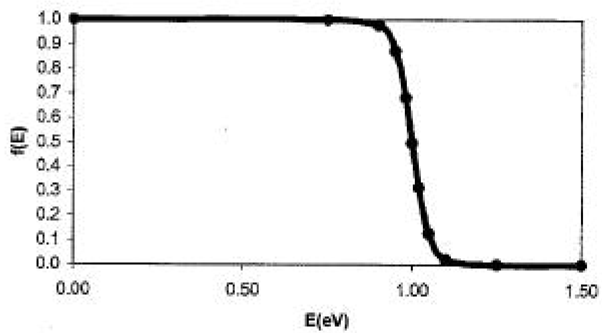
Prob. 3.2

Plot Fermi function for E_F=1eV and show the probability of an occupied state ΔE above E_F is equal to the probability of an empty state ΔE below E_F so $f(E_F+\Delta E)=1-f(E_F-\Delta E)$.

use $f(E)=\frac{1}{1+e^{\frac{E-E_F}{k_BT}}}$ and kT=0.0259eV

E(eV)	(E-EF)/kT	f(E)
0.75	-9.6525	0.99994
0.90	-3.8610	0.97939
0.95	-1.9305	0.87330
0.98	-0.7722	0.68399
1.00	0.0000	0.50000
1.02	0.7722	0.31600
1.05	1.9305	0.12669
1.10	3.8610	0.02061
1.25	9.6525	0.00006



occupation probability above $E_F = f(E_F+\Delta E)=\frac{1}{1+e^{\frac{\Delta E}{kT}}}$

empty probability below $E_F = 1 - f(E_F-\Delta E) = 1-\frac{1}{1+e^{\frac{-\Delta E}{kT}}}$

$$1-f(E_F-\Delta E)=1-\frac{1}{1+e^{\frac{-\Delta E}{kT}}}=\frac{e^{\frac{-\Delta E}{kT}}}{1+e^{\frac{-\Delta E}{kT}}}=\frac{1}{e^{\frac{\Delta E}{kT}}+1}=\frac{1}{1+e^{\frac{\Delta E}{kT}}}=f(E_F+\Delta E)$$

This shows that the probability of an occupied state ΔE above E_F is equal to the probability of an empty state ΔE below E_F.

Prob. 3.3

Calculate electron, hole, and intrinsic carrier concentrations.

$E_g=1.1eV \quad N_C=N_V \quad n=10^{15}\frac{1}{cm^3} \quad E_C\text{-}E_d=0.2eV \quad E_C\text{-}E_F=0.25eV \quad T=300K$

$n=10^{15}\frac{1}{cm^3}$

$$n = N_C \cdot e^{-\frac{E_C-E_F}{kT}} \rightarrow N_C = n \cdot e^{\frac{E_C-E_F}{kT}} = 10^{15}\frac{1}{cm^3} \cdot e^{\frac{0.25eV}{0.0259eV}} = 1.56 \cdot 10^{19}\frac{1}{cm^3}$$

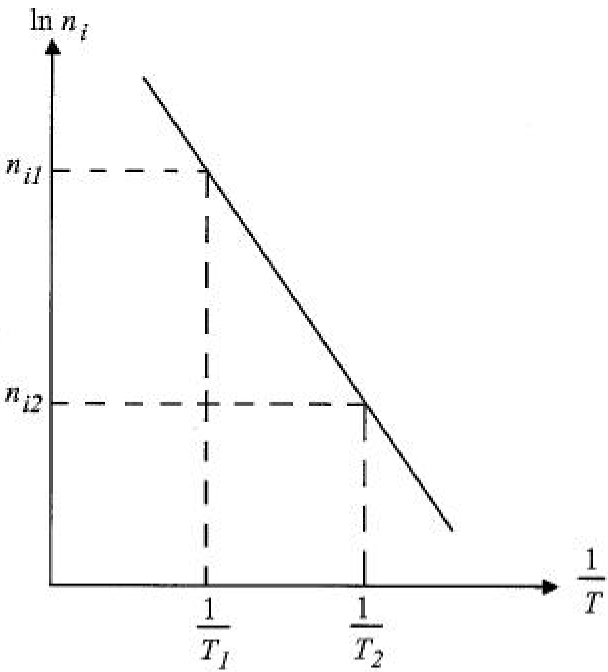
$$N_V = 1.56 \cdot 10^{19}\frac{1}{cm^3}$$

$$p = N_V \cdot e^{-\frac{E_F-E_V}{kT}} = 1.56 \cdot 10^{19}\frac{1}{cm^3} \cdot e^{-\frac{0.85eV}{0.0259eV}} = 8.71 \cdot 10^{4}\frac{1}{cm^3}$$

$$n_i = \sqrt{n \cdot p} = 9.35 \cdot 10^{9}\frac{1}{cm^3} \left(\text{note: } n_i = \sqrt{N_C \cdot N_V} \cdot e^{-\frac{Eg}{2kT}} \text{ may also be used}\right)$$

Prob. 3.6

Find Eg for Si from Figure 3-17.



for n_{i1} and n_{i2} on graph

$$n_{i1} = 3 \cdot 10^{14} \qquad \frac{1}{T_1} = 2 \cdot 10^{-3}\,\frac{1}{K}$$

$$n_{i2} = 10^{8} \qquad \frac{1}{T_2} = 4 \cdot 10^{-3}\,\frac{1}{K}$$

This result is approximate because the temperature dependences of N_C, N_V, and E_g are neglected.

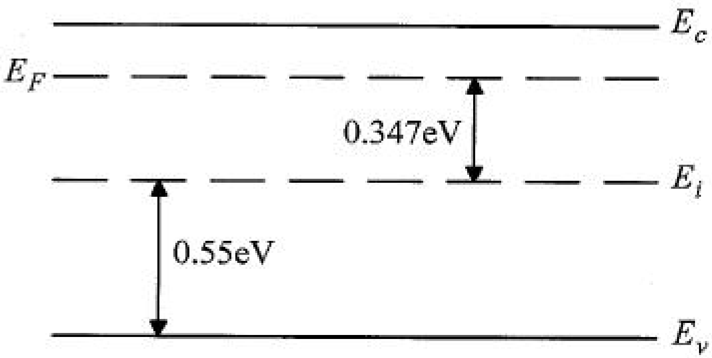
$$n_i = \sqrt{N_C N_V} \cdot e^{-\frac{E_g}{2kT}} \rightarrow E_g = -2kT \cdot \ln \frac{n_i}{\sqrt{N_C N_V}} \rightarrow \ln n_i = -\frac{E_g}{2kT} + \ln\sqrt{N_C N_V}$$

$$\ln \frac{n_{i1}}{n_{i2}} = \ln n_{i1} - \ln n_{i2} = \left(-\frac{E_g}{2kT_1} + \ln\sqrt{N_C N_V}\right) - \left(-\frac{E_g}{2kT_2} + \ln\sqrt{N_C N_V}\right) = \frac{E_g}{2k} \cdot \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

for Si (see above) $\rightarrow$ $E_g = 2k \cdot \left(\dfrac{\ln \frac{n_{i1}}{n_{i2}}}{\frac{1}{T_2} - \frac{1}{T_1}}\right) = 2 \cdot 8.62 \cdot 10^{14} \cdot \left(\dfrac{\ln \frac{3 \cdot 10^{14}}{10^{8}}}{4 \cdot 10^{-3}\,\frac{1}{K} - 2 \cdot 10^{-3}\,\frac{1}{K}}\right) = 1.3\text{eV}$

Prob. 3.8

Show that Equation 3-25 results from Equation 3-15 and Equation 3-19. Find the position of the Fermi level relative to E_i at 300K for $n_o=10^{16}\ cm^{-3}$.



Equation 3-15 → $n_0=N_C\cdot e^{-\frac{(E_C-E_F)}{kT}}$

$$n_0=N_C\cdot e^{-\frac{E_C-E_F}{kT}}=N_C\cdot e^{-\frac{E_C-E_i}{kT}}\cdot e^{\frac{E_F-E_i}{kT}}=n_i\cdot e^{\frac{E_F-E_i}{kT}}$$ using 3-21 yields Equation 3-25a

Equation 3-19 → $p_0=N_V\cdot e^{-\frac{E_F-E_V}{kT}}$

$$p_0=N_V\cdot e^{-\frac{E_F-E_V}{kT}}=N_V\cdot e^{-\frac{E_i-E_V}{kT}}\cdot e^{\frac{E_i-E_F}{kT}}=n_i\cdot e^{\frac{E_i-E_F}{kT}}$$ using 3-21 yields Equation 3-25b

for Fermi level relative to E_i at 300K for $n_o=10^{16}\ cm^{-3}$

$$E_F-E_i=0.0259eV\cdot\ln\frac{1.5\cdot10^{10}}{10^{16}}=0.347eV$$