Chapter 8

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ELEG/PHYS667 Magnetism & Spintronics
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1 Hall effect

When a current moves in a conductor with a perpendicular magnetic field, the Lorentz force deflects the charge to the sides:

$$F = qv \times B \tag{1}$$

$$I = nqv \cdot A \rightarrow v = \frac{I}{nq \cdot A} \tag{2}$$

This build-up of charge is balanced by the repulsive Coulomb field:

$$q\frac{I}{nq \cdot A}B = \frac{qV}{w} \tag{3}$$

$$V_{Hall} = \frac{Iw}{nq \cdot A}B = \frac{Iw}{nq \cdot wd}B = \frac{I}{nqd}B = R_{Hall}I \tag{4}$$

So the transverse voltage is proportional to the magnetic field. Note that the charge density, n, is in the denominator. In metals, $n \approx 10^{23}cm^{-3}$, resulting in an extremely small R_{Hall}.

2 Magneto Resistance

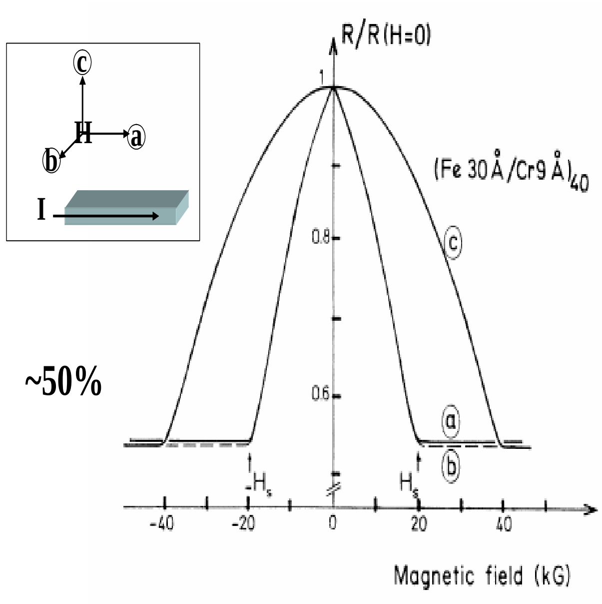
We saw that in the Hall sensor, a magnetic field induces a transverse voltage. There are other magnetic field sensors which change resistance in a magnetic field, known as Magnetoresistive (MR) devices. This effect was first seen in planar films of ferromagnetic metals at about 1857, where the resistance is greater for magnetization parallel to the current direction than perpendicular. While this effect, Anisotropic magnetoresistance (AMR), was the dominant technology in magnetic hard drive read-heads for about 10 years (late 80s to late 90s), it was supplanted by the more sensitive Giant Magneto resistive (GMR) devices which triggered a drastic increase in information density in available hard drives.

3 Giant Magneto Resistance

A GMR device consists of a multilayer of ferromagnetic thin films separated by a spacer material. This can be antiferromagnetic (as in Cr) or "normal" (This essentially means neither ferro- nor antiferromagnetic such as Cu). In the original GMR devices, exchange caused anitferromagnetic alignment of the ferromagnetic magnetizations. When current flowed in-plane, a magnetic field could be used to align the magnetizations and this caused a reduction of resistance of many tens of percent.

4 Resistor Network Theory of GMR

A trivial explanation of this effect involves several simplifications. For instance, for simplicity we consider the current to be perpendicular to the plane. However, a key element is that since the density of states of the two spin species is different in the ferromagnetic layers, the scattering rates and hence the resistivities will be different. We use a Mott



two-current model which assumes the total current flowing in the multilayer consists of parallel components of spin up and spin down. Our task is to calculate the change in resistance when the relative orientations of magnetizations is change between parallel ($\uparrow\uparrow$) and antiparallel ($\uparrow\downarrow$). The resistivity of a spin down current in a film with magnetization $\uparrow$ is ρ^{HI}, and resistivity of a spin up current in a film with magnetization $\uparrow$ is ρ^{LO}. By symmetry, the resistivity of a spin down current in a film with magnetization $\downarrow$ is ρ^{LO}, and resistivity of a spin up current in a film with magnetization $\downarrow$ is ρ^{HI}. Therefore we can calculate the total resistivity, considering the two parallel currents.

$$\frac{1}{\rho_{\uparrow\uparrow}} = \frac{1}{2\cdot\frac{\rho^{HI}}{2}} + \frac{1}{2\cdot\frac{\rho^{LO}}{2}} = \frac{\rho^{HI}+\rho^{LO}}{\rho^{HI}\rho^{LO}} \tag{5}$$

$$\frac{1}{\rho_{\uparrow\downarrow}} = \frac{1}{\frac{\rho^{LO}}{2}+\frac{\rho^{HI}}{2}} + \frac{1}{\frac{\rho^{HI}}{2}+\frac{\rho^{LO}}{2}} = \frac{4}{\rho^{HI}+\rho^{LO}} \tag{6}$$

$$\%MR = \frac{\Delta R}{R} = \frac{\rho_{\uparrow\downarrow}-\rho_{\uparrow\uparrow}}{\rho_{\uparrow\uparrow}} = \frac{\frac{\rho^{HI}+\rho^{LO}}{4} - \frac{\rho^{HI}\rho^{LO}}{\rho^{HI}+\rho^{LO}}}{\frac{\rho^{HI}\rho^{LO}}{\rho^{HI}+\rho^{LO}}} \tag{7}$$

$$= \frac{(\rho^{LO}+\rho^{HI})^2 - 4\rho^{HI}\rho^{LO}}{4\rho^{HI}\rho^{LO}} = \frac{(\rho^{LO}-\rho^{HI})^2}{4\rho^{HI}\rho^{LO}} = \frac{\left(1-\frac{\rho^{HI}}{\rho^{LO}}\right)}{4\frac{\rho^{HI}}{\rho^{LO}}} \tag{8}$$

Defining $\alpha = \frac{\rho^{HI}}{\rho^{LO}}$,

$$\%MR = \frac{(1-\alpha)^2}{4\alpha} \tag{9}$$

Note that α is equivalent to the spin-dependent scattering asymmetry. This result is unsatisfying because it doesn't depend on geometry or any other tunable variable of GMR device we can make.

4.1 Mathon's Theory

Mathon's theory was a first attempt to incorporate some of the features of geometry in the calulation of MR. In this model, we incorporate in-plane transport, and consider that since the electron "samples" different regions of the multilayer, the effective resistivity of the composite is a weighted average of each film. Therefore, a bi-layer with two films of thickness a and b having individual resitivities ρ_a and ρ_b will have effective resitivity of

$$\rho = \frac{a\rho_a + b\rho_b}{a+b} \tag{10}$$

Now imagine we are modeling a 4-layer system with two ferromagnetic layers of thickness M separated by normal metal spacers of thickness N. The resistivity of spin up channel in system with parallel magnetizations:

$$\rho^{\downarrow}_{\uparrow\uparrow} = \frac{N\rho_N + M\rho_{HI} + N\rho_N + M\rho_{HI}}{M + N + M + N} \tag{11}$$

Resistivity of spin down channel in system with parallel magnetizations:

$$\rho^{\uparrow}_{\uparrow\uparrow} = \frac{N\rho_N + M\rho_{LO} + N\rho_N + M\rho_{LO}}{M + N + M + N} \tag{12}$$

For the antiparallel magnetization case, both spin channels see the same resistivity

$$\rho^{\uparrow/\downarrow}_{\uparrow\downarrow} = \frac{N\rho_N + M\rho_{LO} + N\rho_N + M\rho_{HI}}{M + N + M + N} \tag{13}$$

so that

$$\rho_{\uparrow\downarrow} = \frac{\rho^{\uparrow/\downarrow}_{\uparrow\downarrow}}{2} \tag{14}$$

It is simple to define the parameters $\beta = \frac{\rho_{HI}}{\rho_{LO}}$ and $\mu = \frac{\rho_L}{\rho_N}$ giving

$$\rho^{\uparrow}_{\uparrow\uparrow} = \frac{\frac{N}{M\mu} + 1}{(N+M)/M\rho_L} \tag{15}$$

and

$$\rho^{\downarrow}_{\uparrow\uparrow} = \frac{\frac{N}{M\mu} + \beta}{(N+M)/M\rho_L} \tag{16}$$

and

$$\rho^{\uparrow/\downarrow}_{\uparrow\downarrow} = \frac{2\frac{N}{M\mu} + \beta + 1}{4(N+M)/M\rho_L} \tag{17}$$

This gives

$$\rho_{\uparrow\uparrow} = \frac{(\frac{N}{M\mu} + \beta)(\frac{N}{M\mu} + 1)}{(N+M)/M\rho_L \cdot (2\frac{N}{M\mu} + \beta + 1)} \tag{18}$$

and therefore in this model (after a little algebra)

$$\%MR = \frac{\Delta\rho}{\rho} = \frac{1 - \beta^2}{4(\frac{N}{M\mu} + \beta)(\frac{N}{M\mu} + 1)} \tag{19}$$

Note that this β is the same as the previous models' α. The numerator is the same. We can re-write this expression as

$$\%MR = \frac{\Delta\rho}{\rho} = \frac{1 - \alpha^2}{4\alpha + 4\left[\left(\frac{N}{M\mu}\right)^2 + \frac{N}{M\mu}(\alpha + 1)\right]} \tag{20}$$

$$\%MR = \frac{(1-\alpha)^2}{4\alpha + \Delta} \tag{21}$$

Which should be compared to Eqn. 9. The denominator is modified by geometry-dependent variables. This model captures our expectation that the GMR decreases with normal metal thickness (N) and saturates as $\frac{M\mu}{N}$ increases.