Problem Set 4 - Magnetism & Spintronics ELEG/PHYS667

University of Delaware
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Due Wednesday, May 3 2006, in class

1. We can evaluate the tunnel current by integrating strictly in k-space, rather than transforming to energy as Simmons did. The following questions assume a zero-temperature free-electron model for the metals, and a planar device geometry.

   (a) If the momentum vector of a random electron in the emitter metal is (k_x, k_y, k_z), and the junction bias is V_{bias}, what is the momentum vector in the base metal after tunneling through the vacuum barrier?

   (b) Using the above result and considering the fact that electrons in the emitter cannot tunnel into already occupied states in the base, determine the fraction of electrons in the emitter which contribute to the tunneling process at voltage $eV_{bias} < E_F$, where E_F is the emitter metal Fermi energy. What geometrical section of the original Fermi sphere do these electrons come from in momentum space? What fraction contribute at $eV_{bias} \geq E_F$?

   (c) Assume a barrier width, d, and a metal work function, $e\phi$. Determine the tunneling probability for an electron with perpendicular momentum $k_\perp$ at voltage V in the WKB approximation.

   (d) The tunnel current density is an integral of the tunneling transmission coefficient determined above, times the electron flux ($electrons/(Area \cdot time)$). What is the expression, in terms of $k_\perp$, for this flux?

   (e) Using this result, write down the integral expression for the tunnel current density in this model. Explicitly include bounds of integration, but do not attempt to carry out the integration. How does this integral change if the model is extended to include thermal effects ($T > 0$)?

   (f) As the voltage varies when V_{bias} is very small, the values of flux and tunnel probability do not vary much over the integration interval. Use the result of question 2 to determine how the tunnel current density varies with the voltage at small V_{bias}.

   (g) Qualitatively, what happens to the dependence of tunnel current density on voltage at high bias ($eV_{bias} > E_F$)? Why?

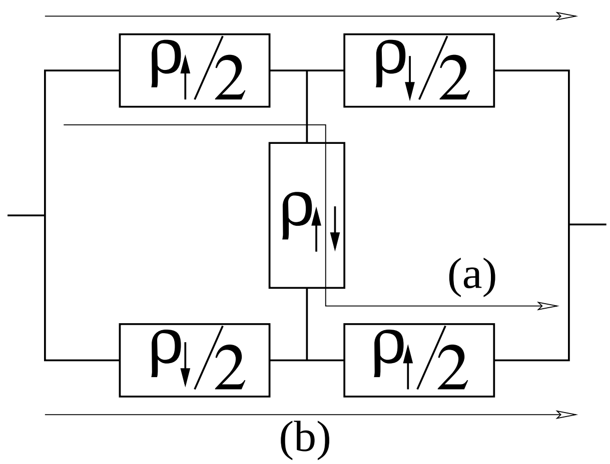
   (h) Sketch the characteristic behavior of I vs. V of a tunnel junction, based on the above.

   (i) Analytically evaluating the expression for the tunnel current (question 5) is difficult. However, numerical integration can be used. Using a software package such as MATLAB, plot the I-V curve for $E_F = e\phi = 5eV$ and $d = 1nm$ from $-5V < V_{bias} < 5V$.

2. The Fert-Campbell equation

$$\rho = \frac{\rho_\uparrow \rho_\downarrow + \rho_{\downarrow\uparrow}(\rho_\uparrow + \rho_\downarrow)}{\rho_\uparrow + \rho_\downarrow + 4\rho_{\uparrow\downarrow}} \tag{1}$$

   was derived from the Boltzmann transport theory for two coupled spin currents. It describes effect of spin-dependent resistivity and spin-flip scattering on total resistance in ferromagnetic metals. Derive this equation from the equivalent circuit:


ρ↑/2
ρ↓/2
ρ↑↓
(a)
ρ↓/2
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(b)