

# Chapter 7

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ELEG/PHYS667 Magnetism & Spintronics  
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## 1 Spin lifetime in metals

Let's revisit electron spin in a magnetic field. The equation of motion is

$$\frac{d\vec{\mu}}{dt} = \frac{g}{\hbar} \vec{\mu} \times \vec{B} \quad (1)$$

When  $\vec{B}$  is along  $\hat{z}$ , this equation has solutions describing Larmor precession around the magnetic field:

$$\mu_x = \mu \sin \theta \cos \omega_L t \quad (2)$$

$$\mu_y = \mu \sin \theta \sin \omega_L t \quad (3)$$

$$\mu_z = \mu \cos \theta \quad (4)$$

However, this is a perfect-case scenario. In reality, there are damping effects which relax the precessing state to a static orientation along the field direction. If there were no such relaxation effects, then the two spin species in a paramagnetic metal would have different Fermi Energies in a magnetic field. We know this situation does not occur.

We can include relaxation effects phenomenologically and separate out the  $x, y, z$  components:

$$\frac{d\mu_z}{dt} = \frac{g}{\hbar} (\vec{\mu} \times \vec{B})_z + \frac{\mu_0 - \mu_z}{T_1} \quad (5)$$

$$\frac{d\mu_x}{dt} = \frac{g}{\hbar} (\vec{\mu} \times \vec{B})_x - \frac{\mu_x}{T_2} \quad (6)$$

$$\frac{d\mu_y}{dt} = \frac{g}{\hbar} (\vec{\mu} \times \vec{B})_y - \frac{\mu_y}{T_2} \quad (7)$$

These are called the Bloch equations, after Felix Bloch who won the Nobel Prize in 1952 for his work on magnetic resonance.  $T_1$  is called the “longitudinal” or “spin-lattice” relaxation time and  $T_2$  is called the “transverse” or “spin-spin” relaxation time.  $T_1$  and  $T_2$  are in general not equal, although  $T_1$  sets an upper bound to  $T_2$ . This is because precessional dephasing can have the same effect to measured transverse component  $\mu_{x,y}$  as coherent decay when measuring an ensemble of moments. This dephasing can be the result of e.g. inhomogeneous magnetic fields resulting in slightly different Larmor precession frequencies across the sample.

Because the relaxation times determine state lifetimes, they also determine the linewidths of excitation and absorption spectra of oscillating fields through the relation

$$\Delta E \delta t = \hbar \quad (8)$$

A rigorous analysis yields the following power absorption spectrum of AC fields as a function of DC field (Electron Spin Resonance - ESR - or Electron Paramagnetic Resonance - EPR):

$$P(B) = \frac{\gamma^2 B \mu_z T_2}{1 + (B_r - B)^2 \gamma^2 T_2^2} B x^2 \quad (9)$$

where  $\gamma = g/\hbar$ ,  $B_r$  is the field at which the Zeeman splitting is resonant with the driving (oscillating) field and  $B_x$  is the magnitude of the driving field. This is a Lorentzian function centered at the resonant field. The HWHM  $\Delta B_{1/2}$  of this Lorentzian is  $\frac{1}{\gamma T_2}$ . In cubic solids,  $T_1 = T_2$ , so

$$T_1 = \frac{1}{\gamma \Delta B_{1/2}}. \quad (10)$$

Since

$$E = \hbar \omega_0 = g \mu B \quad (11)$$

$$g/\hbar = \gamma = \frac{\omega_0}{\mu B} \quad (12)$$

## 1.1 Measuring $T_1$ and $T_2$

Experimental measurement of  $T_1$  can be accomplished with the “inversion-recovery” technique. Moments aligned with a static magnetic field  $\mu_z = \mu_0$  are rotated by  $\pi$  with an RF pulse of proper magnitude and duration to  $\mu_z = -\mu_0$ . The longitudinal component of magnetic moment then recovers to its initial value over the unknown timescale  $T_1$ . During this recovery, after a waiting period  $\tau$ , a  $\pi/2$  pulse rotates this component into the transverse plane where it precesses. The amplitude of the resulting radiated field indicates the longitudinal component at time  $\tau$ . The measurement is repeated for different  $\tau$  and the exponential timescale is determined.

$T_2$  can be measured by the spin-echo technique. Initially aligned states are rotated into the transverse plane and allowed to precess. Dephasing will occur, where fast precessing states will have a phase lead over slower states. After a time  $\tau$ , a  $\pi$  pulse is applied, flipping the order of fast and slow components of the precessing ensemble; now, the fast precessing moments have a phase lag. Over time  $\tau$ , this dispersion is compensated, and all the moments line up again for a short interval. The precession of this partially coherent ensemble radiates a RF signal which can be used to measure its magnitude. Repeating the spin echo sequence with different  $\tau$  can be used therefore to measure  $T_2$ .

## 1.2 Spin diffusion

For Cu,  $T_1 \approx 4 \times 10^{-10} s$  @ 60K. This is a measure of the conduction electron spin relaxation time, so multiplying times its average speed, the Fermi velocity, should give the spin flip mean free path

$$\lambda_{sf} = v_F T_1 = 1.57 \times 10^6 m/s \cdot 4 \times 10^{-10} s = 630 \mu m \quad (13)$$

However, the electron does not travel in a straight line; it diffuses. The relevant length scale is not the mean free path, but the spin flip diffusion length, determined by the modified diffusion equation

$$\frac{\partial P}{\partial t} = D \frac{\partial^2 P}{\partial x^2} - \frac{P}{T_1} \quad (14)$$

where the classical diffusion coefficient  $D = \frac{v_F \lambda}{3}$ . The steady-state solution to this equation is

$$P = e^{-\sqrt{\frac{1}{DT_1}} x} \quad (15)$$

The characteristic length scale of this solution is the sought-after spin diffusion length,

$$\Lambda_{sd} = \sqrt{\frac{\lambda \lambda_{sd}}{3}} \quad (16)$$

But what is  $\lambda$ ? We can recover it from the first successful theory of metals, the Drude Model.

## 1.3 Drude Model

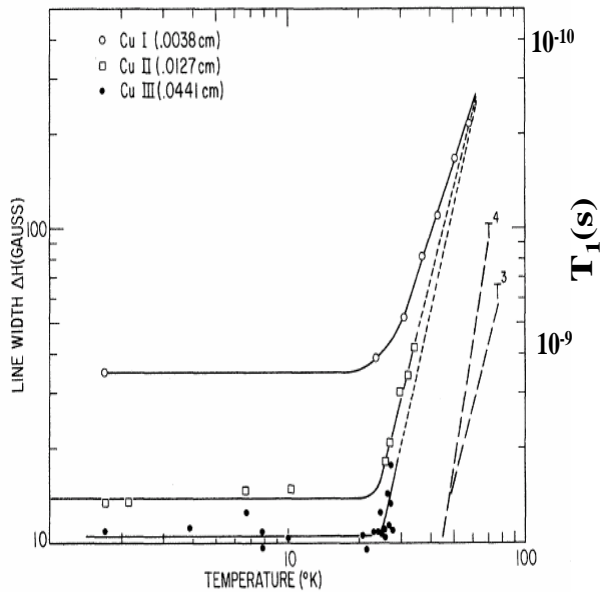
The acceleration of a charge  $e$  in electric field  $E$  is

$$a = \frac{F}{m} = \frac{eE}{m} \quad (17)$$

If  $\tau$  is the scattering time between collisions, the velocity before collision is

$$v_d = a\tau = \frac{eE}{m} \tau \quad (18)$$

However, the current density is given by



(3), (10), (11)

Metal	$\frac{\rho}{(\Delta B)_{\frac{1}{2}}}$ [T <sup>-1</sup> Ω <sup>-1</sup> m <sup>-1</sup> ]	g	n [10 <sup>20</sup> m <sup>-3</sup> ]	$\frac{\tau}{T_1}$
Na	2.6 · 10 <sup>-5</sup>	2.0015	2.65	9.0 · 10 <sup>-6</sup>
K	4.1 · 10 <sup>-6</sup>	1.9996	1.40	0.11 · 10 <sup>-3</sup>
Rb	3.5 · 10 <sup>-7</sup>	1.9991	1.15	1.6 · 10 <sup>-3</sup>
Cs	3.9 · 10 <sup>-8</sup>	1.9904	0.91	17 · 10 <sup>-3</sup>
Cu	6.4 · 10 <sup>-8</sup>	2.033	8.45	1.2 · 10 <sup>-3</sup>
Ag	3.8 · 10 <sup>-8</sup>	1.983	5.85	2.8 · 10 <sup>-3</sup>
Au	1.0 · 10 <sup>-9</sup>	2.10 ± 0.03	5.90	0.22
Be	6.9 · 10 <sup>-4</sup>	2.0014	—	—
Mg	3.3 · 10 <sup>-6</sup>	~2.0	—	—
Al	1.9 · 10 <sup>-7</sup>	1.997	6.02*	0.53 · 10 <sup>-3</sup>

$$j = nev_d \quad (19)$$

where  $n$  is the density of electrons. Therefore,

$$\frac{j}{ne} = \frac{eE}{m} \tau \quad (20)$$

If we use Ohm's law,  $E = j\rho$ ,

$$\frac{j}{ne} = \frac{ej\rho}{m} \tau \quad (21)$$

$$\frac{1}{ne} = \frac{e\rho}{m} \tau \quad (22)$$

$$\tau = \frac{m}{ne^2\rho} \quad (23)$$

Since  $\lambda = v_F\tau$ ,

$$\lambda = \frac{v_F m}{\rho e^2 n} \quad (24)$$

For an order-of-magnitude approximation, we can substitute some values, using  $e^2 \approx 10^{-7} eV \cdot cm$ :

$$\lambda \approx \frac{10^8 cm/s \times 5 \times 10^5 eV/c^2}{10^{-6} \Omega \cdot cm 10^{-7} eV \cdot cm 10^{23} cm^{-3}} \approx 10^{-18} \Omega^{-1} \cdot s \quad (25)$$

But we expect this mean-free path to have units of cm! To reconcile, note that in units of eV, cm, and s, resistance has units of

$$\Omega = \frac{Volt}{Ampere} = \frac{Volt}{Coulomb/second} = \frac{e \cdot Volt \cdot s}{e \cdot Coulomb} = \frac{e \cdot Volt \cdot s}{e^2/1.6 \times 10^{-19}} \quad (26)$$

Since  $e^2 \approx 10^{-7} eV \cdot cm$ ,

$$1\Omega \approx 10^{-12} (cm/s)^{-1} \quad (27)$$

Therefore,

$$\lambda \approx 10^{-6} cm \quad (28)$$

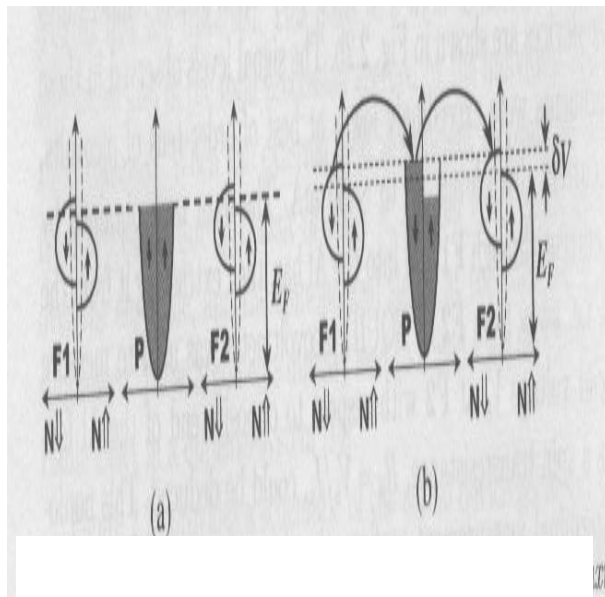
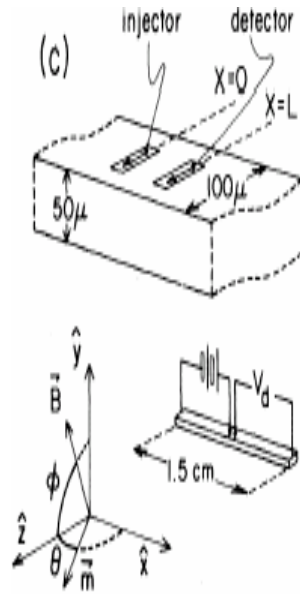
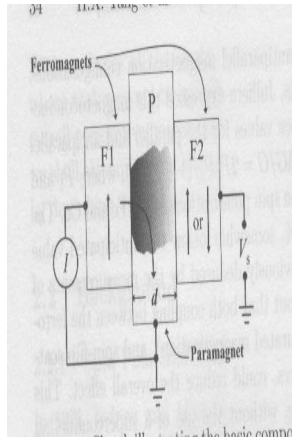
This guess (10nm) is a bit low since for Cu,  $\lambda = 660nm$ . But it gives us a reasonable order of magnitude for the spin diffusion length in metals

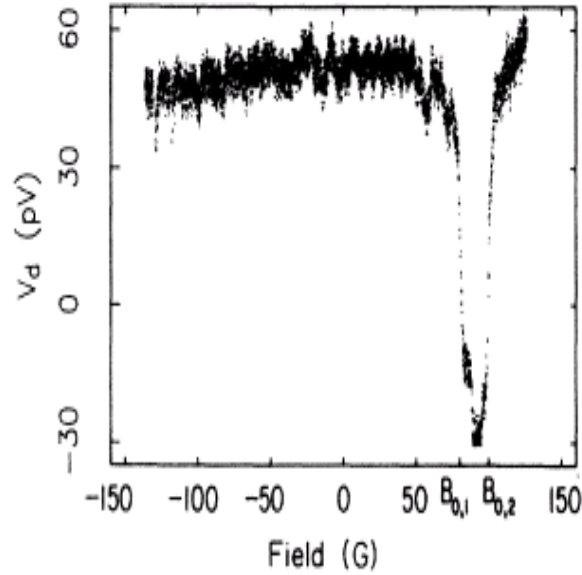
$$\Lambda_{sd} = \sqrt{\frac{\lambda\lambda_{sd}}{3}} \approx 1\mu m \quad (29)$$

## 2 Johnson-Silsbee experiment

In 1985, Silsbee and his graduate student Mark Johnson performed what could be called the first modern spintronics experiment. The motivation of this work was to inject spin polarized electrons from a ferromagnet into a metal (Aluminum) and detect their presence.

Injecting the spins is easy: drive an electron current from one ferromagnet into the Aluminum. We want to detect the presence with another ferromagnet. In our explanation, we approximate both ferromagnets as half-metals (conduction electrons at the Fermi Energy are of only one spin species). Then, the injected polarization is initially 100% at the injection point. These electrons can only fill states in the appropriate spin band, so the density of electrons in that spin band rises. Consequently, the Fermi level of that band also rises. Since total charge is conserved, the density and hence Fermi energy of the opposite spin band decreases. (This is because spins of both types are being carried away by the other contact to the Aluminum). If no current flows through the interface between the Aluminum and the half-metallic ferromagnet used as spin detector, equilibrium demands that its Fermi energy equal the Fermi energy of the spin band with compatible spin in the Aluminum. Measuring potential (voltage) between the ferromagnet and a point far away from the injection (where the Fermi levels have equilibrated) then

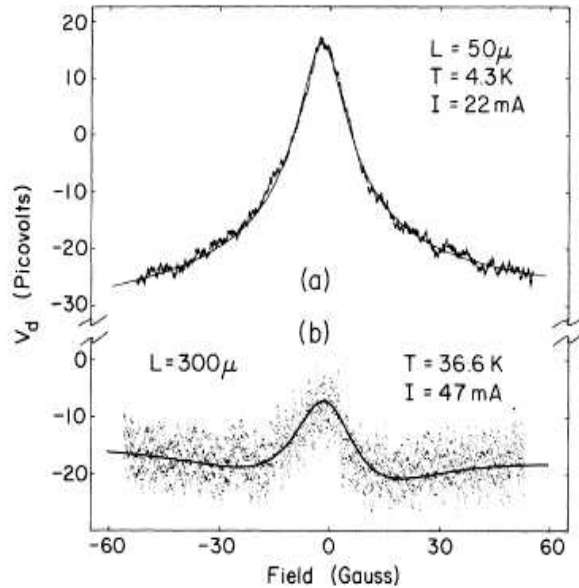




tells us that spin is present. This is called a “non-local” measurement and it is done to avoid measuring voltages due to Ohmic losses in regions where charge currents flow.

Furthermore, by sweeping an external magnetic field, the relative magnetizations of injector and detector can be changed, and the measured voltage will change sign.

By using a magnetic field perpendicular to the magnetization (and hence spin axis), precession will cause the signal to decay. This is called the Hanlé effect.



## 2.1 Calculation of signal magnitude

It is clear that the voltage measured will be proportional to the spin present, but how big is that voltage? Signal magnitude can be determined by calculating magnetization current

$$I_m = \frac{I}{e} \mu_B. \quad (30)$$

The rate of magnetization increase in the Aluminum is

$$\frac{dM}{dt} |_{injection} = \frac{I_m}{A \cdot d} = \frac{I \mu_B}{eA \cdot d} \quad (31)$$

At steady state, this must match the spin decay rate

$$\frac{dM}{dt} |_{relaxation} = \frac{M}{T_1} \quad (32)$$

yeilding an equilibrium magnetization

$$M_{equilibrium} = \frac{\mu_B I T_1}{eA \cdot d}. \quad (33)$$

This is the consequence of filling states above the Fermi Energy

$$\frac{M_{equilibrium}}{\mu_B} = \frac{I T_1}{eA \cdot d} = D(E_F) eV \quad (34)$$

So we can define a “resistance” which tells us the ratio of the measured voltage to the injection current:

$$R = \frac{V}{I} = \frac{T_1}{e^2 A \cdot d D(E_F)} \quad (35)$$

For the free electron gas, we know

$$D(E_F) = \frac{3n}{2E_F} \quad (36)$$

Therefore,

$$R = \frac{2T_1 E_F}{3ne^2 A \cdot d} \quad (37)$$

What is the magnitude of this expression? For a device  $100\mu m$  on a side, we have

$$R = \frac{10^{-9} s \cdot 1eV}{10^{23} cm^{-1} \cdot 10^{-7} eV \cdot cm \cdot 10^{-6} cm^3} = 10^{-19} (cm/s)^{-1} \quad (38)$$

Again, this value is a resistance but is given in units of  $(cm/s)^{-1}$ , not  $\Omega$ . Since we have derived before

$$1\Omega \approx 10^{-12} (cm/s)^{-1} \quad (39)$$

Our expected signal will be

$$\frac{10^{-19} (cm/s)^{-1}}{10^{-12} (\Omega \cdot cm/s)^{-1}} = 100n\Omega \quad (40)$$

This is consistent with the signal of approximately 60pV at 22mA.