Chapter 6

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ELEG/PHYS667 Magnetism & Spintronics
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1 Antiferromagnetism

Another type of magnetic material important to spintronics is antiferromagnets. Experimentally, it was found that above a critical temperature, the inverse susceptibility was shifted to the left (with a negative temperature intercept). As the mechanism behind this was a mystery, other efforts were undertaken to probe the crystal structure. Neutron diffraction revealed that below the critical temprature, greater symmetry appeared.

Neel proposed that the experimental behavior was the result of two interpenetrating lattices with magnetic moments which tend to anti-align below the critical temperature. To model this effect, he proposed that the lattices, A and B, were subject to two different molecular fields $H_A = -wM_B$ and $H_B = -wM_A$. Note that the field acting on the A sublattice is dependent on the magnetization of the B sublattice, and opposite in sign.

We know that each sublattice independently must follow

$$M_A = \frac{N}{2}\mu \tanh \alpha_A \tag{1}$$

and

$$M_B = \frac{N}{2}\mu \tanh \alpha_B \tag{2}$$

and also must obey

$$\alpha_A = \frac{\mu(H + H_A)}{k_B T} = \frac{\mu(H - wM_B)}{k_B T} \tag{3}$$

and

$$\alpha_B = \frac{\mu(H + H_B)}{k_B T} = \frac{\mu(H - wM_A)}{k_B T} \tag{4}$$

Once again, we can examine the high-temeperature behavior when the first order expansion of $\tanh \alpha$ is a good approximation.

$$M_A = \frac{N\mu^2}{2k_B T}(H - wM_B) \tag{5}$$

$$M_B = \frac{N\mu^2}{2k_B T}(H - wM_A) \tag{6}$$

These are two coupled linear equations which can easily be solved with some algebra. Defining

$$M_A = \frac{N\mu^2}{2k_B T}(H - wM_B) = \frac{C}{2T}(H - wM_B) \tag{7}$$

and

$$M_B = \frac{N\mu^2}{2k_B T}(H - wM_A) = \frac{C}{2T}(H - wM_A) \tag{8}$$

where

$$C = \frac{N\mu^2}{k_B} \tag{9}$$

Substituting M_A into the expression for M_B:

$$M_B = \frac{C}{2T}(H - w(\frac{C}{2T}(H - wM_B)) \tag{10}$$

Solving:

$$\frac{2T}{C}M_B = (H - \frac{Cw}{2T}(H - wM_B)) \tag{11}$$

$$\frac{2T}{C}M_B = H(1 - \frac{Cw}{2T}) - \frac{Cw^2}{2T}M_B \tag{12}$$

$$M_B(\frac{2T}{C} - \frac{Cw^2}{2T}) = H(1 - \frac{Cw}{2T}) \tag{13}$$

$$M_B = H\frac{(1 - \frac{Cw}{2T})}{(\frac{2T}{C} - \frac{Cw^2}{2T})} \tag{14}$$

$$M_B = H\frac{(1 - \frac{Cw}{2T})}{(\frac{2T}{C} - \frac{Cw^2}{2T})} \cdot \frac{2TC}{2TC} \tag{15}$$

$$M_B = H\frac{2TC - C^2w}{4T^2 - C^2w^2} \tag{16}$$

$$M_B = H\frac{C(2T - Cw)}{(2T - Cw)(2T + Cw)} \tag{17}$$

$$M_B = H\frac{2C}{T + \frac{Cw}{2}} \tag{18}$$

Therefore,

$$\chi = \frac{C}{T + T_N} \tag{19}$$

T_N is the Néel temperature, the critical ordering temperature for ferromagnets. Néel won the Nobel prize for his work on magnetism in 1970.

2 Interface exchange bias

As we will see, Spintronics is dependent on being able to independently control the magnetization state of individual ferromagnetic layers in stacks of multilayers. To do this, the magnetization of some layers must be held fixed or pinned while others are free to rotate.

The exchange coupling between a ferromagnet and antiferromagnet can be used to pin the magnetization. Consider Figure 2 which shows an antiferromagnetic particle (AFM) on the surface of a ferromagnet (FM). For simplicity, assume the anisotropy easy axes of both materials coincide.

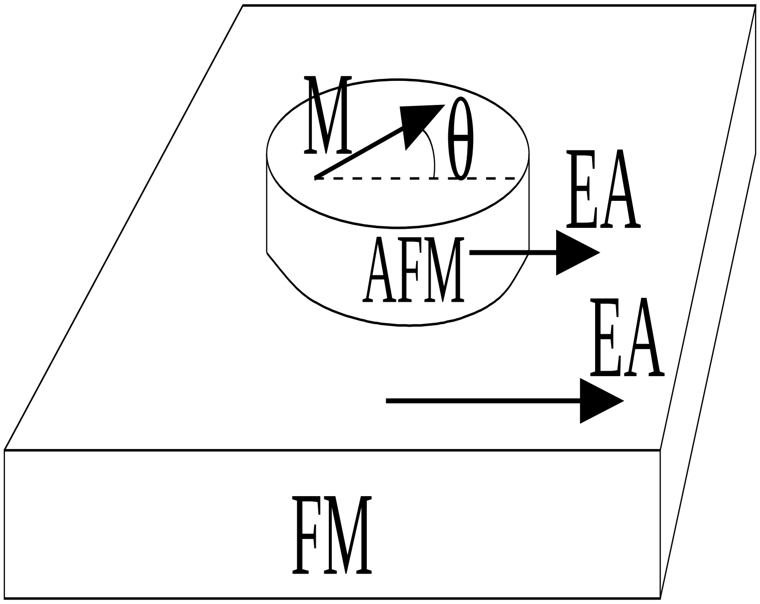
The energy of the AFM is the sum of the anisotropy energy and the exchange energy between its interface magnetic moment and the FM magnetic moment:

$$E_{AFM} = Kad\sin^2\theta + JMma\cos\theta \tag{20}$$

$$e = e_a\sin^2\theta + e_x\cos\theta \tag{21}$$

$$e_a = \frac{Kad}{k_B} \tag{22}$$

$$e_x = \frac{JMma}{k_B} \tag{23}$$



This energy profile has a minimum at $\theta = 0$ (state 1) and $\theta = \pi$ (state 2), with a barrier in between. In equilibrium, the Boltzmann factor determines the fraction of the ensemble in each minimum state. In other words, if we have N particles, N_1 of them are in state 1 and N_2 of them are in state 2. The fractional occupation is $n_1 = N_1/N$ and $n_2 = N_2/N$ for states 1 and 2 respectively. We want to determine how fast it takes to get to equilibrium determined by the conditions at $t = \infty$:

$$n_1(\infty) = \frac{e^{-\frac{2e_x}{T}}}{1 + e^{-\frac{2e_x}{T}}} = \frac{1}{1 + e^{\frac{2e_x}{T}}} \tag{24}$$

and

$$n_2(\infty) = \frac{1}{1 + e^{-\frac{2e_x}{T}}} = \frac{e^{\frac{2e_x}{T}}}{1 + e^{\frac{2e_x}{T}}} \tag{25}$$

where

$$n_1 + n_2 = 1 \tag{26}$$

(i.e. all particles must be in either state 1 or state 2).
Since

$$m(t) = n_1(t) - n_2(t) = n_1(t) - (1 - n_1(t)) = 2n_1(t) - 1, \tag{27}$$

the equilibrium magnetization is given by

$$\begin{aligned} m(\infty) = 2n_1(\infty) - 1 = 2\frac{1}{1 + e^{\frac{2e_x}{T}}} - 1 &= \frac{2 - (1 + e^{\frac{2e_x}{T}})}{1 + e^{\frac{2e_x}{T}}} \\ = \frac{1 - e^{\frac{2e_x}{T}}}{1 + e^{\frac{2e_x}{T}}} = \frac{e^{-\frac{e_x}{T}} - e^{\frac{e_x}{T}}}{e^{-\frac{e_x}{T}} + e^{\frac{e_x}{T}}} &= \tanh \frac{e_x}{T} \end{aligned} \tag{28}$$

3 Rate Equation

Now we determine how fast the system gets to the equilibrium condition. This is clearly determined by the height of the barrier between the states, so we need to calculate it by finding the maximum:

$$\frac{de}{d\theta} = 2e_a \cos\theta \sin\theta + e_x \sin\theta = 0 \tag{29}$$

$$\theta = \cos^{-1}\frac{e_x}{2e_a} = \cos^{-1} h \tag{30}$$

where

$$h = \frac{e_x}{2e_a} \tag{31}$$

the energy at the maximum is

$$e_0 = e_a(1 - \cos^2\theta) + e_x \cos\theta \tag{32}$$

$$e_0 = e_a(1 - h^2) + e_x h \tag{33}$$

$$e_0 = e_a(1 - \left(\frac{e_x}{2e_a}\right)^2) + \frac{e_x^2}{2e_a} \tag{34}$$

$$e_0 = e_a - \frac{e_x^2}{4e_a} + \frac{e_x^2}{2e_a} = e_a + \frac{e_x^2}{4e_a} = e_a\left(1 + \frac{e_x^2}{4e_a^2}\right) = e_a(1 + h^2) \tag{35}$$

The height of the barrier from states 1 (at $\theta = 0$) and 2 ($\theta = \pi$) are

$$q_1 = e_0 - e_x = e_a(1 + h^2) - e_x \tag{36}$$

and

$$q_2 = e_0 + e_x = e_a(1 + h^2) + e_x \tag{37}$$

3.1 Approach to equilibrium

Consider an ensemble of N AFM particles. N_1 is the number of these particles in minima 1 at energy q_1 below the barrier, and $N_2 = N - N_1$ is the number in minima 2 at energy q_2 below the barrier.

From basic statistical mechanics, we know that the thermal energy will determine the rates that the particles in minimum 1 will be leaving and the rate at which particles from minimum 2 will be entering minimum 2. The total rate of change is the difference in these two rates:

$$\frac{dN_1}{dt} = \nu_0\left(N_2 e^{-\frac{q_2}{T}} - N_1 e^{-\frac{q_1}{T}}\right) \tag{38}$$

where ν_0 is the attempt frequency.

$$\frac{dN_1}{dt} = \nu_0\left((N - N_1)e^{-\frac{q_2}{T}} - N_1 e^{-\frac{q_1}{T}}\right) \tag{39}$$

$$\frac{dN_1}{dt} = \nu_0 N e^{-\frac{q_2}{T}} - \nu_0 N_1\left(e^{-\frac{q_2}{T}} + e^{-\frac{q_1}{T}}\right) \tag{40}$$

$$\frac{1}{\nu_0}\frac{dn_1}{dt} = e^{-\frac{q_2}{T}} - n_1\left(e^{-\frac{q_2}{T}} + e^{-\frac{q_1}{T}}\right) \tag{41}$$

this is a first-order linear differential equation in the form of

$$An' + Bn = C \tag{42}$$

which has general solution

$$n = C_0 e^{-\frac{B}{A}t} + D \tag{43}$$

where $A = \frac{1}{\nu_0}$, $B = e^{-\frac{q_2}{T}} + e^{-\frac{q_1}{T}}$. We can use the boundary condition at $t = \infty$ to determine the unknown coefficients C_0 and D.

The characteristic rate $1/\tau = B/A$ in Equation 43 is

$$\nu = \nu_0 \left(e^{-\frac{q_2}{T}} + e^{-\frac{q_1}{T}} \right) \tag{44}$$

$$\nu = \nu_0 \left(e^{-\frac{e_a}{T}(1+h^2)+\frac{e_x}{T}} + e^{-\frac{e_a}{T}(1+h^2)-\frac{e_x}{T}} \right) \tag{45}$$

$$\nu = \nu_0 e^{-\frac{e_a}{T}(1+h^2)} \left(e^{\frac{e_x}{T}} + e^{-\frac{e_x}{T}} \right) \tag{46}$$

$$\nu = 2\nu_0 e^{-\frac{e_a}{T}(1+h^2)} \cosh(\frac{e_x}{T}) \tag{47}$$

Note the *exponential* dependence on temperature. If this rate ν is slower than the rate f at which we change an external field, the system will not have enough time to relax to equilibrium, and the magnetization is effectively pinned to its initial value. This only occurs below a sufficiently low temperature T_B such that $\nu = f$, called the blocking temperature.