

Chapter 5

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ELEG/PHYS667 Magnetism & Spintronics
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By postulating the molecular field, Weiss accounted for the spontaneous magnetic moment found in e.g. lodestone. However, it did not account for why ferromagnetic metals such as iron and steel are often found in a nonmagnetized state, but can be magnetized by a strong field. To explain this, Weiss postulated that on a small scale, ferromagnetic ordering always dominated, but on larger scales the system breaks up into domains. The magnetization of nearby domains often takes antiferromagnetic ordering, and the far-field is reduced. Heuristically speaking, the system will reduce its total energy by confining the field to the surface, so it likes to close its flux lines by forming domains. Of course, the process of continuous subdivision would continue indefinitely unless the domain wall formation energy cost outweighed the benefit of formation. Therefore, we should look at the formation energy of domain walls:

1 Non-Uniform Magnetization

We have an understanding of exchange with a nearest-neighbor coupling energy

$$E = -\beta\mu_1\mu_2 \cos \theta, \quad (1)$$

so we can examine the result of a non-uniform magnetization:

$$E = -\beta\mu^2 \sum_{i,j} \cos \theta_{ij} \quad (2)$$

since the exchange energy is strong, it keeps the nearest-neighbor angle θ_{ij} small. Therefore,

$$\cos \theta \approx 1 - \frac{\theta^2}{2} \quad (3)$$

$$E = C + \frac{\beta\mu^2}{2} \sum_{i,j} \theta_{ij} \quad (4)$$

Since we can always re-define zero energy, and if $\theta_{ij} = \delta\theta$ for all i and j , then the total energy is the number of pairs of moments in a unit-cell volume times the pair energy:

$$\epsilon = \frac{\beta\mu^2}{2a^3} \delta\theta^2 \quad (5)$$

This is an energy density. Since we can always write

$$\delta\theta = a \frac{d\theta}{dx} \quad (6)$$

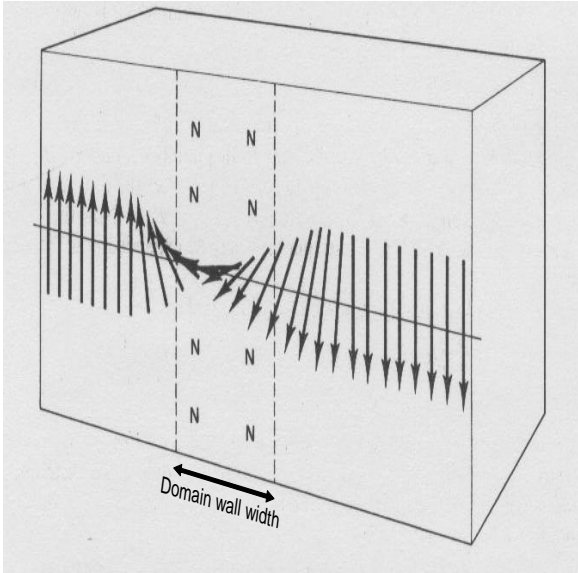
we have

$$\epsilon = \frac{\beta\mu^2}{2a} \left(\frac{d\theta}{dx} \right)^2 = A \left(\frac{d\theta}{dx} \right)^2 \quad (7)$$

where A is the “exchange constant”.

If the wall separating regions of well-defined magnetic moment (the domains) has a width w , N lattice constants (a) thick,

$$w = Na \quad (8)$$



$$\delta\theta = \frac{\pi}{N} \quad (9)$$

$$\epsilon = \frac{A}{a^2} (\delta\theta)^2 = \frac{A}{a^2} \left(\frac{\pi}{N}\right)^2 \quad (10)$$

This is the energy per wall volume. The unit energy per wall area is then

$$\epsilon \cdot w = \gamma = \frac{A\pi^2}{aN} \quad (11)$$

1.1 Anisotropy Energy

This exchange energy competes with an *anisotropy energy* that tries to maintain preferred directions of magnetization in the ferromagnet. There are many sources of anisotropy, among them:

1. magnetocrystalline anisotropy
2. shape anisotropy
3. magnetostriction

The first, *magnetocrystalline anisotropy*, is due to the electrostatic crystal field which aligns the electron orbitals along the crystal axes. Since the electron spin is coupled to the orbital angular momentum, a higher field is required to overcome the effect of the crystal symmetry.

The second, *shape anisotropy*, is due to the reduction in field energy by single domains with magnetization along the longest length of geometry. Heuristically this can be explained by the reduction of free poles on the surface and the corresponding reduction in demagnetization field within the sample.

In reality, the domain magnetizations never form cyclical patterns because of the third source of anisotropy *magnetostriction*, the coupling of magnetization and lattice spacing in the crystal. Typically, the lattice spacing is either elongated or reduced along the magnetization axis, and this causes mechanical stress within the material, resulting in anisotropy. One notable exception to this rule is Permalloy, an alloy of Ni and Fe, where a suitable ratio (about 80% Ni and 20% Fe) of the two elements creates a material with zero magnetostriction. Another significant material made from nickel and iron is called Invar, where the lattice thermal expansion and the temperature-dependent magnetostriction cancel each other. The elemental ratios are 64% Fe and 36% Ni. This discovery, of importance in establishing the standard length scale of the day, won Charles Edouard Guillaume the Nobel Prize in 1920.

2 Domain wall thickness

Since the anisotropy energy which determines the axis of alignment or anti-alignment of domain magnetizations is proportional to the wall thickness, $E_{anisotropy} = Kw = KNa$, we can minimize the total energy (Anisotropy + Exchange) with respect to the number of lattice spacings in the wall:

$$\frac{d}{dN} \left(KNa + \frac{A\pi^2}{Na} \right) = 0 \quad (12)$$

$$Ka - \frac{A\pi^2}{N^2a} = 0 \quad (13)$$

$$N = \sqrt{\frac{A\pi^2}{Ka^2}} \quad (14)$$

Since $w = Na$, the width of the wall is

$$w = Na = \sqrt{\frac{A\pi^2}{K}} \quad (15)$$

A typical value of the exchange constant A is $\approx 10^6 eV/cm$ and the anisotropy constant K is $\approx 10^{18} eV/cm^3$, giving typical domain wall thicknesses of

$$w = Na = \sqrt{\frac{10^6 \pi^2}{10^{18}}} \approx 10^{-6} cm = 10nm \quad (16)$$

If the anisotropy is large, the wall will be thin so that most moments are aligned parallel or antiparallel to the anisotropy axis. If the anisotropy is comparatively small, the wall will be wide to minimize the exchange energy.

3 Domain Size

We can therefore see how the process of subdivision to reduce surface poles is counteracted by the energy necessary for domain formation. Imagine a ferromagnet with total volume V . If there are N domains, then each domain will have volume V/N and surface area $\propto \left(\frac{V}{N}\right)^{2/3}$. The total domain wall area is then

$$\propto N \cdot \left(\frac{V}{N}\right)^{2/3} \quad (17)$$

and the total energy is

$$E_{total} = E_{Demag} + E_{DomainWall} = C_{Demag} \frac{V}{N} + C_{walls} N \cdot \left(\frac{V}{N}\right)^{2/3} \quad (18)$$

This energy is of the form

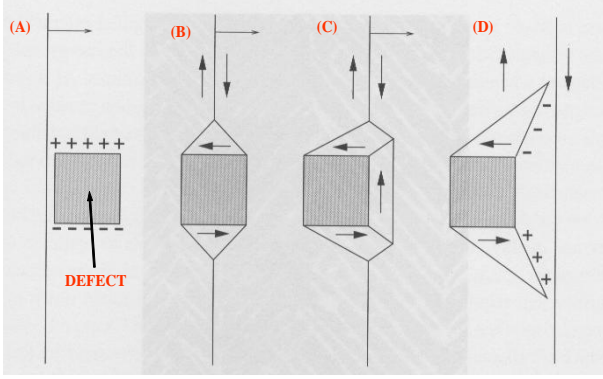
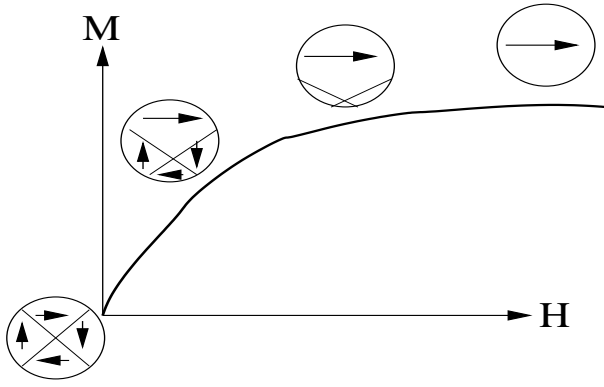
$$E_{total} = AN^\alpha + BN^{-\beta} \quad (19)$$

which always has a minimum for some finite, nonzero N .

4 Magnetization Mechanics

The formation of domains explains the lack of substantial spontaneous macroscopic magnetic moment in ferromagnetic materials without an external field. But what happens when an external field is applied and the ferromagnet is magnetized?

Two things may occur: The first is domain wall motion, where the size of domains changes in response to the field, and the other is coherent rotation.



4.1 Domain wall motion

Refer to Figure 3. As the external field increases, the domain with moment aligned parallel to the field direction increases its size while the domain antiparallel to the field decreases in size, until the particle is saturated and the domain wall is pushed to the boundary.

In reality, things are not so simple, because there are always defects in the crystal which pin the domain walls. When the field increases enough to pull the domains away from the trapping defects, a large, virtually instantaneous, change in the moment occurs. This effect can be detected by a pickup coil wrapped around a ferromagnetic sample in a slowly varying field, and is called Barkhausen noise, which was the first evidence of domain formation before domains were microscopically imaged by Bitter in the 1930s.

4.2 Coherent rotation

The other means of magnetizing a sample is coherent magnetization rotation in a single domain. We consider our ferromagnetic material an ensemble of oblate spheroid domains, a volume of revolution created by rotating an ellipse around one of its primary axes. Since the demagnetization field will be higher when the magnetization is along the minor axis, this system has a shape anisotropy keeping the magnetization along the major axis.

The energy of an arbitrary magnetization state is then determined by the degree of demagnetization induced by the orientation:

$$E = \frac{1}{2}M^2 (D_a \cos^2 \psi + D_b \sin^2 \psi) - HM \cos \phi \quad (20)$$

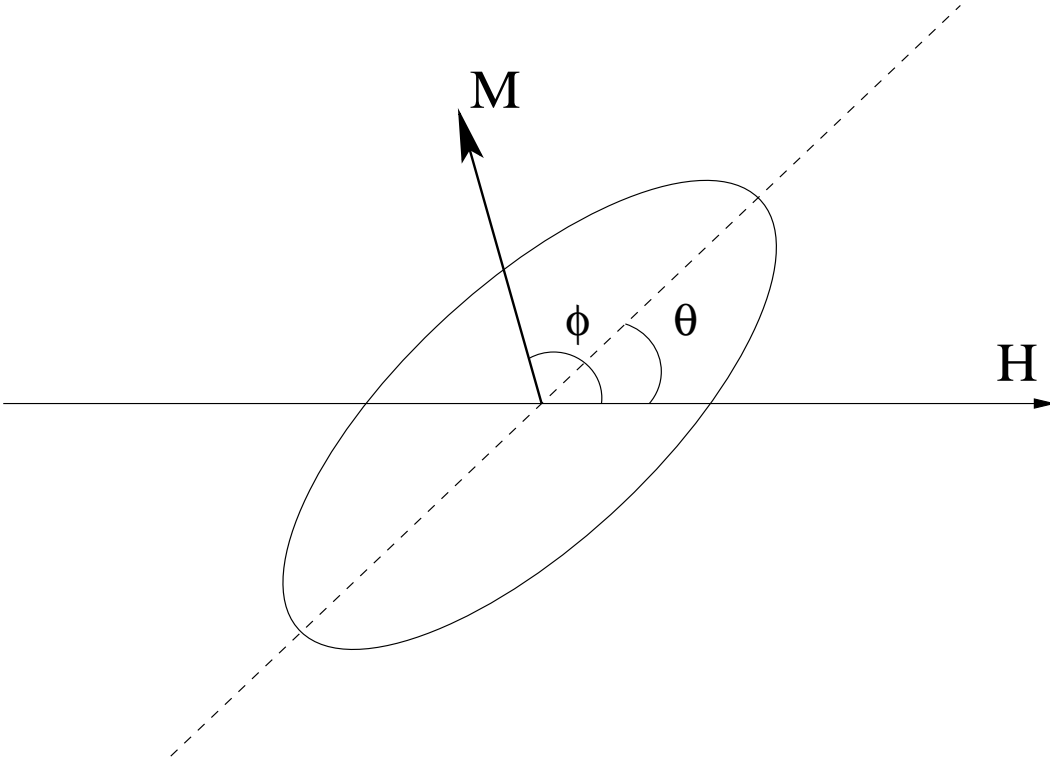
where $\psi = \phi - \theta$. The first two terms are the demagnetization energies along both axis, and the third term is the “Zeeman” energy due to the moment interacting with the field.

We can re-write this expression (using $\cos^2(x) = 1/2(1 + \cos(2x))$ and $\sin^2(x) = 1/2(1 - \cos(2x))$) as

$$E = \frac{1}{4}M^2(D_a + D_b) - \frac{1}{4}M^2(D_b - D_a) \cos 2\psi - HM \cos \phi \quad (21)$$

Now we can normalize to get a dimensionless energy

$$\eta = \frac{E}{(D_b - D_a)M^2} = \frac{D_a + D_b}{4(D_b - D_a)} - \frac{1}{4} \cos 2\psi - \frac{H}{(D_b - D_a)M} \cos \phi \quad (22)$$



We can drop the first term since it is a constant:

$$\eta = -\frac{1}{4} \cos 2\psi - \frac{H}{(D_b - D_a)M} \cos \phi = -\frac{\cos 2\psi}{4} - b \cos \phi \quad (23)$$

where $b = \frac{H}{(D_b - D_a)M}$, the “reduced field”. The orientation of the magnetization will always correspond to the minimum of this expression for b and θ fixed, i.e.

$$\frac{d\eta}{d\phi} = \frac{1}{2} \sin (2(\phi - \theta)) + b \sin \phi = 0 \quad (24)$$

For example, when $\theta = 0$, the spheroid’s major axis is aligned with the field. We expect that at a sufficiently high reduced field, the magnetization will spontaneously switch direction to align itself with the external field. At lower field values, the magnetization orientation corresponding to minimum energy will be two valued since the system could be parallel or antiparallel to the field at the same energy.

The minimization condition becomes

$$\frac{d\eta}{d\phi} = \frac{1}{2} \sin 2\phi + b \sin \phi = 0 \quad (25)$$

This equation has two solutions: $\phi = 0$ and $\phi = \pi$, independent of b . This corresponds to a magnetization along \mathbf{H} of $\cos(\phi) = \pm 1$.

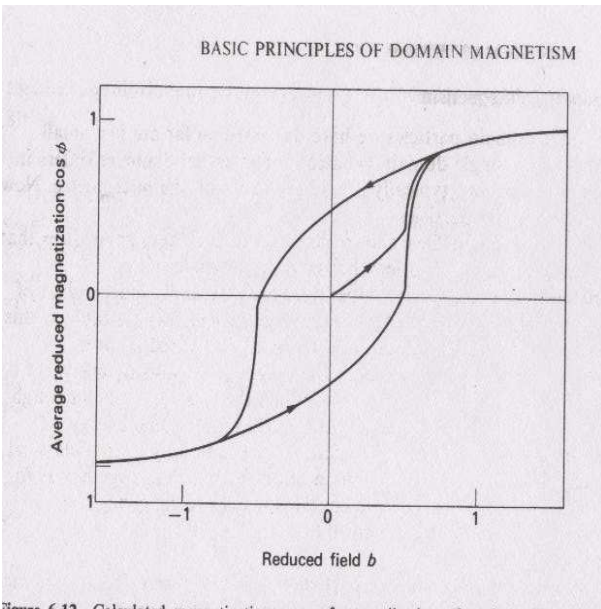
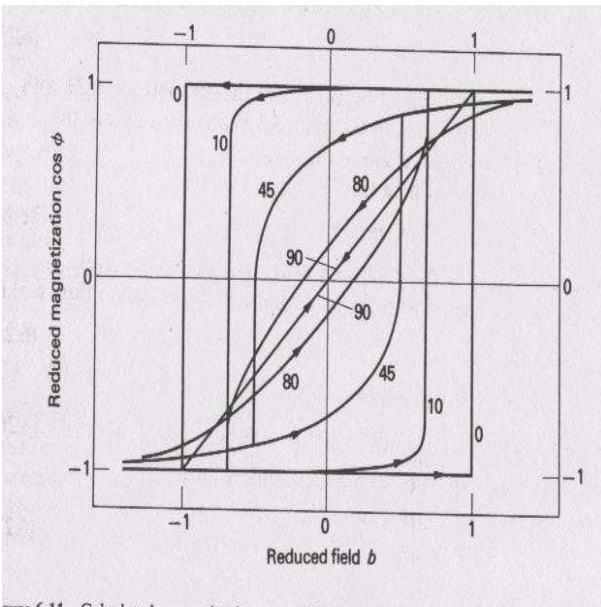
As another example, consider when $\theta = \pi/4$. In this case, the coherent rotation does not occur spontaneously. The transition to zero moment occurs at $b = \pm \frac{1}{2}$. This can be seen by solving the minimization equation

$$\frac{d\eta}{d\phi} = -\frac{1}{2} \sin (2(\phi - \frac{\pi}{4})) - \frac{1}{2} \sin \phi = 0 \quad (26)$$

$$\sin (2(\phi - \frac{\pi}{4})) = \sin \phi \quad (27)$$

$$2(\phi - \frac{\pi}{4}) = \phi \quad (28)$$

$$\phi = \frac{\pi}{2} \quad (29)$$



since $\cos \frac{\pi}{2} = 0$, the projection of the magnetization along the field axis is zero and the magnetization vanishes. For $\theta = \pi/2$, the field is aligned with the minor axis. In this case,

$$1/2 \sin (2(\phi - \frac{\pi}{2})) = -b \sin \phi \tag{30}$$

$$1/2 \sin (2\phi - \pi) = -b \sin \phi \tag{31}$$

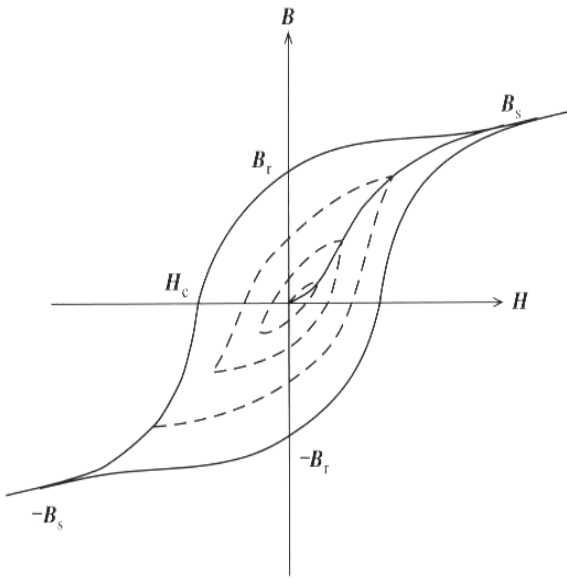
$$-1/2 \sin 2\phi = -b \sin \phi \tag{32}$$

$$\sin \phi \cos \phi = b \sin \phi \tag{33}$$

$$\cos \phi = b \tag{34}$$

In other words, the magnetization along H is proportional to H; it forms a straight line.

Figure 4.2 shows the hysteresis curves for various values of θ , and Figure 4.2 shows the distribution-averaged plot.



The value of magnetization approached asymptotically as the field reaches higher values is called the *saturation magnetization*, the value at zero field is called the *remnant magnetization*, and the point where this hysteresis curve crosses the field axis is called the *coercive field*.

The average behavior of our distribution points out how one can demagnetize a ferromagnetic sample with many domains. By cycling a magnetic field with progressively smaller and smaller magnitude, the domains with coercive (switching fields) that have not been attained are effectively randomized. When the magnetic field vanishes, all the domains are randomized and the spontaneous macroscopic magnetization is eliminated.