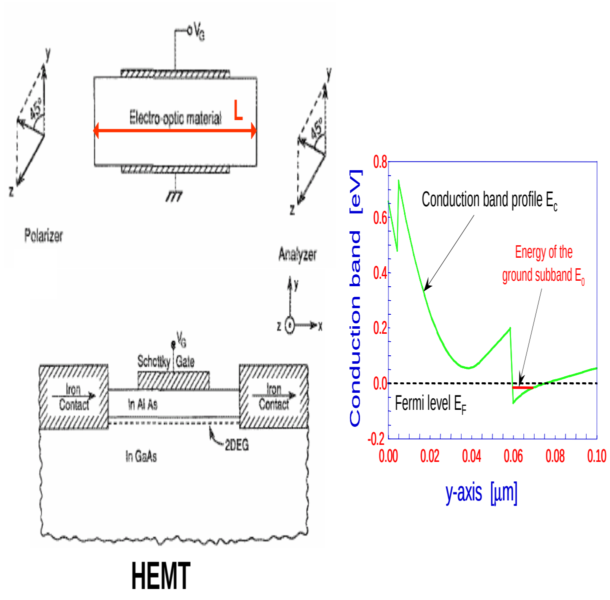
Chapter 12

University of Delaware
Electrical and Computer Engineering Department
ELEG/PHYS667 Magnetism & Spintronics
Prof. Ian Appelbaum

1 Datta and Das Spin FET

The 1990 paper by Datta and Das “Electronic Anologue of Electro-Optic Modulator” (APL **56** 665) catalyzed interest in the use of electron spin in solid-state semiconductor devices. First let’s learn how the electro optic modulator works:

1.1 electro-optic modulator



Consider a beam of photons incident on one side of our birefringent device. It is polarized with an initial polarization state at 45 degrees to the optical axis. We can write this initial state as

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{1}$$

which simply says that the polarization is a coherent superposition of equal parts vertical and horizontal polarization.

In the birefringent electro-optic material, different polarizations have different indices of refraction, so wavevectors k_1 (for the x polarization) and k_2 (for the y polarization) are different. Therefore, each component accumulates a different phase by passing through the material. The final state after a path of length L is then

$$\Psi_f = e^{ik_1L} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{ik_2L} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2}$$

Then, we analyze the polarization in the initial 45 degree axis. This means we must project this state onto the 45^0 basis and take the square norm:

$$| \left[\begin{array}{cc} 1 & 1 \end{array} \right] \cdot \left[\begin{array}{c} e^{ik_1 L} \\ e^{ik_2 L} \end{array} \right] |^2 = \left(e^{-ik_1 L} + e^{-ik_2 L}\right)\left(e^{ik_1 L} + e^{ik_2 L}\right) = 2 + e^{i(k_1 - k_2)L} + e^{-i(k_1-k_2)L} = 4\cos^2 \frac{(k_1 - k_2)L}{2} \quad (3)$$

Since $(k_1 - k_2)$ is controlled by an electrostatic gate to the electro-optic material, we can turn the modulator on and off.

1.2 Electron Analogue

When electron spin replaces electric field polarization, we must have a physical mechanism to impose different phases for spin up and spin down. Clearly, a magnetic field would do this because spin up and spin down have energies $\pm\mu B$, resulting in phases over time t of

$$\pm \frac{\mu B}{\hbar} t \quad (4)$$

However, we want to control our device electrostatically. Fortunately, the spin-orbit interaction can be used to transform an electric field into an effective magnetic field in the electron's rest frame.

1.3 spin-orbit interaction

There are three types of spin-orbit effects in semiconductors, with causes:

1. Rashba Effect: Bychkov and Rashba, JETP Lett., 56 665 (1984): electric fields caused by charge accumulation at hetero-interfaces
2. Dresselhaus Effect: Dresselhaus Phys. Rev. 100 580 (1955): Bulk inversion asymmetry like in zincblende materials such as GaAs
3. Native interface asymmetry: Occurs at heterointerfaces with different cations like at the GaSb/InAs interface

1.4 The device

Datta and Das proposed that a low-power, nonvolatile field-effect transistor could be made by analogy with the electro-optic modulator by using the gate to tune the Rashba spin-orbit effect of spin-polarized carriers injected and detected by ferromagnetic contacts into a 2-dimensional electron gas (2DEG) formed at the buried interface between two planar semiconductors. This is similar to a High Electron Mobility Field Effect Transistor (HEMT). The FM contacts are like the polarizer and analyzer in the electro-optic modulator.

The form of the Rashba spin-orbit Hamiltonian is (from an expansion of the Dirac Equation):

$$H_{SO} = \frac{\hbar^2}{2m^2c^2r}\frac{dV}{dr} \cdot (l \cdot s) \quad (5)$$

Since $l = r \times p$,

$$l \cdot s = r \times p \cdot s = r \cdot (p \times s). \quad (6)$$

We can evaluate

$$p \times s = \hbar \left| \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ k_x & k_y & k_z \\ \sigma_x & \sigma_y & \sigma_z \end{array} \right| \quad (7)$$

Since $\frac{dV}{dr} = \vec{\mathcal{E}}$ is in the $-\hat{y}$ direction, only one non-zero term in the above expression survives, resulting in

$$H_{SO} = \frac{\hbar^2}{2m^2c^2r} r\mathcal{E}_y [k_x\sigma_z - k_z\sigma_x] = \eta[k_x\sigma_z - k_z\sigma_x] \quad (8)$$

where η is only dependent on the magnitude of the field perpendicular to the 2DEG. Since

$$\sigma_z = \left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right] \quad (9)$$

and

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{10}$$

our full Hamiltonian looks like

$$H = H_{KE} + H_{SO} = \begin{bmatrix} -\frac{\hbar^2}{2m^*}(k_x^2 + k_z^2) + \eta k_x & -\eta k_z \\ -\eta k_z & -\frac{\hbar^2}{2m^*}(k_x^2 + k_z^2) - \eta k_x \end{bmatrix} \tag{11}$$

If we impose $\mathbf{k_z = 0}$ (Datta and Das propose using a lateral potential to confine the electrons to a waveguide – essentially forming a 1DEG), the eigenstates are

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{12}$$

with unequal energies $E_0 + \eta k_x$ and $E_0 - \eta k_x$. This is like an effective magnetic field forcing precession frequency

$$\omega_\pm = \pm \frac{\eta k_x}{\hbar} \tag{13}$$

Therefore, the electron will accumulate a phase over its path along $\hat{x}$

$$\phi_\pm = \omega_\pm t = \omega_\pm \left(\frac{L}{\frac{\hbar k_x}{m^*}} \right) = \pm \frac{\eta m^* L}{\hbar^2} \tag{14}$$

and the difference between the phases accumulated for spin up and down, ϕ_+ and ϕ_- is

$$\Delta\phi = \frac{2\eta m^* L}{\hbar^2} \tag{15}$$

Note that this is independent of k_x, so the wide distribution of momenta does not wash out the effect. Since we can engineer a phase difference, we can modulate the current through the channel from source to drain with our electrostatic gate, much like we can modulate the amount of light through the electrooptic modulator.

2 Spin injection from FM into Semiconductors

Many groups tried to perform the experiment proposed in Datta and Das' paper, but no convincing evidence of spin injection and detection (let alone current modulation by Rashba spin-orbit interaction) was found for many years. The reason for this was first explained by Schmidt in 2000 (PRB **62** 4790), ten years of fruitless labor after Datta and Das' pioneering paper.

2.1 "Fundamental Obstacle to Electrical Spin Injection..."

Schmidt solved the spin-dependent transport equations defined from the bulk spin polarization

$$\beta = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} \tag{16}$$

where the σs are spin-dependent conductivities, the current spin polarization

$$\alpha = \frac{j^\uparrow - j^\downarrow}{j^\uparrow + j^\downarrow} \tag{17}$$

where the js are spin-dependent particle flux densities, the spin dependent "Ohm's law"

$$\bigtriangledown \mu^{\uparrow,\downarrow} = \frac{e j^{\uparrow,\downarrow}}{\sigma^{\uparrow,\downarrow}} \tag{18}$$

where the μs are the spin-dependent electrochemical potentials, spin-dependent charge continuity

$$\bigtriangledown \cdot j^{\uparrow,\downarrow} = 0 \tag{19}$$

and the diffusion equation

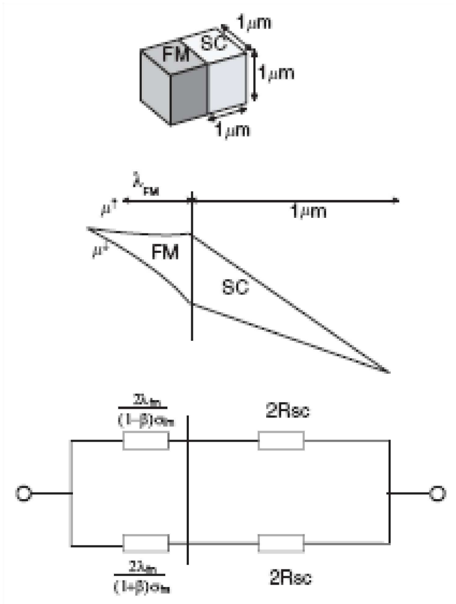


Figure 1: Schmidt et al, J. Phys. D: Appl. Phys. **38** R107 (2005)

$$\frac{(\mu^{\uparrow} - \mu \downarrow)}{\tau_{sf}} = D \bigtriangledown^2 (\mu^{\uparrow} - \mu \downarrow) \tag{20}$$

where we use the relaxation-time approximation for the time partial derivative, with spin-flip time τ_{sf}. In one dimension, this equation is a second order linear ordinary differential equation, with solutions

$$(\mu^{\uparrow} - \mu \downarrow) = C_0 e^{\pm \frac{x}{\lambda}} \tag{21}$$

where $\lambda = \sqrt{D\tau_{sf}}$, the spin-flip diffusion length determines the length scale over which splittings in electrochemical potential decay. The other equations (for instance Ohm's law), permit solutions for $\mu^{\uparrow}$ and $\mu^{\downarrow}$ with linear and constant components. Boundary conditions at interfaces between dissimilar materials require continuous μs and solutions typically look like the figure, with boundary conditions at infinity imposed so that

$$(\mu^{\uparrow} - \mu \downarrow) \mid_{\pm\infty} = 0 \tag{22}$$

2.2 Example

In a later paper (J. Phys. D **38** R107), Schmidt gives the following example to illustrate the "fundamental obstacle":

Consider ferromagnetic metal (FM) in contact with a semiconductor of dimensions $1\mu m \times 1\mu m \times 1\mu m$. The resistivity of the semiconductor is $1m\Omega \cdot cm$ and of the metal $\rho_F M = 1\mu\Omega \cdot cm$. Assume a bulk spin polarization in the FM of $\beta = 40\%$ and spin flip length $\lambda = 10$nm.

Now, consider an electron current of $10\mu A$ flowing from the FM into the semiconductor. If we want this current to have a modest current spin polarization of 10%, the boundary condition at the interface must be consistent on both sides – the electrochemical splitting is the same when approacing the interface from the semiconductor or the FM.

2.2.1 semiconductor side

Ohm's law tells us that in the semiconductor,

$$V = IR = I\frac{\rho L}{Area} = 10^{-5}A \cdot \frac{10^{-3}\Omega \cdot cm \times 10^{-4}cm}{10^{-4}cm \times 10^{-4}cm} = 10^{-5}A \cdot 10\Omega = 10^{-4}V. \tag{23}$$

Since the two spin channels have the same conductivity, in a parallel configuration, the voltage drops across two equal resistances of 20Ω each. Because we want a current spin polarization of 10%, we have

$$\alpha = \frac{j^{\uparrow} - j^{\downarrow}}{j^{\uparrow} + j^{\downarrow}} = 0.10 \tag{24}$$

and

$$j^{\uparrow} + j^{\downarrow} = 10\mu A \tag{25}$$

giving a current difference between the spin channels of $1\mu A$. Since each channel sees 20Ω, the elctrochemical splitting at the interface must be $IR = 1\mu A \cdot 20\Omega = 20\mu V$.

2.2.2 FM side

Any electrochemical splitting at the interface must decay to zero over the spin flip length $\lambda = 10nm$. If we calculate the effective resistance for each spin channel,

$$R_{FM}^{\uparrow,\downarrow} = \frac{L}{\sigma^{\uparrow,\downarrow} \cdot Area} \tag{26}$$

we can determine what current is needed to drive a necessary splitting at the interface. In other words, if we determine from the semiconductor analysis that we need $20\mu V$ splitting at the interface, we can calculate how much current must flow through the FM to establish this splitting on the other side.

From the bulk spin polarization and the total conductivity, we can determine the spin-dependent conductivity

$$\sigma^{\uparrow,\downarrow} = \frac{2\sigma^{\uparrow,\downarrow}}{2(\sigma^{\uparrow} + \sigma^{\downarrow})}(\sigma^{\uparrow} + \sigma^{\downarrow}) \tag{27}$$

$$= \frac{(\sigma^{\uparrow} + \sigma^{\downarrow}) \pm (\sigma^{\uparrow} - \sigma^{\downarrow})}{2(\sigma^{\uparrow} + \sigma^{\downarrow})}(\sigma^{\uparrow} + \sigma^{\downarrow}) \tag{28}$$

$$= \frac{(\sigma^{\uparrow} + \sigma^{\downarrow})}{2}(1 \pm \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}) \tag{29}$$

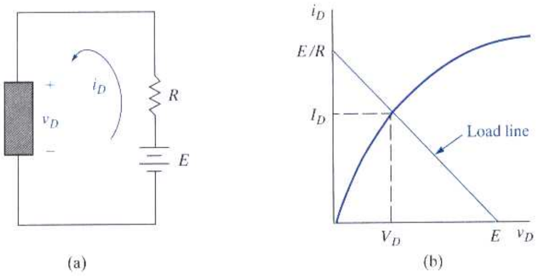
$$= \frac{\sigma_{FM}}{2}(1 \pm \beta) \tag{30}$$

where σ_{FM} is the total conductivity of the FM ($\sigma_{FM} = \frac{1}{\rho_{FM}}$). This results in

$$R_{FM}^{\uparrow,\downarrow} = \frac{\lambda}{\frac{\sigma_{FM}}{2}(1 \pm \beta) \cdot (1\mu m \times 1\mu m)} \tag{31}$$

and substituting appropriate values gives $333\mu\Omega$ for spin up and $143\mu\Omega$ for spin down, and approximate spin-dependent difference of about $200\mu\Omega$. Since we need $20\mu V$ splitting at the interface, we should drive $I = V/R \approx 20\mu V\ 200\mu\Omega = 100mA$. But that's clearly inconsistent with our initially assumed condition of $10\mu A$ current flowing in the semiconductor! Clearly, the problem is an impedance mismatch between the highly resistive semiconductor and the highly conductive FM.

2.3 Load line



(a) (b)

Schmidt also suggested a simple means of evaluating the effectiveness of spin injection by making use of an engineering technique called the *load-line*. This is a graphical means of evaluating the current and voltage dropped across a nonlinear device when in series with a voltage source and a load resistor.

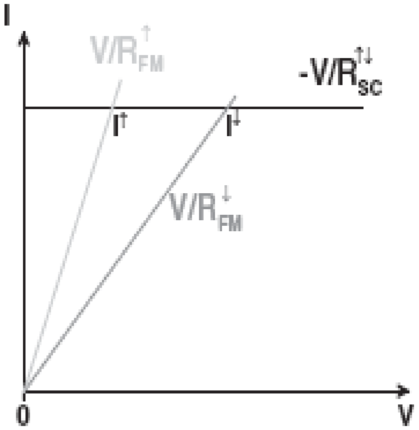
The loop equation tells us that the voltage drop across the device and the resistor must equal the applied voltage:

$$V_{applied} = IR + V_{device} \tag{32}$$

Since we have a series circuit, the current through the device and the resistor are the same. We can write

$$I_{device} = -V_{device}\frac{1}{R} + \frac{V_{applied}}{R} \tag{33}$$

Which defines the load-line. In our case, the "device" is the I-V curve for the spin-dependent resistances in the FM, and the load is the spin-independent semiconductor resistance. Since the resistance of the load is so much higher than the spin-up and spin-down FM "devices", the difference in currents carried by each spin is negligible and the current is essentially unpolarized.



2.3.1 Interfacial Resistance

The solution to this problem of course is to increase the interfacial resistance. This lowers the slope of the device I-V and allows a greater asymmetry in the spin-dependent currents. Since the resistance in a tunnel-junction contact can be arbitrarily large, that is obviously a candidate for an effective method of improvement. Also, the resistance can be thought of as proportional to the density of states. Since the DOS in the FM is asymmetric, there is a corresponding asymmetry in the spin-dependent resistance. This method has been used with much success experimentally in recent years.