

1. A token is placed at the origin on a piece of graph paper. A coin biased to heads is given, $P(H) = 2/3$. If the result of a toss is heads, the token is moved one unit to the right, and if it is a tail the token is moved one unit to the left. Repeating this 1200 times, what is a probability that the token is on a unit N , where $350 \leq N \leq 450$? Simulate the system and plot the histogram using 10,000 realizations.
2. Random variable X is characterized by cdf $F_X(x) = (1 - e^{-x})U(x)$ and event C is defined by $C = \{0.5 < X \leq 1\}$. Determine and plot $F_X(x|C)$ and $f_X(x|C)$.
3. Prove that the characteristic function for the univariate Gaussian distribution, $N(\eta, \sigma^2)$, is

$$\phi(\omega) = \exp\left(j\omega\eta - \frac{\omega^2\sigma^2}{2}\right)$$

Next determine the moment generating function and determine the first four moments.

4. Let $Y = X^2$. Determine $f_Y(y)$ for:
 - (a) $f_X(x) = 0.5 \exp\{-|x|\}$
 - (b) $f_X(x) = \exp\{-|x|\}U(X)$
5. Given the joint pdf $f_{XY}(x, y)$

$$f_{XY}(x, y) = \begin{cases} 8xy, & 0 < y < 1, 0 < x < y \\ 0, & \text{otherwise} \end{cases}$$

Determine (a) $f_x(x)$, (b) $f_Y(y)$, (c) $f_Y(y|x)$, and (d) $E[Y|x]$.

6. Let W and Z be RVs defined by

$$W = X^2 + Y^2 \quad \text{and} \quad Z = X^2$$

where X and Y are independent; $X, Y \sim N(0, 1)$.

- (a) Determine the joint pdf $f_{WZ}(w, z)$.
- (b) Are W and Z independent?