

Last Time:

- Carrier concentration
- Product of DOS and Probability of occupation
- DOS 3-D $\propto E^{1/2}$

- Occupation Probability
conservation of Energy + Equilibrium
 \Rightarrow Boltzmann distribution
+ Pauli Exclusion Principle \Rightarrow Fermi-Dirac

This Time:

properties of F-D
carrier concentration, E_F 'Fermi Energy'
doping

Fermi-Dirac Distribution:

$$\frac{1}{1 + e^{(E-E_F)/kT}}$$

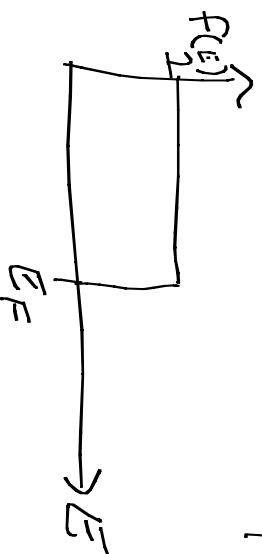
$$T \equiv 0$$

$$E - E_F > 0$$

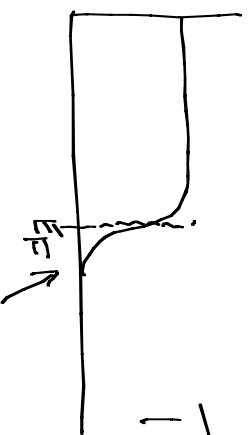
$$\frac{1}{1 + e^{+\infty}} \sim \frac{1}{1 + e^{\infty}} \Rightarrow 0$$

$$E - E_F < 0$$

$$\frac{1}{1 + e^{-\infty}} \Rightarrow 1$$



$$T \neq 0$$



$$\frac{1}{1 + e^{0/kT}} = \frac{1}{1 + 1} = \frac{1}{2}$$

$$\frac{(E - E_F) \gg kT}{kT}$$

$$\frac{1}{1 + e^{(E-E_F)/kT}}$$

$$\sim e^{-(E-E_F)/kT}$$

~ Fermi-Dirac Approx

Boltzmann Distribution!!

Carrier Concentration for parabolic band in 3-D:

$$n = \int_0^{\infty} \left[\frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^3} E^{1/2} \right] e^{-(E-E_F)/kT} dE = \int N(E) f(E) dE$$

$$= \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^3} e^{E_F/kT} \int_0^{\infty} E^{1/2} e^{-E/kT} dE$$

$x = E/kT$

$$dx = \frac{dE}{kT}$$

$$= \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^3} e^{E_F/kT} \int_0^{\infty} (x kT)^{1/2} e^{-x} kT dx$$

$$= \left(\frac{kT \hbar^2}{2m} \right)^{3/2} \underbrace{\int_0^{\infty} x^{1/2} e^{-x} dx}_{\frac{\sqrt{\pi}}{2}}$$

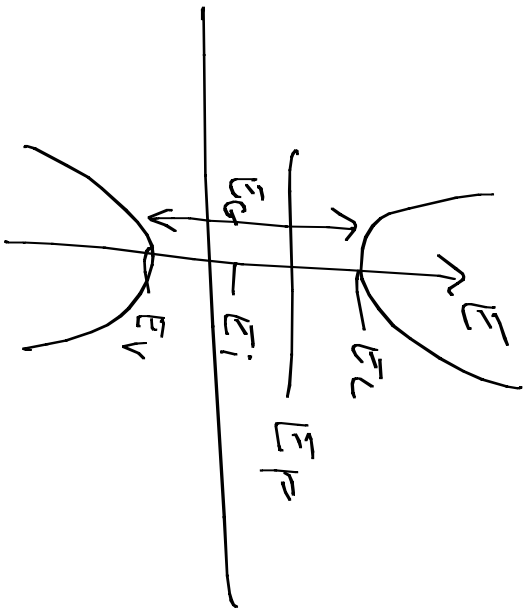
$$= 2 \left(\frac{kT m}{2 \hbar^2 \pi} \right)^{3/2} e^{E_F/kT}$$

$$= 2 \left(\frac{2 \pi kT m}{\hbar^2} \right)^{3/2} e^{(E_F - E_c)/kT}$$

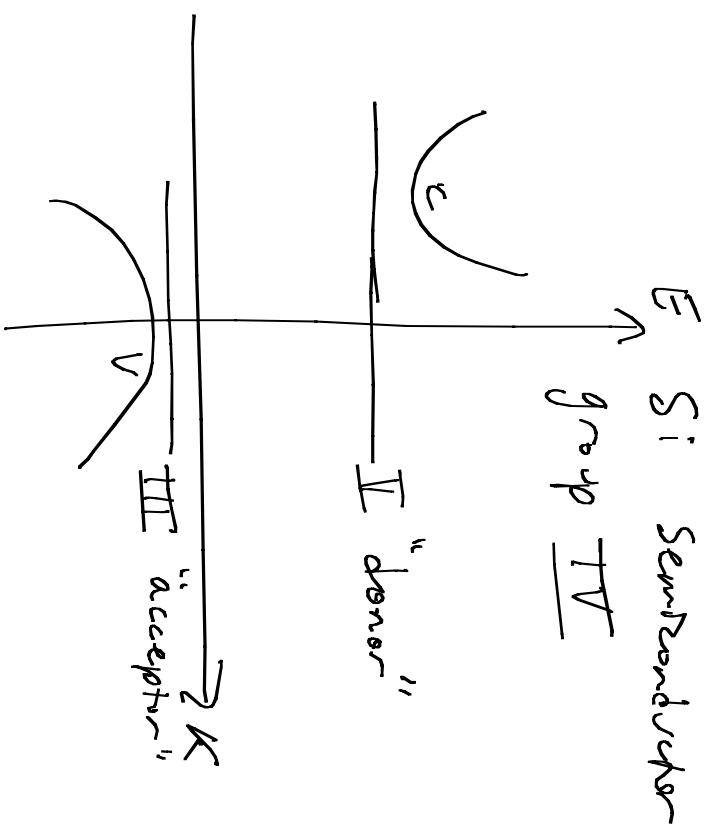
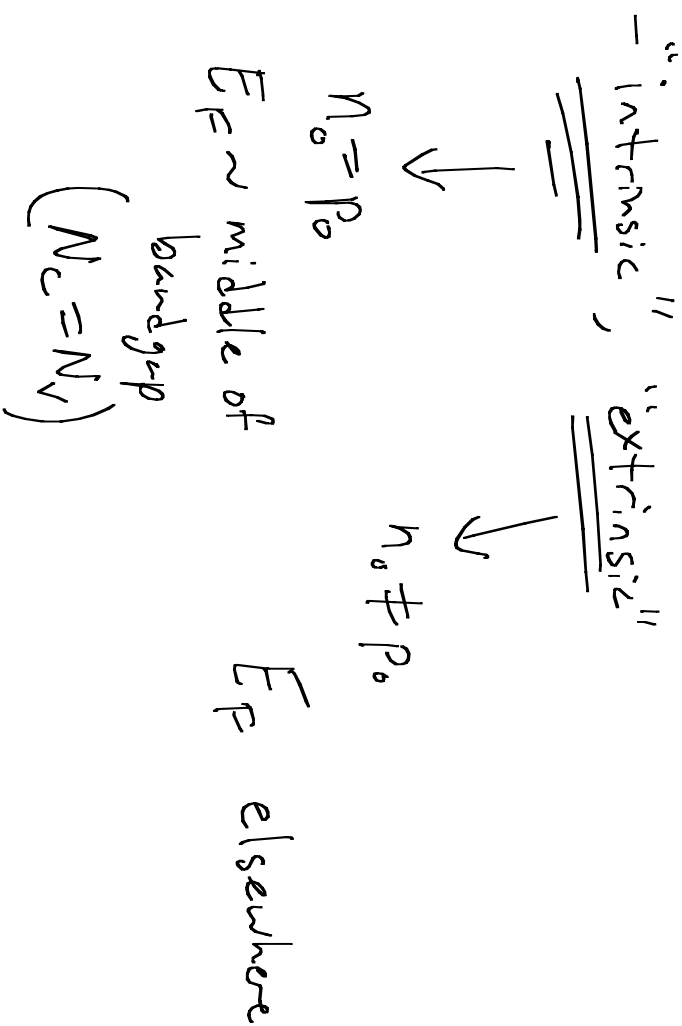
\downarrow

$$n = N_c e^{(E_F - E_c)/kT}$$

$$P = N_V e^{(E_V - E_F)/kT}$$



Fermi Energy Controls Carrier Concentration:



- Carrier Concentration Applet
- doping w/ impurities to control position of $E_F \rightarrow$ "compensation"

"Law of Mass action":

$$N_0 = N_c e^{(\epsilon_F - \epsilon_c)/kT}$$

$$P_0 = N_v e^{(\epsilon_v - \epsilon_F)/kT}$$

$$N_0 P_0 = N_c N_v e^{-(\epsilon_c - \epsilon_v)/kT} = N_c N_v e^{-\epsilon_g/kT} = n_i^2$$

$$N_0 P_0 = n_i^2$$

" $n_i^2 = N_A n_i$ "

$$N_0 = N_c e^{(\epsilon_F - \epsilon_c)/kT} e^{\epsilon_i/kT} e^{-\epsilon_i/kT} = N_c e^{(\epsilon_F - \epsilon_i)/kT} e^{(\epsilon_i - \epsilon_c)/kT}$$

$$= N_c e^{(\epsilon_i - \epsilon_c)/kT} e^{(\epsilon_F - \epsilon_i)/kT}$$

$$N_0 = n_i e^{(\epsilon_F - \epsilon_i)/kT}$$

"Drude" Conductivity (Section 3.4)



$$J = nqv = nqav = nq \left(\frac{E}{m} \right) v = \left(\frac{nq^2 v^2}{m} \right) E = \sigma E$$

$$J = \sigma E \quad \text{Ohm's Law}$$

↑ Conductivity

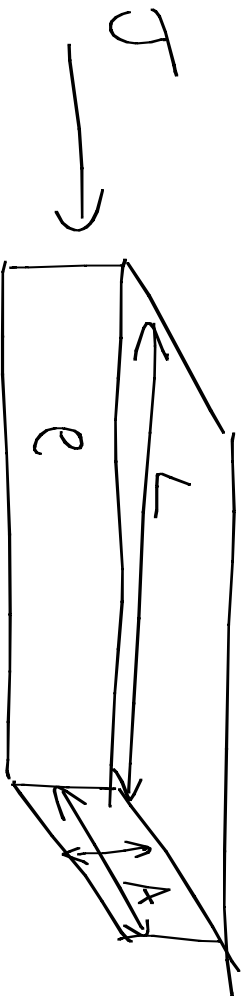
Ohm's Law :

$$J = \sigma E$$

$$V = IR$$

$$J = \frac{I}{A} = \sigma \frac{V}{L}$$

$$V = \frac{L}{\sigma A} I = \left(\frac{L \rho}{A} \right) I = IR$$



Mobility

$$\sigma = \frac{nq^2\tau}{m^*} = nq \left(\frac{q\tau}{m^*} \right) = nq\mu \quad \leftarrow \text{mobility}$$

$$\mu = \frac{q^2\tau}{m^*} = \frac{q^2\tau}{m^*} = \frac{V}{E} = \frac{\text{cm}^2}{\text{S}\cdot\text{V}}$$

$$J = \sigma E = nq\mu E$$