1. Show that if $\beta_x(t) = f_x(t|\mathbf{x} > t)$, $\beta_y(t) = f_y(t|\mathbf{y} > t)$ and $\beta_x(t) = k\beta_y(t)$
then $1 - F_x(x) = [1 - F_y(x)]^k)$

2. Express the density $f_x(y)$ of the RV $\mathbf{y} = g(\mathbf{x})$ in terms of $f_x(x)$ if (a) $g(x) = |x|$; (b) $g(x) = e^{-x}U(x)$.

3. The RVs $\mathbf{x}$ and $\mathbf{y}$ are independent with exponential densities
$f_x(x) = \alpha e^{-\alpha x}U(x), f_y(y) = \beta e^{-\beta y}U(y)$
Find the densities of the following RVs: $\mathbf{2x} + \mathbf{y}$; $\frac{\mathbf{x}}{\mathbf{y}}$

4. The RVs $\mathbf{x}$ and $\mathbf{y}$ are $N(0;\sigma)$ and independent. Show that, if $\mathbf{z} = |\mathbf{x} - \mathbf{y}|$, then

$$E\{\mathbf{z}\} = 2\sigma/\sqrt{\pi}, E\{\mathbf{z}^2\} = 2\sigma^2.$$

5. Use the moment generating function, show that the linear transformation of a Gaussian random vector is also Gaussian.

6. Let $\{x_k(n)\}_{k=1}^4$ be four IID random variables with exponential distribution with α = 1.

$$y_k(n) = \sum_{l=1}^{k} x_l(n), 1 \leq k \leq 4$$

(a) Determine and plot the pdf of $y_2(n)$
(b) Determine and plot the pdf of $y_3(n)$
(c) Determine and plot the pdf of $y_4(n)$
(d) Compare the pdf of $y_4(n)$ with that of the Gaussian density.

7. The mean and covariance of a Gaussian random vector $\mathbf{x}$ are given by, respectively,

$$\mu_x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and

$$\mathbf{\Gamma}_x = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

Plot the 1σ, 2σ, and 3σ concerntration ellipses representing the contours of the densitiy function in the $(x1, x2)$ plane. $Hinits$: The radius of an ellipse with major axis a (along $x1$) and minor axis $b < a$ (along $x2$) is given by

$$r^2 = \frac{a^2b^2}{a^2 sin^2\theta + b^2 cos^2\theta}$$

where $0 \leq \theta \leq 2\pi$. Compute the 1 σ ellipse specified by $a = \sqrt{\lambda_1}$ and $b = \sqrt{\lambda_2}$ and then rotate and translate each point $\mathbf{x}^(i) = [x_1^{(i)} x_2^{(i)}]$ using the transformation $\mathbf{w^{(i)}} = \mathbf{Q_x x^i} + \mu_\mathbf{x}$.