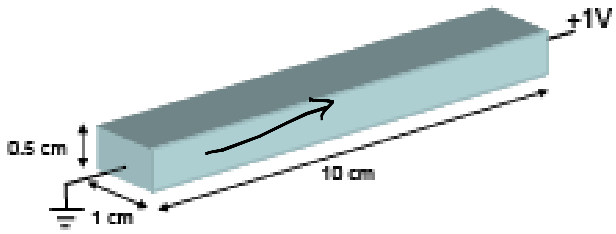
1. 1 V is applied across the longest dimension of a 0.5cm x 1 cm x 10cm n-type semiconductor. Assume electron and hole mobilities are equal, and intrinsic carrier concentration is 10^{10} cm^{-3}.



(a) Indicate the direction of electron flow on the figure. (5)

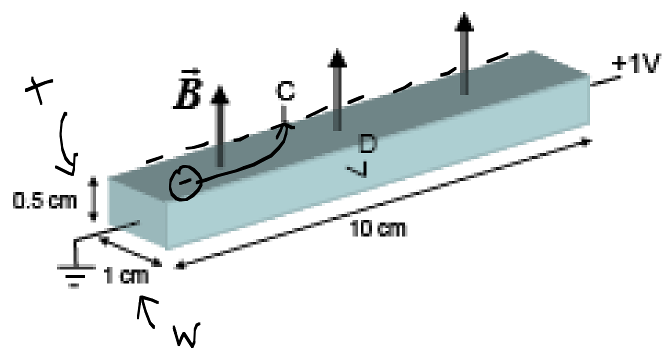
negatively-charged electrons are attracted to positive potential.

(b) If 50 mA current flows, what is the conductivity? (5)

$$J = \sigma \mathcal{E} \qquad I = JA$$

$$\sigma = \frac{J}{\mathcal{E}} = \frac{\left(\frac{5\times10^{-2}\,A}{5\times10^{-1}cm \cdot 1cm}\right)}{\left(\frac{1V}{10cm}\right)} = \frac{10^{-1}}{10^{-1}} \frac{A}{V\,cm} = 1\ \Omega^{-1}cm^{-1}$$

Now, a magnetic field with strength 10^{-4} V·s/cm^2 (1 Tesla) perpendicular to the flow of current is applied, and a Lorentz force $q\vec{v}\times\vec{B}$ acts on the flowing charge carriers. Equilibrium is established when the transverse Coulomb force $q\mathcal{E}$ equals this Lorentz force.



(c) Show that the magnitude of the transverse voltage (between points C and D) is equal to $\frac{IB}{nqt}$, where t is the thickness of the semiconductor. (5)

$$q\mathcal{E}_x = qvB \qquad J = \sigma\mathcal{E} = nq\mu\mathcal{E} = nqv$$

$$\frac{V}{w} = \frac{J}{nq}B = \frac{IB}{wtnq}$$

$$V = \frac{IB}{nqt}$$

(d) Using the result of part (a), is the polarity of the voltage at point D (relative to point C) positive or negative?(5) [Remember the right hand rule, and the sign of the charge carrier!]

negatively-charged electrons move toward C, So D is at a relatively more positive Voltage

(e) Using the expression in part (c), if the voltage measured transverse to the current flow from point C to point D is 10 mV, what is the carrier concentration of electrons, n? (use q~10^{-19} Coulomb)(5)

$$10^{-2}\,V = \frac{5\times10^{-2}A \quad 10^{-4}\,\frac{Vs}{cm^2}}{n\,10^{-19}C \quad 5\times10^{-1}cm}$$

$$n = \frac{10^{-6}}{10^{-22}}\;\frac{\cancel{A}\,\cancel{V}\cancel{s}}{\cancel{V}\cancel{C}\,cm^3} \qquad \left(1A = 1\frac{C}{s}\right)$$

$$= 10^{16}\,cm^{-3}$$

(f) Use the result of part (e) to find the carrier concentration of holes. (5)

$$n_o p_o = n_i^2$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(10^{10})^2}{10^{16}} = 10^4\,cm^{-3}$$

(g) Use the result of parts (b) and (e) to find the mobility of electrons.(5)

$$\sigma = nq\mu$$

$$\mu = \frac{\sigma}{nq} = \frac{1\,\Omega^{-1}cm^{-1}}{10^{-19}C\cdot10^{16}cm^{-3}} = 10^3\,\frac{A\,cm^2}{V\,C}$$

$$= 10^3\,\frac{cm^2}{Vs} \qquad \left(1A = \frac{1C}{s}\right)$$

(h) How far from the intrinsic level does the Fermi energy lie at room temperature? (5)

$$n_o = n_i\, e^{(E_F - E_i)/kT}$$

$$kT \ln\frac{n_o}{n_i} = E_F - E_i$$

$$26\,meV \cdot \ln\frac{10^{16}}{10^{10}} = E_F - E_i$$

$$E_F - E_i = 26\,meV \cdot 6 \cdot \ln 10 = \sim 350\,meV$$

$$(\ln 10 \sim 2.3)$$

2. If the conduction band density of states (DOS) is 10 times larger than valence band DOS, how much energy separates the intrinsic level from the middle of the bandgap? Is the intrinsic level closer to the conduction band or valence band? (15)

for intrinsic $n_o = p_o$

$$n_0 = N_c e^{(E_F - E_c)/kT} = N_v e^{(E_v - E_F)/kT} = p_o$$

$$\frac{N_c}{N_v} = e^{[E_v - E_F - (E_F - E_c)]/kT}$$

average of two is middle value "E_m"

$$kT \ln \frac{N_c}{N_v} = E_v + E_c - 2E_F = 2\left(\frac{E_v + E_c}{2}\right) - 2E_F$$

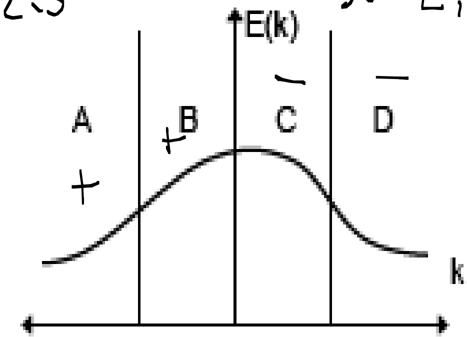
For intrinsic, $E_F = E_i$:

$$E_m - E_i = \frac{kT}{2} \ln \frac{10 N_v}{N_v} \cong \frac{26\,\text{meV}}{2} \cdot 2.3 \sim 30\,\text{meV}$$

positive, so $E_i < E_m$

3. For the given bandstructure, indicate whether the group velocity is positive or negative for each of the four regions shown. (10 -- 2.5 each)

$V_{group} \propto$ slope



4. Using only eV, V, s, K (Kelvin), and cm, what are the units of (15 -- 2 each):

mobility $\frac{cm^2}{V \cdot s}$ from $\mu = \frac{v}{\mathcal{E}}$

mass $eV s^2/cm^2$ from $E = mc^2$

Planck constant (h) $eV \cdot s$ from $E = \hbar\omega$

effective density of states cm^{-3} from $n_0 = N_c e^{(E_F - E_c)/k_B T}$

wavevector cm^{-1} from $e^{\underbrace{ikx}_{\text{unitless}}}$

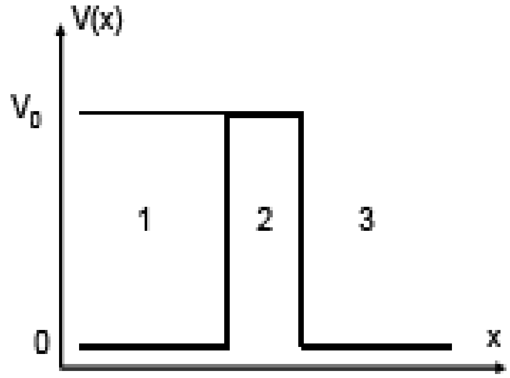
Boltzmann constant (k_B) eV/K from $k_B T = 26\,\text{meV}$ @ RT

electric field V/cm from $V = -\int \mathcal{E} \cdot dl$

5. The time-independent Schrödinger equation for an electron in a 1-dimensional potential is

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi + V(x)\varphi = E\varphi .$$

Assuming E<V_0, What is the general form of the solution φ to this differential equation in region 2 shown? (10)

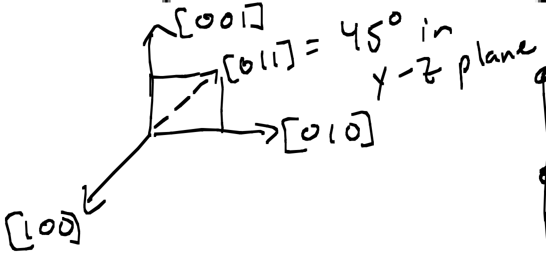


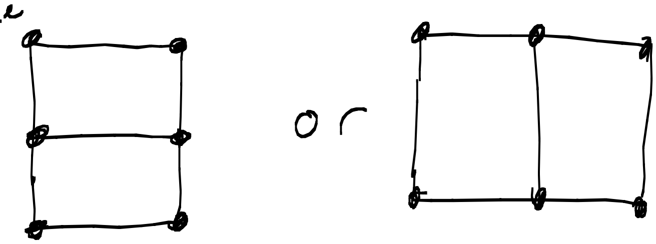
$$\varphi'' = -\frac{2m}{\hbar^2}(E - V_0)\varphi$$

$$= \frac{2m}{\hbar^2}(V_0 - E)\varphi$$

$$\varphi = C_1 e^{+Kx} + C_2 e^{-Kx} \qquad K^2 = \frac{2m(V_0-E)}{\hbar^2} > 0$$

6. Draw the *simple cubic* unit cell as viewed along the [011] direction. (10)





EXTRA CREDIT:

Solve for the ***general*** solution to the following linear, ordinary differential equations: (2 pts each)

(a) $\frac{df}{dz} = Af$ $\quad f(z) = C_0 e^{Az}$

(b) $\frac{d^2 f}{dr^2} = Af$ $\quad f(r) = C_1 e^{-\sqrt{A}r} + C_2 e^{+\sqrt{A}r}$

(c) $\frac{d^2 f}{dt^2} = -Af$ $\quad f(t) = C_1 e^{-i\sqrt{A}t} + C_2 e^{+i\sqrt{A}t}$

(d) $\frac{d^2 f}{dy^2} = 0$ $\quad f(y) = C_1 y + C_2$

(e) $\frac{d^2 f}{dx^2} = A$ $\quad f(x) = \frac{A}{2}x^2 + C_1 x + C_2$