1. (30 pts) The partial differential diffusion equation in space and time for excess holes in the absence of an electric field is

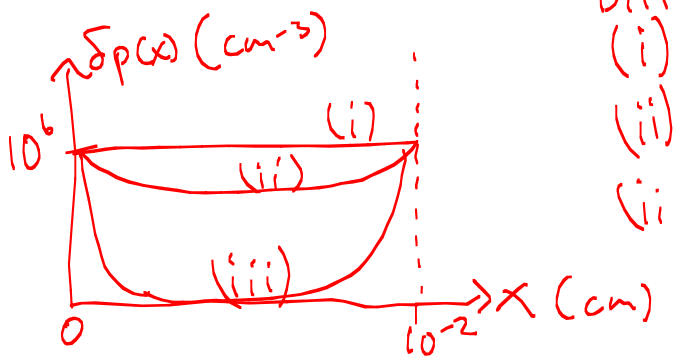
$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

(a) What is the difference between p and δp? (5)

$$p = p_0 + \delta p$$

(b) If the boundaries are at x=0 and x=100μm, and $\delta p(x=0)=10^6\text{cm}^{-3}$ and $\delta p(x=100\mu m)=10^6\text{cm}^{-3}$ for all *t*, use the diffusion equation above (no electric field) to draw δp(x) for (i) $D\tau_p \sim 100\text{cm}^2$ (ii) $D\tau_p \sim 10^{-4}\text{ cm}^2$ and (iii) $D\tau_p \sim 10^{-6}\text{cm}^2$ for times *long after initial conditions*. ~~(15)~~ → steady-state, $\frac{\partial p}{\partial t} = 0$

$p'' = \frac{\delta p}{D\tau}$ $\quad \delta p = Ae^{x/L} + Be^{-x/L}$, $\quad L = \sqrt{D\tau_p}$



Diffusion length:

(i) $L = \sqrt{100\text{cm}^2} = 10\text{ cm}$

(ii) $L = \sqrt{10^{-4}\text{cm}^2} = 10^{-2}\text{cm}$

(iii) $L = \sqrt{10^{-6}\text{cm}^2} = 10^{-3}\text{cm}$

(c) The Drift-Diffusion equation for holes is

$$J_p = (q\mu p)\mathcal{E} - qD_p\frac{dp}{dx}$$

Use this to determine the direction of hole diffusion current at the boundaries for part (b). **Explain your answer.** (10)

@ x=0, Jp is in positive direction.

@ x=10⁻²cm, Jp is in negative direction.

holes must be injected from boundaries to maintain steady-state because they are being lost to recombination.

2. (10 pts) Show that the slope of the bands in a band diagram is proportional to electric field.

potential energy $= (-q)V$

slope of bands $= \frac{d}{dx}(-q)V = -q\frac{dV}{dx}$

since $V = -\int \varepsilon \cdot dl$, $\frac{dV}{dx} = -\varepsilon$. so slope $= q\varepsilon$

3. (20 pts) A 3cm-long, 1mm-thick intrinsic semiconductor of bandgap 0.62eV (intrinsic concentration at 300K=$\boxed{10^{12}\text{cm}^{-3}}$) is illuminated with 10^{16} photons/sec of wavelength 1μm at x=1cm and 3μm at x=2cm, respectively, in spots of diameter 1mm. The carrier lifetime for electrons and holes is 10μs.

n_i

(a) What is the steady-state carrier concentration at x=1cm and x=2cm? What is the difference between quasi-Fermi energies and intrinsic Fermi level for both carriers and at both places? (10)

@ 1cm, $E = \frac{hc}{\lambda} = \frac{1.24\,\mu m\, eV}{1\,\mu m} = 1.24\,eV > E_g$

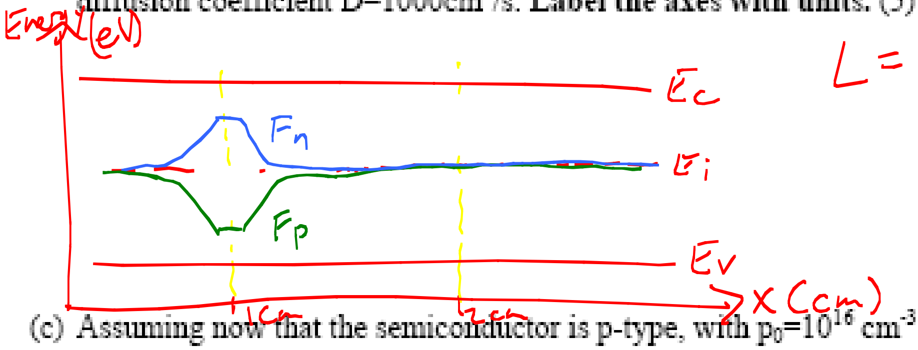
@ 2cm, $E = \frac{hc}{\lambda} = \frac{1.24\,\mu m \cdot eV}{3\,\mu m} = 0.41\,eV < E_g$ (no EHPs)

$\frac{dn}{dt} = 0 = \frac{10^{16}\ EHPs}{1mm \cdot (1mm)^2} - \frac{\delta n}{10^{-5}s} \Rightarrow \delta n = \delta p = 10^{14}\ cm^{-3}$ @ 1cm

@1cm, $p = n \sim 10^{14} cm^{-3}$, $|F_{n,p} - E_i| = KT \ln\frac{10^{14}}{n_i} \sim 26meV \cdot 2 \cdot 2.3 \sim 120meV$

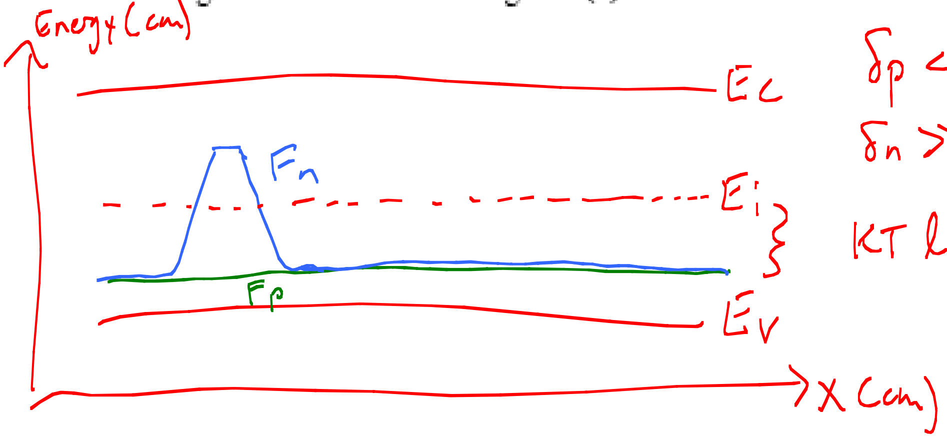
@2cm, $p = n = n_i$, so $|F_{n,p} - E_i| = 0$

(b) Draw the band diagram, showing quasi-Fermi levels for electrons and holes, assuming diffusion coefficient D=1000cm^2/s. **Label the axes with units.** (5)



$L = \sqrt{D\tau} = \sqrt{10^3 \frac{cm^2}{s} \cdot 10^{-5}s}$
$= 10^{-1}cm$

(c) Assuming now that the semiconductor is p-type, with p$_0$=10^{16} cm^{-3}, re-draw the quasi-Fermi energies on another band diagram. (5)



$\delta p << p_o$ so little change in F_p

$\delta n >> n_0 = \frac{n_i^2}{p_o}$

$KT \ln\frac{10^{16}}{10^{12}} \simeq 200meV$

4. (10 pts) An otherwise intrinsic semiconductor sample at room temperature is doped with acceptors from one side so that $N_a = N_0 exp(-ax)$, where $n_i << N_0$, and x is distance from the surface toward the bulk.

(a) Determine the electric field that exists in the sample in equilibrium. Show that your answer has the right units. (5)

$$J = q\mu p\mathcal{E} - qD\frac{dp}{dx} = 0$$

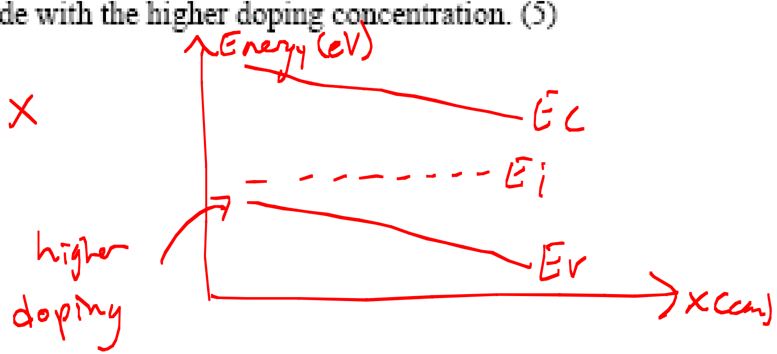
$$\mathcal{E} = \frac{D}{\mu}\frac{1}{p}\frac{dp}{dx} = \frac{D}{\mu}\frac{-ae^{-ax}}{e^{-ax}} = -\frac{aD}{\mu} \Rightarrow \frac{\frac{1}{cm}\frac{cm^2}{s}}{\frac{cm^2}{V\cdot s}} = \frac{V}{cm}$$

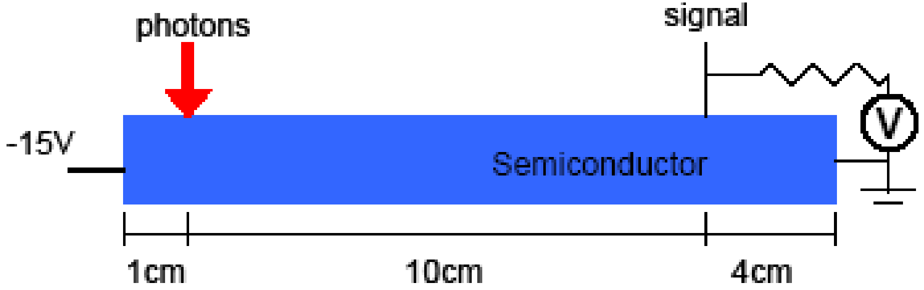
✓

(b) Find the electrostatic potential associated with this field and draw the band diagram with appropriately labeled scale and units. Label the side with the higher doping concentration. (5)

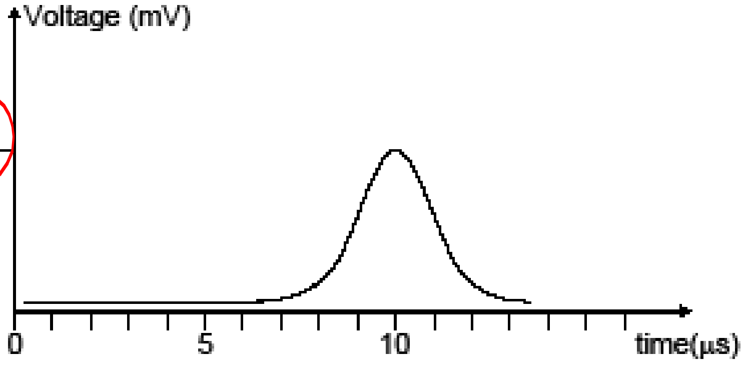
$$V = -\int \mathcal{E}\cdot d\ell = +\frac{aD}{\mu}x$$

$$\text{Potential energy} = (-q)V = -\frac{qaD}{\mu}x$$





5. (30 pts) Data from the Haynes-Shockley experiment, using the configuration above, is shown below.



Should be -5. If this influenced your answer, come see me!

From it, find

(a) mobility (5)

$$\mu = \frac{v}{\mathcal{E}} = \frac{10\,\text{cm}/10\,\mu\text{s}}{15\,\text{V}/15\,\text{cm}} = 10^6\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}$$

(b) diffusion coefficient (5)

$$D = \frac{L^2}{\tau} = \frac{(v\cdot\Delta\tau)^2}{\tau} \simeq \frac{(10^6\,\text{cm/s}\cdot 1\,\mu\text{s})^2}{10\,\mu\text{s}} = 10^5\,\frac{\text{cm}^2}{\text{s}}$$

(c) What type is the semiconductor? (5)

carriers move away from negative voltage → electrons

Haynes-Shockley measures properties of minority carriers, so p-type

(d) Are the above properties of electrons or holes? (5)

(e) How could you measure the carrier lifetime? What other measurement is necessary? (5)

Change voltage → This changes transit time. Then integrate signal + fit to exponential decay e^{-t/τ_n} to determine τ_n.

(f) What temperature was the experiment performed at? (5)

$$\frac{D}{\mu} = \frac{kT}{q} = 26\,\text{meV} \;@\; 300\,\text{K}$$

Here, $\frac{D}{\mu} = \frac{10^5\,\text{cm}^2/\text{s}}{10^6\,\frac{\text{cm}^2}{\text{V}\cdot\text{s}}} = 100\,\text{meV}$, so $T \cong 1200\,\text{K}$