n)$ is a nonconvex function of the filter weights

- The cost function can have numerous local minima

To evaluate the convolution noise, recall

$$u(n) = \sum_k h_k x(n-k)$$

or

$$u(n-i) = \sum_k h_k x(n-i-k)$$

using this in

$$y(n) = \underbrace{\sum_i w_i u(n-i)}_{x(n)} + \underbrace{\sum_i [\hat{w}_i(n) - w_i] u(n-i)}_{v(n)}$$

where

$$w_i \overset{\Delta}{=} \text{perfect equalizer weights}$$

$$\hat{w}_i(n) \overset{\Delta}{=} \text{Finite approximate equalizer with}$$

$$\hat{w}_i(n) = 0 \quad for \quad |i| > L$$

Then

$$v(n) = \sum_i [\hat{w}_i(n) - w_i] u(n-i)$$

$$= \sum_i \sum_k h_k [\hat{w}_i(n) - w_i] x(n-i-k)$$

Letting $n - i - k = l$

$$v(n) = \sum_l x(l) \nabla(n-l)$$

where

$$\nabla(n) = \sum_k h_k [\hat{w}_{n-k}(n) - w_{n-k}]$$

$$\nabla(n) = \sum_k h_k [\hat{w}_{n-k}(n) - w_{n-k}]$$

is the residual impulse response of the channel due to imperfect equalization.

- $\nabla(n)$ is small in value, but long and oscillatory

Since the convolution noise is given by

$$v(n) = \sum_l x(l)\nabla(n-l)$$

$$E\{v(n)\} = \sum_l E\{x(l)\}\nabla(n-l)$$

$$= 0$$

Also

$$E\{v(n)v(n-j)\} = E\left\{\sum_l x(l)\nabla(n-l)\sum_m x(m)\nabla(n-m-j)\right\}$$

$$= \sum_l \sum_m \nabla(n-l)\nabla(n-m-j)E\{x(l)x(m)\}$$

$$= \sum_l \nabla(n-l)\nabla(n-l-j)$$

Since $\nabla(n)$ is long and oscillatory

$$E\{v(n)v(n-j)\} = \sum_l \nabla(n-l)\nabla(n-l-j)$$

tends to average to 0 for $j \neq 0$. Thus

$$E\{v(n)v(n-j)\} = \begin{cases} \sigma^2 & j = 0 \\ 0 & j \neq 0 \end{cases}$$

where

$$\sigma^2(n) = \sum_l \nabla^2(n-l)$$

Also, since $v(n) = \sum_l x(l)\nabla(n-l)$ is a weighted sum of i.i.d. RVs. By the central limit theorem $v(n)$ is Gaussian. Lastly,

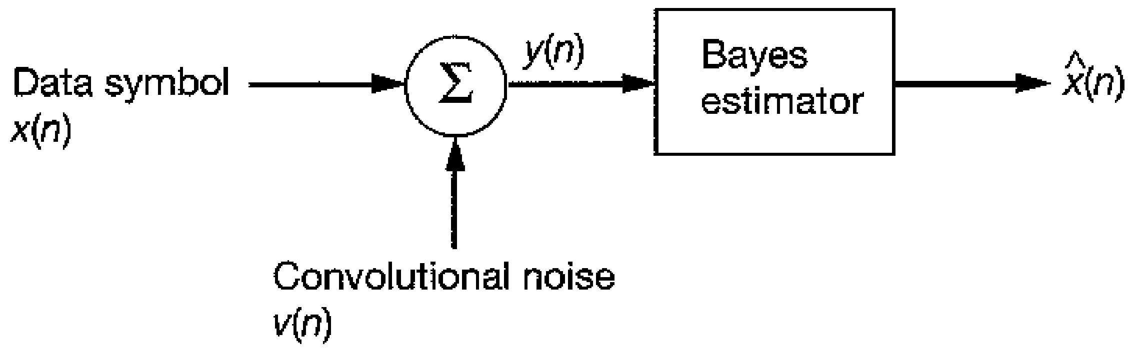
$$E\{x(n)v(n-j)\} = E\{x(n)\sum_l x(l)\nabla(n-l-j)\}$$

$$= \sum_l \nabla(n-l-j)E\{x(n)x(l)\}$$

$$= \nabla(-j) << \sum_l \nabla^2(n-l) = \sigma^2(n)$$

Thus we can say $x(n)$ and $v(n)$ are essentially independent.

Now we must consider the nonlinear estimation of $x(n)$



Where:

1) $x(n)$ is uniformly distributed with zero mean and unit variance.
2) $v(n)$ is white Gaussian noise with zero mean and variance $\sigma^2(n)$, which is independent of $x(n)$.

We will use Bayes estimation technique, which requires knowledge of the distributions.

Define the estimation risk as

$$R = E\{c(x,\hat{x}(n)\}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(x,\hat{x}(n)) f_{xy}(x,y)dydx$$

if $c(x,\hat{x}(n)) = (x-\hat{x}(n))^2$ is the squared error cost, then the risk is minimized if,

$$\hat{x} = \int_{-\infty}^{\infty} x f_x(x \mid y)dx$$
$$= E\{x \mid y\} \qquad \text{(conditional expectation)}$$

where we can use the fact that

$$f_x(x \mid y) = \frac{f_y(y \mid x) f_x(x)}{f_y(y)}$$

Using the current model,

$$y = c_0 x + v$$

where $c_0 < 1$ is a scaling factor included to ensure $E\{y^2\} = 1$. Also, x and v are independent. Thus,

$$f_y(y \mid x)) = f_v(y - c_0 x)$$

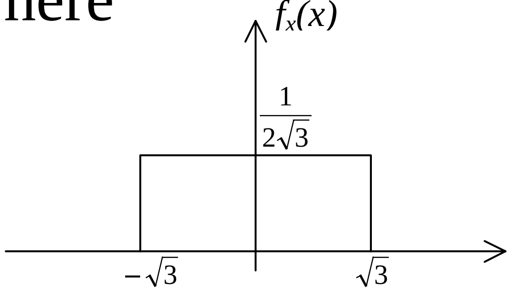
and

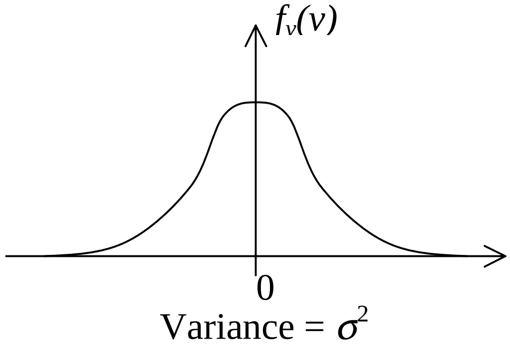
$$\hat{x} = \int_{-\infty}^{\infty} x f_x(x \mid y) dx$$

$$= \frac{1}{f_y(y)} \int_{-\infty}^{\infty} x f_y(y \mid x) f_x(x) dx$$

$$= \frac{1}{f_y(y)} \int_{-\infty}^{\infty} x f_v(y - c_0 x) f_x(x) dx$$

where





Evaluating this yields (Bellini, 1988)

$$\hat{x} = \frac{1}{c_0 y} - \frac{\sigma}{c_0} \frac{Z(y_1) - Z(y_2)}{Q(y_1) - Q(y_2)}$$

where

$$y_1 = \frac{1}{\sigma}(y + \sqrt{3}c_0)$$

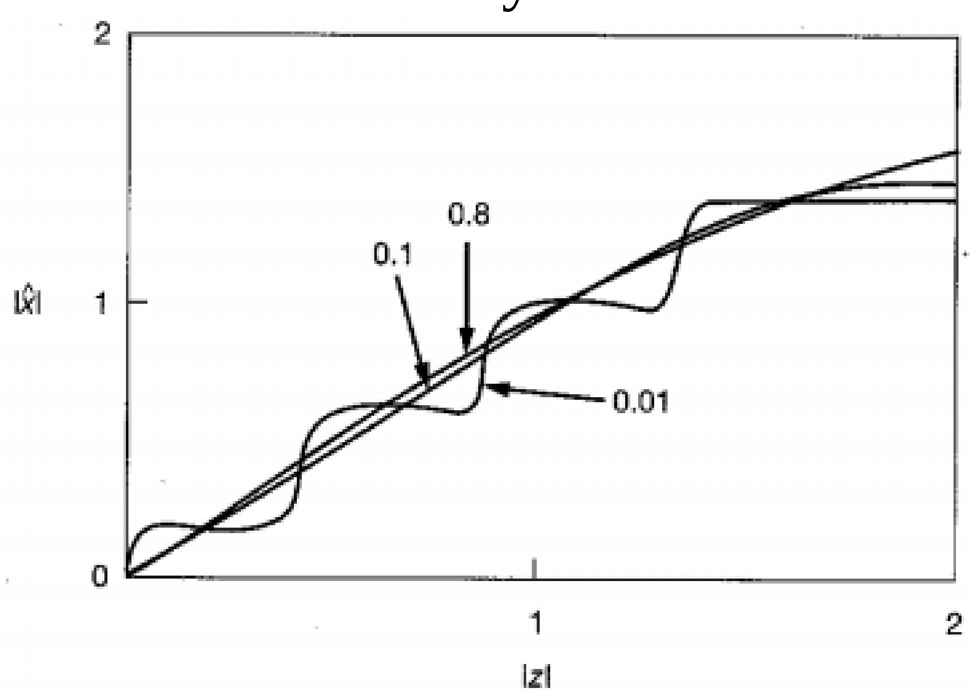
$$y_2 = \frac{1}{\sigma}(y - \sqrt{3}c_0)$$

and

$$Z(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$$

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^{\infty} e^{-\frac{u^2}{2}} du$$

For an 8 level PAM system:



When the blind equalizer has converged, the algorithm is switched to decision-directed mode.

In the case of binary symbol,

$$x(n)=\begin{cases}+1 & \text{for symbol } 1\\ -1 & \text{for symbol } 2\end{cases}$$

we have

$$\hat{x}(n)=\operatorname{sgn}(y(n))=\begin{cases}+1 & \text{if } y(n)\geq 0\\ -1 & \text{else}\end{cases}$$

and

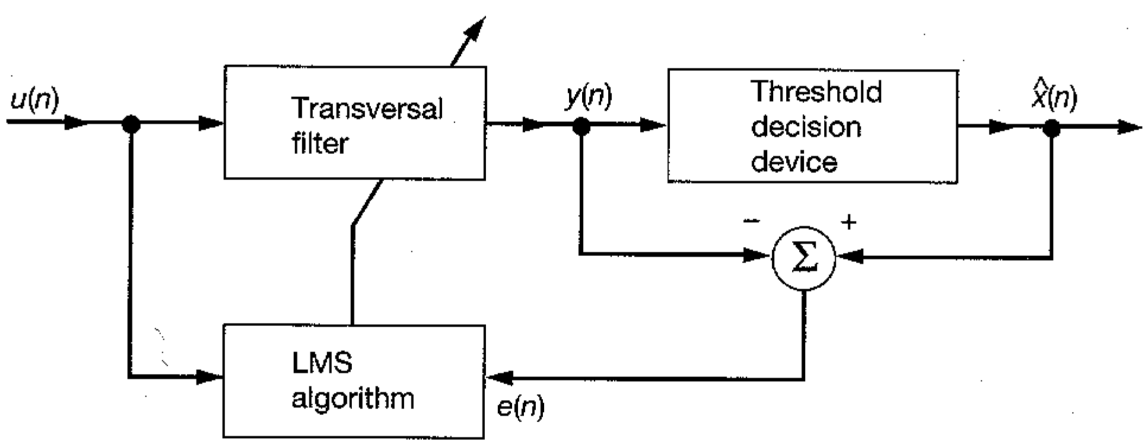


Figure 18.8 Block diagram of decision-directed mode of operation.