The Importance of Blocking and Multiplicity/Diversity

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This is digital era. Thanks to Shannon who invented the systematic theory of digital communications in the 1940s and made it possible.

Recalling the Shannon digital communication theory, one important step is blocking, i.e., a sequence of data is blocked to process (encoding and decoding) and most of the theory becomes profound when the block length goes large.

One example of blocking is to combat fading in wireless communications. When a block of data sequence is precoded together and decoded jointly at the receiver, it achieves so-called signal space diversity in a single antenna system [1, 2]. This is recently and frequently termed as good for Doppler effects.

Although blocking itself is not time-invariant, a single input and single output (SISO) linear time-invariant system after blocking becomes and is equivalent to a multi-input and multi-output (MIMO) system whose system function matrix is pseudo-circulant [3]. This plays an important role in vector OFDM systems with single transmit antenna [4] that can achieve signal space diversity and/or multipath diversity for channels with Doppler and/or time spreads.

By replacing the pseudo-circulant system function matrix with a general system function matrix, it becomes a general MIMO system, which is still linear but may not be time-invariant anymore. This leads to multiplicity and diversity. In this case, the single antenna signal space diversity may correspond to the spatial diversity, for example, in diagonal space-time coded MIMO systems.

Diversity (or/and multiplicity) helps in many ways. The above mentioned spatial diversity is one of them. Below I will list two more.

We know that the inverse of a finite impulse response (FIR) SISO system is no longer FIR unless it is only a delay. Interestingly, an FIR MIMO system most likely has an FIR inverse, which makes filterbanks much easier to design, at least, in theory [3].

Another interesting example is about co-primeness. Two positive integers are more likely not co-prime, comparing with that two integer matrices are more likely co-prime. For a given fixed value range, there are more available pairwisely co-prime integer matrices than pairwisely co-prime numbers. This helps the reconstruction of a vector of large values from its multiple remainder vectors of much smaller values modulo multiple matrices. This can be done by using multidimensional (MD) Chinese remainder theorem (CRT), MD-CRT, or its robust version, for vectors [5]. It turns out that (robust) MD-CRT may have applications in multiple modulo or folding or self-reset analog-to-digital convertors (ADC) for complex-valued signals [6].

References

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