COMPRESSIVE MATCHED SUBSPACE DETECTION

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ABSTRACT

In this paper, matched subspace detectors based on the framework of Compressive Sensing (CS) are developed. The proposed approach, called compressive matched subspace detectors, exploits the sparsity model of the signal-of-interest in the design of the random projection operator. By tailoring the CS measurement matrix (projection operator) to the subspace where the signal-of-interest is known to lie, the compressive matched subspace detectors can effectively capture the signal energy while the interference and noise effects are mitigated at sub-Nyquist rate. The proposed detection approach is particularly suitable for detection of wideband signals that emerge in modern communication systems that demand high-speed ADCs. The performance of the subspace compressive detectors are studied by analytically deriving closed-form expressions for the detection probability and through extensive simulations.

1. INTRODUCTION

Optimum matched subspace detectors that maximize the detection probability for a given probability of false alarm in the presence of additive noise and interference have been developed under different models for the signal, noise and interference [1, 2, 3, 4]. These detectors, in general, rely on the assumption that the received signal can be sampled at least at Nyquist rate to derive sufficient statistics as a function of the sampled signal. In many emerging application areas, however, such as impulse radio ultra-wideband communications, the signal-of-interest to be detected has ultra-short duration at the (sub)nanosecond scale demanding the use of high-speed analog-to-digital converters (ADCs) with sampling frequency on the order of several GHz. Such formidable sampling rates can be achieved with high-speed ADCs at the expense of high power consumption, limited resolution and demanding very fast digital hardware for subsequent data processing.

Compressive sensing (CS) emerges as a potential approach that maps the original signal to a new domain where the transformed signal can be sampled at Sub-Nyquist rates [5, 6]. Interestingly, it has been shown that by using a random basis the relevant information about the signal is preserved in the mapping operation. Furthermore, if the projections are carried out in the analog domain, sampling the randomly projected signal at sub-Nyquist rate leads to a reduced set of samples (random projections) that convey the salient information about the original signal without demanding high-speed ADCs.

Much like a reduced set of compressed measurements provides sufficient information for signal reconstruction [6], these compressed measurements can be suitably used for other statistical inference tasks such as signal detection [7, 8]. Signal detection using compressive measurements has been recently addressed in [8] where a matched filter-like test statistic has been derived as a function of the random projections. Thus, the compressive signal detection reduces to randomly projecting the received signal followed by a matched filter operation on the random samples. Although the random measurement scheme for compressive detection proposed in [8] provides universality for detecting signals in the N-dimensional space, it fails to exploit the signal structure that may be known *a priori* [1].

In this paper, we propose a new approach to address the subspace signal detection problem under the framework of Compressive Sensing. It is shown that the detection performance based on compressive measurements can be significantly improved by exploiting the underlying signal structure in the random projection stage. More precisely, by designing a projection matrix tailored to the subspace where the signal-of-interest is known to lie, the signal energy can be captured more efficiently, and at the same time mitigating the effect of noise and interference, resulting in a much better detector performance with a reduced set of random measurements. A new class of sub-Nyquist sampling rate matched subspace detectors are derived based on compressive measurements which yield comparable performances to those obtained with conventional (Nyquist sampling) detectors [1] but without demanding expensive ADC resources. Our work is a refinement of [8], where we take advantage of the known signal subspace, and the resulting detection performance is much enhanced. This new class of matched subspace detectors includes subspace compressive detectors for the detection of a known signal in a known subspace where the signal-of-interest is embedded in additive white Gaussian noise or in additive white Gaussian noise and interference. Interestingly, the new class of matched subspace detectors defines statistical decision rules in the subspace domain leading to test statistics that depend on the random measurements, therefore signal reconstruction is not required for signal detection. This builds on the fact that generally, far fewer measurements and less computational complexity are needed for signal detection than for signal reconstruction [7, 8]. Further details of this work as well as the extension to subspace detection of unknown sparse signals can be found in [9].

2. PRELIMINARIES

2.1 The detection Problem

Given a continuous-time signal $x(t)$, the goal is to decide which of two possible models is more suitable to describe the observation signal, $x(t)$. In a first model, the observation signal is a random realization of a noise process, whereas for the second model, $x(t)$ is the superposition of a signal-of-interest and noise. Assume for now that the signal of interest and the noise are bandlimited to $|f| \leq B$ and that it is possible to sample the observation signal at the Nyquist rate over some interval $0 \leq t < T$. Let $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ be the discrete-time representation of the signal $x(t)$ where $x_i = x(t_i)$. This decision problem reduces to distinguish between the hypotheses $\mathscr{H}_0$ and $\mathscr{H}_1$,

$$\begin{array}{lcl} \mathscr{H}_0 & : & \mathbf{x} = \mathbf{w} \\ \mathscr{H}_1 & : & \mathbf{x} = \mathbf{s} + \mathbf{w} \end{array} \tag{1}$$

where $\mathbf{s}$ represents the signal of interest that is assumed to be K-sparse in some dictionary $\mathbf{\Psi} = [\psi_1, \psi_2, \psi_3, \ldots, \psi_Z]$ with $\psi_i \in \mathscr{R}^N$. Thus, $\mathbf{s}$ can be represented by a linear combination of K vectors from Ψ, with $K << N$. Namely, $\mathbf{s} = \mathbf{H}\Theta$ where $\mathbf{H} = [\psi_{n_1}, \psi_{n_2}, \ldots, \psi_{n_K}]$, with $n_i \in \{1, 2, \ldots, Z\}$ for $i = 1, 2, \ldots, K$. Θ is a K-dimensional vector with all non-zero entries that defines $\mathbf{s}$ in the signal subspace spanned by the columns of $\mathbf{H}$. Thus, the signal of interest resides in the K-dimensional subspace spanned by the columns of the $N \times K$ matrix $\mathbf{H}$. In Eq. (1), $\mathbf{w}$ denotes the noise vector that is assumed to be i.i.d. obeying $\mathscr{N}(0, \sigma^2\mathbf{I})$. Furthermore, we assume that the signal subspace $\langle \mathbf{H} \rangle$ is known or it can be learned from a secondary data set [2, 4, 9, 10, 11].

2.2 Conventional Matched Subspace Detector

In order to introduce the notation and terms that will be used hereafter, let's consider the detection of a known sparse signal in white Gaussian noise with known variance σ^2. The detection problem reduces to distinguishing between the hypotheses

$$\begin{array}{lcl} \mathscr{H}_0 & : & \mathbf{x} = \mathbf{w} \\ \mathscr{H}_1 & : & \mathbf{x} = \mathbf{H}\Theta + \mathbf{w} \end{array} \tag{2}$$

The aim is to find an expression as a function of the observation vector $\mathbf{x}$ that used as a test statistic allow us to decide whether the signal of interest is present or not in the observation vector. According to the Neyman-Pearson (NP) criterion, the optimum decision strategy that maximizes the probability of detection (P_d) while keep the probability of false alarm P_{fa} under a certain value is given by the likelihood ratio test (LRT) defined as [12]:

$$\Lambda(\mathbf{x}) = \frac{f_1(\mathbf{x}; \Theta, \sigma^2 | \mathscr{H}_1)}{f_0(\mathbf{x}; \Theta, \sigma^2 | \mathscr{H}_0)} \underset{\mathscr{H}_0}{\overset{\mathscr{H}_1}{\gtrless}} \eta \tag{3}$$

where $f_1(\mathbf{x}; \Theta, \sigma^2 | \mathscr{H}_1)$ is the probability of observing $\mathbf{x}$ under $\mathscr{H}_1$ and $f_0(\mathbf{x}; \Theta, \sigma^2 | \mathscr{H}_0)$ is the probability of observing $\mathbf{x}$ under $\mathscr{H}_0$. The threshold parameter η is chosen to achieve a desired probability of false alarm.

It is well known that the LRT for detecting a known signal in white Gaussian noise with known variance yields as sufficient statistic the matched filtering operation [12]:

$$t(\mathbf{x}) = \Theta^T \mathbf{H}^T \mathbf{x} = \mathbf{s}^T \mathbf{x} \tag{4}$$

where $\mathbf{s} = \mathbf{H}\Theta$ is the signal of interest and ^{T} denotes the transpose operator. Since the matched filter output is a linear combination of Gaussian random variables, $t(\mathbf{x})$ obeys the following distribution

$$t(\mathbf{x}) \sim \begin{cases} \mathscr{N}(0, \sigma^2 \mathbf{s}^T \mathbf{s}) & \text{under } \mathscr{H}_0 \\ \mathscr{N}(\mathbf{s}^T \mathbf{s}, \sigma^2 \mathbf{s}^T \mathbf{s}) & \text{under } \mathscr{H}_1 \end{cases}$$

where we use the notation $\sim$ to mean "is distributed as" and $\mathscr{N}(\mu, \sigma_0^2)$ to denote normal distribution with mean μ and variance σ_0^2.

To evaluate the performance of the matched filter detector, the false alarm probability and the detection probability are found based on the statistic of the matched filter output. Namely,

$$\begin{array}{lcl} P_{fa} & = & \text{Prob}\,(\text{select } \mathscr{H}_1 \mid \mathscr{H}_0) \;=\; \text{Prob}\left(t(\mathbf{x}) > \eta' \mid \mathscr{H}_0\right) \\ P_d & = & \text{Prob}\,(\text{select } \mathscr{H}_1 \mid \mathscr{H}_1) \;=\; \text{Prob}\left(t(\mathbf{x}) > \eta' \mid \mathscr{H}_1\right) \end{array}$$

Setting the probability of false alarm to a fixed value, say α, the P_d can be found as a function of P_{fa} as follows [12]:

$$P_d(\alpha) = Q\left(Q^{-1}(\alpha) - \sqrt{\frac{\mathbf{s}^T\mathbf{s}}{\sigma^2}}\right) \tag{5}$$

where $Q(x) \triangleq (2\pi)^{-\frac{1}{2}} \int_x^\infty e^{-x^2/2} dx$ and $Q^{-1}(\cdot)$ denotes the inverse of the $Q(\cdot)$ function. Equation (5) is known as the receiver operating characteristics (ROC) for the matched filter detector and it fully describes the performance of the detection as a function of the false alarm probability and the signal-to-noise ratio $SNR = \frac{\mathbf{s}^T\mathbf{s}}{\sigma^2}$.

3. SUBSPACE COMPRESSIVE DETECTION OF A KNOWN SPARSE SIGNAL

CS framework has emerged as a promising approach to reduce ADC resources by randomly transforming the original observation signal to a new domain where the transformed signal is sampled at Sub-Nyquist rates preserving the salient information about the original signal. In the new domain, a random projection based test statistic must be derived to allow us to decide whether the signal of interest is present or not and where the samples are obtained at sub-Nyquist sampling rates.

Consider the detection problem described in Eq. (1), where the signal of interest is known to lie in the signal subspace spanned by the columns of $\langle \mathbf{H} \rangle$. Furthermore, assume that the signal parameter Θ and the noise variance are known. Under the subspace CS framework, the detection problem reduces to distinguish between two hypotheses $\mathscr{H}_0$ and $\mathscr{H}_1$:

$$\begin{array}{lcl} \mathscr{H}_0 & : & \mathbf{y} = \Phi\mathbf{w} \\ \mathscr{H}_1 & : & \mathbf{y} = \Phi(\mathbf{s} + \mathbf{w}) \end{array} \tag{6}$$

where Φ is an $M \times N$ random projection matrix, called the measurement matrix, that is suitably designed to exploit the *a prior* knowledge about the signal sparsity model. That is, unlike the conventional CS framework where the projection matrix, Φ, are realizations of i.i.d. random variables following a normal distribution, in the application at hand, we want to exploit in the projection operation the inherent structure of the sparse signal to be detected. More precisely, if we know *a priori* that the signal of interest follows a linear model, i.e., $\mathbf{s} = \mathbf{H}\Theta$, the measurement matrix can be tailored to the signal model leading to fewer random projections, and, therefore less demanding ADC resources.

In this work, the proposed subspace measurement matrix is defined as:

$$\Phi = \mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T \tag{7}$$

where $\mathbf{G}$ is an $M\times K$ i.i.d. random matrix whose entries follow a $\mathscr{N}(0,1/K)$ distribution with $M\leq K$. Note that under $\mathscr{H}_1$ the random projection reduces to $\mathbf{y}=\mathbf{G}\left(\Theta+(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}\mathbf{w}\right)$, hence, there is no loss of signal energy while the noise power is greatly reduced. Thus, good detection performance is expected even with a small number of measurements.

Note in Eq. (6) that the observation data $\mathbf{y}$ is an M-dimensional vector whose entries are random projections of the underlying signal. As a result, a new test statistic and a decision rule have to be derived to decide whether or not $\mathbf{s}$ is present in the random measurement $\mathbf{y}$. Since $\mathbf{w}\sim\mathscr{N}(0,\sigma^2\mathbf{I})$ and the measurement matrix is known, it follows that the likelihood ratio test reduces to:

$$\Lambda(\mathbf{y})=\frac{\exp\left[-(\mathbf{y}-\Phi\mathbf{H}\Theta)^T(\sigma^2\Phi\Phi^T)^{-1}(\mathbf{y}-\Phi\mathbf{H}\Theta)\right]}{\exp\left[-\mathbf{y}^T\left(\sigma^2\Phi\Phi^T\right)^{-1}\mathbf{y}\right]} \quad (8)$$

Taking the logarithm on Eq. (8) and after some algebraic manipulations, it follows that the sufficient statistic is given by:

$$\bar{t}(\mathbf{y})=\mathbf{y}^T(\Phi\Phi^T)^{-1}\Phi\mathbf{s}=\mathbf{y}^T[\mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{G}^T]^{-1}\mathbf{G}\Theta. \quad (9)$$

Thus, the signal is detected if the test statistic, $\bar{t}(\mathbf{y})$ is greater than a given threshold η.

Furthermore, since $\bar{t}(\mathbf{y})$ is a weighted sum of Gaussian random variables, it is distributed according to:

$$\bar{t}(\mathbf{y})\sim\begin{cases}\mathscr{H}_0: & \mathscr{N}(0,\Theta^T\mathbf{C}^{-1}\Theta)\\ \mathscr{H}_1: & \mathscr{N}(\Theta^TG^T[\mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{G}^T]^{-1}\mathbf{G}\Theta,\Theta^T\mathbf{C}^{-1}\Theta)\end{cases}$$

where $\mathbf{C}=[\mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{G}^T]^{-1}\mathbf{G}$. This allow us to find a closed-form expression to characterize the performance of the subspace CS detection. It can be shown that for a given $P_{fa}=\alpha$, the detection probability is given by[9]:

$$P_d(\alpha)=Q\left[Q^{-1}(\alpha)-\sqrt{\frac{\mathbf{s}^T\mathbf{P}_\Phi\mathbf{s}}{\sigma^2}}\right], \quad (10)$$

where $\mathbf{P}_\Phi=\Phi^T(\Phi\Phi^T)^{-1}\Phi$. Furthermore, we have that

$$\begin{aligned}\mathbf{s}^T\mathbf{P}_\Phi\mathbf{s} &= \mathbf{s}^T\Phi^T(\Phi\Phi^T)^{-1}\Phi\mathbf{s}\\ &= \Theta^T\mathbf{G}^T[\mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{G}^T]^{-1}\mathbf{G}\Theta.\end{aligned}$$

When $M=K$, $\mathbf{G}$ is invertible with probability 1 leading to

$$\mathbf{s}^T\mathbf{P}_\Phi\mathbf{s}=\Theta^T\mathbf{H}^T\mathbf{H}\Theta=\mathbf{s}^T\mathbf{s}. \quad (11)$$

Note that when the number of random projections equals the sparsity of the signal, i.e., $M=K$, the proposed subspace CS detection has the same performance as the conventional detector given by Eq. (5). Note that $\mathbf{G}\mathbf{G}^T\approx K\mathbf{I}_{M\times M}$ and $\mathbf{G}^T\mathbf{G}\approx M\mathbf{I}_{K\times K}$ if $M,K\gg 1$. Further, if the signal of interest is sparse on an orthonormal basis, it can be shown that:

$$\mathbf{s}^T\mathbf{P}_\Phi\mathbf{s}\approx(M/K)\mathbf{s}^T\mathbf{s}. \quad (12)$$

Then, with $M\leq K$, the detector performance can be approximated as:

$$\bar{P}_d(\alpha)\approx Q\left[Q^{-1}(\alpha)-\sqrt{M/K}\sqrt{\frac{\mathbf{s}^T\mathbf{s}}{\sigma^2}}\right]. \quad (13)$$

The approximation of $P_d(\alpha)$ by Eq. (13) also holds when ψ_i's in Ψ have unit energy and are approximately orthogonal to each other.

Comparing the proposed subspace CS detection to that introduced in [8], we notice several differences. First, in [8] the signal model is not taken into account in designing the measurement matrix. Second, the number of measurements has to be greater than the number of basis elements that defines the signal of interest, while in our approach only $M\leq K$ measurements are required. Finally, comparing the performance of the subspace CS detection to that in [8], it can be noticed that the proposed detector provides better detection performance with much fewer number of measurements.

Much like the CS detector in [8] provides a universal detection scheme for signals in the N-dimensional space, the proposed subspace compressive detector provides a universal detection scheme for all signals that lie in the same subspace.

4. COMPRESSIVE SUBSPACE DETECTION IN NARROWBAND INTERFERENCE

The proposed subspace compressive detection can be naturally extended to the case where interference signals are present in the observations. In particular, we are interested in detecting signals embedded in additive Gaussian noise in the presence of narrowband interference (NBI) that lies in a known interference subspace. With the knowledge of the interference subspace, an effective subspace compressive detector can be designed for subspace signal detection and, at the same time, interference rejection.

Consider the interference modeled by $\mathbf{z}=\mathbf{S}\varphi$ where $\mathbf{S}$ is an $N\times J$ matrix whose columns span the interference subspace $\langle\mathbf{S}\rangle$ and $\varphi\in\mathscr{R}^J$ is the interference parameter that localizes the interference in the subspace $\langle\mathbf{S}\rangle$ with $J\ll N-K$. In our study, we assume that the interference subspace $\mathbf{S}$ is available but the interference coefficient φ is unknown. That is, we know the subspace where the interference lies but we do not know its exact location because φ is unknown. Furthermore, we assume that the signal subspace and the interference subspace do not overlap.

The detector aims to distinguish between two hypotheses, $\mathscr{H}_0$: interference + noise and $\mathscr{H}_1$: signal-of-interest + interference + noise. Formally:

$$\begin{aligned}\mathscr{H}_0 &: \mathbf{x}'=\mathbf{S}\varphi+\mathbf{w},\\ \mathscr{H}_1 &: \mathbf{x}'=\mathbf{H}\Theta+\mathbf{S}\varphi+\mathbf{w}.\end{aligned} \quad (14)$$

Under the subspace compressive framework, we want to design a measurement matrix that on the one hand cancels out the interference and on the other hand exploits the fact that the signal-of-interest lies in a known subspace spanned by the columns of $\mathbf{H}$. It turns out that by letting the projection matrix be

$$\check{\Phi}=\mathbf{G}(\check{\mathbf{H}}^T\check{\mathbf{H}})^{-1}\check{\mathbf{H}}^T=\mathbf{G}(\mathbf{H}^T\mathbf{P}_S^{\perp}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{P}_S^{\perp} \quad (15)$$

where $\check{\mathbf{H}}=\mathbf{P}_S^{\perp}\mathbf{H}$ and $\mathbf{P}_S^{\perp}=\mathbf{I}_N-\mathbf{S}(\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T$ both goals are jointly achieved in the projection stage. Note that $\mathbf{P}_S^{\perp}$ is the orthogonal projection matrix for the NBI null space. $\mathbf{G}$ is, as before, an $M\times K$ random matrix whose entries are i.i.d. random variables following a Gaussian probability density function with zero-mean and variance $1/K$. Thus, the hypothesis problem (14) reduces to:

$$\begin{aligned}\mathcal{H}_0 &: \quad \breve{\mathbf{y}} = \breve{\Phi}(\mathbf{S}\varphi + \mathbf{w}), \\ \mathcal{H}_1 &: \quad \breve{\mathbf{y}} = \breve{\Phi}(\mathbf{H}\Theta + \mathbf{S}\varphi + \mathbf{w}).\end{aligned} \tag{16}$$

Upon closer examination of Equations (15) and (16), it can be noticed that the proposed detector first projects the received signal onto the NBI null space rejecting the interference and then the resultant signal is projected onto the signal subspace reducing the noise effect. Note also that the design of the subspace measurement matrices according to Eq. (15) is similar in spirit to the principle of the zero-forcing equalizer [13] widely used in digital communications. The interference, if present, is completely eliminated by projecting the received signal onto the interference null space.

In order to derive a test statistic expression as a function of the measurements $\breve{\mathbf{y}}$, notice that under $\mathcal{H}_0$, $\breve{\mathbf{y}}$ follows a normal distribution with mean $\breve{\Phi}\mathbf{S}\varphi$ and variance $\sigma^2\breve{\Phi}\breve{\Phi}^T$, whereas under $\mathcal{H}_1$, $\breve{\mathbf{y}}$ is normally distributed with mean $\breve{\Phi}(\mathbf{H}\Theta + \mathbf{S}\varphi)$ and variance $\sigma^2\breve{\Phi}\breve{\Phi}^T$. Therefore, it can be shown that the LRT yields as sufficient statistic [9]:

$$\breve{t}(\breve{\mathbf{y}}) = \breve{\mathbf{y}}^T(\breve{\Phi}\breve{\Phi}^T)^{-1}\breve{\Phi}\mathbf{s} = \breve{\mathbf{y}}^T[\mathbf{G}(\mathbf{H}^T\mathbf{P}_S^{\perp}\mathbf{H})^{-1}\mathbf{G}^T]^{-1}\mathbf{G}\Theta. \tag{17}$$

Consequently, the detector performance is given by:

$$P_D(\alpha) = Q\left(Q^{-1}(\alpha) - \sqrt{\frac{\mathbf{s}^T\mathbf{P}_{\breve{\Phi}}\mathbf{s}}{\sigma^2}}\right), \tag{18}$$

where $\mathbf{s} = \mathbf{H}\Theta$, $\alpha = P_{fa}$ and $\mathbf{P}_{\breve{\Phi}}$ is the orthogonal projection onto the subspace spanned by the rows of $\breve{\Phi}$, i.e., $\mathbf{P}_{\breve{\Phi}} = \breve{\Phi}^T(\breve{\Phi}\breve{\Phi}^T)^{-1}\breve{\Phi}$.

Note that the detector performance depends on the term: $\varepsilon = \mathbf{s}^T\mathbf{P}_{\breve{\Phi}}\mathbf{s} = ||\mathbf{s}_{\breve{\Phi}}||^2$, which is the signal energy that the detector can collect in the random subspace $\breve{\Phi}$. Furthermore, for $M = K$, it can be shown that ε reduces to $\mathbf{s}^T\mathbf{P}_S^{\perp}\mathbf{s}$ [13]. Thus, if the number of measurements is equal to the sparsity of the signal-of-interest, the energy collected by the detector turns out to be the signal energy after it has been passed through the null-steering operator $\mathbf{P}_S^{\perp}$.

It should be pointed out that a natural extension of the compressive detection approach developed in [8] to address the problem of signal detection in presence of interference and additive white Gaussian noise can be obtained by observing that the received signal can be passed through an interference rejecting filter [1] and then projected using an $M \times N$ random projection matrix with entries obeying a normal distribution. Following this line of thought, the detection problem (14) reduces to:

$$\begin{aligned}\mathcal{H}_0 &: \quad \tilde{\mathbf{y}} = \tilde{\Phi}\mathbf{P}_S^{\perp}(\mathbf{S}\varphi + \mathbf{w}), \\ \mathcal{H}_1 &: \quad \tilde{\mathbf{y}} = \tilde{\Phi}\mathbf{P}_S^{\perp}(\mathbf{H}\Theta + \mathbf{S}\varphi + \mathbf{w})\end{aligned} \tag{19}$$

where $\mathbf{P}_S^{\perp} = \mathbf{I}_N - \mathbf{S}(\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T$ and $\tilde{\Phi}$ is an $M \times N$ i. i. d. matrix with $M > K$. Note that unlike to the subspace compression detection where the *a priori* knowledge is exploited in the design of the measurement matrix, in this latter CS based detection approach, the *a priori* knowledge about the interference subspace is exploited in the null steering operator $\mathbf{P}_S^{\perp}$ while the CS universality is kept in the random projection.

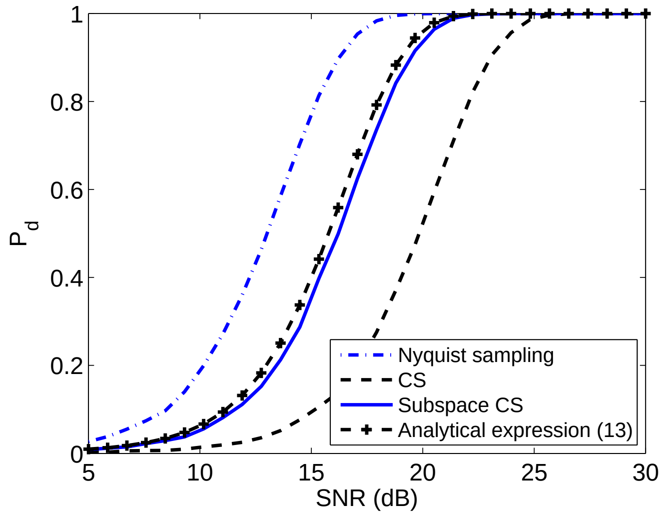

Figure 1: Performance of the various signal detection algorithms for detecting a known signal in additive white Gaussian noise

Following a similar approach to that used in deriving equations (17) - (18), it can be shown that the sufficient statistics and the detection performance for this case are given, respectively, by:

$$\tilde{t}(\tilde{\mathbf{y}}) = \tilde{\mathbf{y}}^T(\tilde{\Phi}\mathbf{P}_S^{\perp}\tilde{\Phi}^T)^{-1}\tilde{\Phi}\mathbf{P}_S^{\perp}\mathbf{H}\Theta \tag{20}$$

$$P_d = Q\left[Q^{-1}(P_{fa}) - \sqrt{\frac{\Theta^T\mathbf{H}^T\mathbf{P}_S^{\perp}\tilde{\Phi}^T(\tilde{\Phi}\mathbf{P}_S^{\perp}\tilde{\Phi}^T)^{-1}\tilde{\Phi}\mathbf{P}_S^{\perp}\mathbf{H}\Theta}{\sigma^2}}\right]$$

5. SIMULATIONS

In this section, the performance of the proposed subspace compressive detectors are evaluated through several simulations and compared to the performances yielded by the conventional subspace detectors (Nyquist sampling) and to the corresponding extension of the CS detector [8]. For all the simulations, the sparse signal $\mathbf{s}$ is given by: $\mathbf{s} = \mathbf{H}\Theta$, where $\mathbf{H} \in \mathcal{R}^{N \times K}$, $\Theta \in \mathcal{R}^{K \times 1}$. The narrowband interference $\mathbf{z}$ is given by $\mathbf{z} = \mathbf{S}\varphi$, where $\mathbf{S} \in \mathcal{R}^{N \times T}$, $\varphi \in \mathcal{R}^{T \times 1}$. All the entries of $\mathbf{H}$, Θ, $\mathbf{S}$ and φ are drawn from an i.i.d. normal distribution. The *SNR* is defined as: $SNR = \frac{\mathbf{s}^T\mathbf{s}}{\sigma^2}$.

First, the various detectors are tested in the detection of a known signal in additive white Gaussian noise. The signal subspace matrix $\mathbf{H}$ has dimensions $N = 2000$ and $K = 200$. The number of measurements for the CS detector [8] is 600, whereas for the proposed approach it is 120. Figure 1 depicts the detection probabilities as a function of the SNR for a $P_{fa} = 10^{-4}$. As can be seen, with much fewer measurements subspace compressive detector outperforms the CS detector that does not use signal subspace information.

In Fig. 1, the approximation to the subspace detection probability given by Eq. (13) is also shown. As can be seen, the approximation is quite close to the empirical results, hence validating the assumption made to obtain this approximation. Furthermore, note that the subspace compressive detector yields a detection performance relatively close to that of conventional detector with just 6% of the Nyquist sampling rate. It should be pointed out that all the closed-form

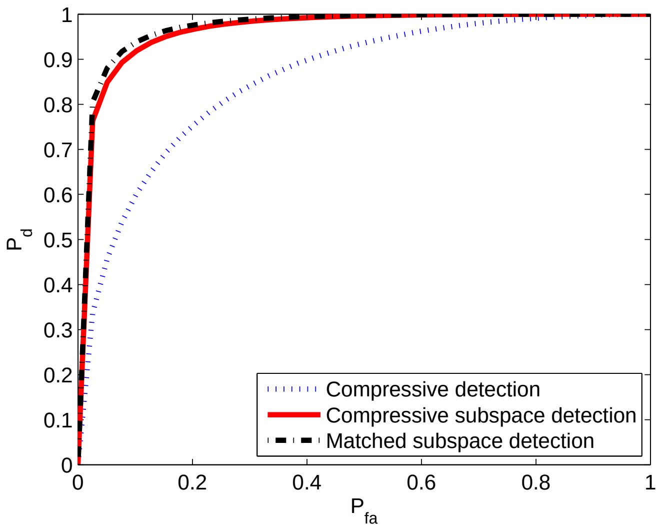

Figure 2: Performance of the proposed approach for detection of known signal embedded in additive white Gaussian noise and interference.

expressions found to characterize the performances of the CS based detectors match very well the empirical results. They are not shown in the plots to avoid overloading the graphs.

In the next simulation, the performance of the proposed subspace compressive detector under narrowband interference is evaluated. The subspace matrix $\mathbf{H}$ has dimensionality $N = 2000$ and $K = 200$. The dimension of the interference subspace is 2000×20. $\mathbf{H}$ and $\mathbf{S}$ are assumed to be known. The signal-to-noise ratio is 9 dB. The number of measurements for the subspace compressive detector is 180. The resulting ROC curve is compared to that yielded by the compressive detector where an interference rejecting filter is followed by an universal random projection. The number of i.i.d. random measurements for this latter detection approach is 600. The simulation results are shown in Fig. 2. As can be seen, by jointly exploiting signal subspace information and interference rejection in the projection stage, the compressive matched subspace detector outperforms the results found by projecting the received signal into the interference null subspace and then applying compressive detector [8]. For comparison purpose, the performance of the conventional matched subspace detector [1] is also shown in Fig. 2. As can be seen, the proposed approach has a detection performance quite similar to the one yielded by the conventional matched subspace detection but using just 9% of the original sampling rate.

6. CONCLUSIONS.

In this paper, we derived new matched subspace detectors under the framework of compressive sensing. We show that with just a few random projections the compressive matched subspace detector yields competitive performance compared with the conventional Nyquist rate detector but without demanding high ADC resources. The proposed approach leads to a new class of subspace compressive sensing that includes LRT detector for interference-free and interference environments. Extensive simulations and analytical expressions show that by exploiting the *a priori* information about the signal subspace in the projection stages, the proposed approach outperforms universal sampling CS detectors. Furthermore, the proposed approach can be generalized to the detection of an unknown signal embedded in additive white Gaussian noise with or without interference. These are report in [9].

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