2.5

a.

$$\begin{aligned}\mathbf{R}_u &= E\{[\alpha(n)\mathbf{s}(n)+\mathbf{v}(n)][\alpha(n)\mathbf{s}(n)+\mathbf{v}(n)]^H\}\\ &= \sigma_\alpha^2\mathbf{s}(n)\mathbf{s}^H(n)+\mathbf{R}_v\\ &= \sigma_\alpha^2\mathbf{I}+\mathbf{R}_v\end{aligned}$$

Sine $\mathbf{s}(n)$ is Hermitian symmetric.

b. $\mathbf{p}=0 \Rightarrow \mathbf{w}=\mathbf{R}_u^{-1}\mathbf{p}=0$

c. $\sigma_\alpha^2=0 \Rightarrow \mathbf{R}_u=\mathbf{R}_v$. Thus $\mathbf{w}=\mathbf{R}_v^{-1}\mathbf{p}$, where

$$\mathbf{p}=[r_v(k),r_v(k-1),\ldots,r_v(k-M+1)]^T$$

d.

$$\begin{aligned}\mathbf{p} &= E\{[\alpha(n)\mathbf{s}(n)+\mathbf{v}(n)][\alpha(n)e^{-j\omega\tau}]\\ &= \sigma_\alpha^2\mathbf{s}^*(n)e^{j\omega\tau}\\ &= \sigma_\alpha^2\begin{bmatrix}e^{j\omega\tau}\\ e^{j\omega(\tau-1)}\\ \vdots\\ e^{j\omega(\tau-M+1)}\end{bmatrix}\\ \Rightarrow \mathbf{w} &= [\mathbf{I}+\frac{1}{\sigma_\alpha^2}\mathbf{R}_v]^{-1}\begin{bmatrix}e^{j\omega\tau}\\ e^{j\omega(\tau-1)}\\ \vdots\\ e^{j\omega(\tau-M+1)}\end{bmatrix}\end{aligned}$$

2.10 $J_{\min}=\sigma_d^2-\mathbf{p}^H\mathbf{R}\mathbf{p}=1,0.17,0.32,0.31,0.30$ for $M=0,1,\ldots,4$.

4.1

a. $\lambda_{\max}=1.5 \Rightarrow 0<\mu<1.\bar{3}$

b. Substituting in and rearranging,

$$\begin{aligned}w_1(n+1) &= 0.5-0.5w_2(n)]\\ w_1(n+1) &= 0.25-0.5w_1(n)]\end{aligned}$$

c. The transformed weight vector error recursion is

$$\begin{aligned}v_1(n) &= (1-0.5\mu)^n v_1(0)\\ v_2(n) &= (1-1.5\mu)^n v_2(0)\end{aligned}$$

Thus $0<\mu<2/3 \Rightarrow$ damped trajectory while $2/3<\mu<4/3 \Rightarrow$ oscillating trajectory.

4.5

$$\begin{aligned}\mathbf{w}(n+1) &= \mathbf{R}^{-1}(n+1)\mathbf{p} \\ &= \mu\sum_{k=0}^{n}(\mathbf{I}-\mu\mathbf{R})^k\mathbf{p} \\ &= \mu\mathbf{p}+(\mathbf{I}-\mu\mathbf{R})\mu\sum_{k=0}^{n-1}(\mathbf{I}-\mu\mathbf{R})^k\mathbf{p} \\ &= \mu\mathbf{p}+(\mathbf{I}-\mu\mathbf{R})\mathbf{R}^{-1}(n)\mathbf{p} \\ &= \mu\mathbf{p}+(\mathbf{I}-\mu\mathbf{R})\mathbf{w}(n) \\ &= \mathbf{w}(n)+\mu(\mathbf{p}-\mathbf{R}\mathbf{w}(n))\end{aligned}$$

4.12

$$\begin{aligned}\begin{bmatrix} r(0) & r(1) & r(2) \\ r(1) & r(0) & r(1) \\ r(2) & r(1) & r(0)\end{bmatrix}\begin{bmatrix}-0.5\\-0.5\\0.5\end{bmatrix} &= \begin{bmatrix} r(1)\\ r(2)\\ r(3)\end{bmatrix} \\ r(0)+0.5r(1)+0.5r(2)-0.5r(3) &= 1 \\ \Rightarrow \mathbf{p} &= \begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 1 & -0.5 \\ -0.5 & -0.5 & 1\end{bmatrix} \\ \Rightarrow \lambda &= 0, 1.5, 1.5 \\ &\Rightarrow 0<\mu<1.\bar{3}\end{aligned}$$

5.8

$$\begin{aligned}J(n) &= |e(n)|^2+\alpha||\mathbf{w}(n)||^2 \\ \Rightarrow \nabla J(n) &= -2\mathbf{x}(n)e^*(n)-2\alpha\mathbf{w}(n) \\ \Rightarrow \mathbf{w}(n+1) &= \mathbf{w}(n)-\frac{1}{2}\mu[-2\mathbf{x}(n)e^*(n)-2\alpha\mathbf{w}(n)] \\ &= (1-\mu\alpha)\mathbf{w}(\mathbf{n})+\mu\mathbf{x}(n)e^*(n) \\ \Rightarrow E\{\mathbf{w}(n+1)\} &= E\{(1-\mu\alpha)\mathbf{w}(n)+\mu\mathbf{x}(n)(d^*(n)-\mathbf{x}^H(n)\mathbf{w}(n))\} \\ \Rightarrow \lim_{n\to\infty}E\{\mathbf{w}(n)\} &= (1-\mu\alpha)E\{\mathbf{w}(\mathbf{n})\}+\mu\mathbf{p}-\mu\mathbf{R}E\{\mathbf{w}(n)\} \\ &= (\mathbf{R}+\alpha\mathbf{I})^{-1}\mathbf{p}\end{aligned}$$

Thus simply add zero mean, α variance i.i.d. noise samples to the input.
5.13 Recall $\mathbf{K}(n)=E\{\varepsilon^H(n)\varepsilon(n)\}$ and that trace$[\mathbf{K}(n)]=E\{||\varepsilon(n)||^2\}$. Thus from the previously derived recursion

$$\mathbf{K}(n+1) = (\mathbf{I}-\mu\mathbf{R})\mathbf{K}(n)(\mathbf{I}-\mu\mathbf{R})+\mu^2J_{\min}\mathbf{R}$$

For n small, the errors are large and the RHS is dominated by the first term. Utilizing the given correlation matrix yields

$$\begin{aligned}\mathbf{K}(n+1) &\approx (\mathbf{I}-\mu\sigma_u^2\mathbf{I})\mathbf{K}(n)(\mathbf{I}-\mu\sigma_u^2\mathbf{I}) \\ &= (1-\mu\sigma_u^2)^2\mathbf{K}(n)\end{aligned}$$

Taking the trace of both sides yields the desired result.